MANIFEST

(MECHANICS)

QUANTUM NON-LOCALITY IN QUANTUM THEORY, QUANTUM FIELD THEORY & QUANTUM GRAVITY

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(FLM) "FROM YANG-MILLS TO QUANTUM GRAVITY"

1706.03305 & 1707.00312

⊕ (1307.7080; 1405.3949; 1502.08005; 1606.01829)

⊕ METAPARTICLES (FLM ⊕ JERZY KOWALSKI-GLIKMAN) ETC.
(WROCLAW)

FLM

THE ESSENCE OF THIS TALK:

(RELATIVE LOCALITY; BORN GEOMETRY)

1) QUANTUM THEORY (MECHANICS) IN A MANIFESTLY NON-LOCAL FORMULATION (QUANTUM)

VIA MODULAR VARIABLES & MODULAR (QUANTUM) SPACE-TIME GEOMETRY

(SCHWINGER; AHARONOV)

$\downarrow$

- PURELY QUANTUM MEASUREMENTS
- NEW CORRELATIONS / PHASES

CAUSALITY (BORN) (QUANTUM GEOMETRY)

"$\underline{SO}(D-1,1) = Sp(2D) \cap \underline{SO}(D,D) \cap \underline{SO}(2,2(D-1))$"

(QUANTUM) NON-LOCALITY

2) QUANTUM FIELD THEORY IN A MANIFESTLY NON-LOCAL FORMULATION

$\phi(x) \to \phi(x,\tilde{x})$ & $[x,\tilde{x}] = 2\pi i \lambda^2$

$\left.\begin{bmatrix} \omega \to Sp(2D) \\ \eta \to O(D,D) \\ H \to O(2,2(D-1)) \end{bmatrix}\right\}$ BORN GEOMETRY

- NON-LOCAL QUANTA : "METAPARTICLES"
- NON-COMMUTATIVE STRUCTURE HIDDEN IN QFT

(NON-COMMUTATIVE GEOMETRY OF STANDARD MODEL) ← (CONNES) ... ⇝ (D.M TAKEUCHI AYDEMIR SUN) (EXACT)

? • (ORIGIN OF DUALITIES BTW QFTs ; ORIGIN OF S-AMPLITUDES)

3) QUANTUM GRAVITY = "GRAVITIZING THE QUANTUM"

("STRING THEORY") METASTRINGS

(DYNAMICAL QUANTUM GEOMETRY)

$\omega_{AB}(\mathbb{X}),\ \eta_{AB}(\mathbb{X}),\ H_{AB}(\mathbb{X})$

$\omega_{AB}(\mathbb{X}) = -\omega_{BA}(\mathbb{X}),\quad \eta_{AB}(\mathbb{X}) = \eta_{BA}(\mathbb{X}),\quad H_{AB}(\mathbb{X}) = H_{BA}(\mathbb{X})$

(DEEP)

ANALOGY WITH RELATIVISTIC SPACE-TIME PHYSICS
(EINSTEIN)

1) SPECIAL RELATIVITY: MANIFEST COVARIANCE (POINCARÉ; LORENTZ;)
(ABSOLUTE CAUSALITY) FLAT SPACE-TIME GEOMETRY (MINKOWSKI)
RELATIVITY OF SPACE & TIME

2) CLASSICAL RELATIVISTIC FIELD THEORY: MANIFEST COVARIANCE
(LOCALITY ⊕ CAUSALITY) (REPS. OF LORENTZ GROUP)

3) GENERAL RELATIVITY: "GRAVITIZE SPECIAL RELATIVITY"
("LOCAL LORENTZ") (DYNAMICAL SPACE-TIME GEOMETRY)
BACKGROUND INDEPENDENCE (DYNAMICAL CAUSALITY)

IN WHAT FOLLOWS WE REVEAL THE QUANTUM GEOMETRY
BEHIND QUANTUM-NON-LOCALITY USING
(WEYL) SCHWINGER-AHARONOV'S MODULAR VARIABLES
AND THEN WE MAKE THIS QUANTUM GEOMETRY DYNAMICAL
(ABSOLUTE LOCALITY $\longrightarrow$ RELATIVE LOCALITY)
LEADING TO QUANTUM GRAVITY ("STRING THEORY")
METASTRINGS
GENERIC PREDICTION: METAPARTICLES

MODULAR VARIABLES & QUANTUM NON-LOCALITY

WEYL; SCHWINGER
(AHARONOV ET AL
FREIDEL, LEIGH, D.M.)

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INTERFERENCE WITHOUT ψ!

TWO-SLIT EXP

SCHRÖDINGER: $\psi(x) + e^{i\theta}\psi(x+R) \Rightarrow$ ESSENTIAL QUANTUM NON-LOCALITY

CLASSICAL(!) LABELS

AHARONOV'S QUESTION: WHAT QUANTUM OBSERVABLES CAPTURE THIS INTERFERENCE?

($[q,p] = i\hbar \Rightarrow$ NO POLYNOMIAL $P(\hat{q},\hat{p})$ WORKS!)

NEED SOMETHING THAT SHIFTS $x \to x+R \Rightarrow V(\hat{p}) = e^{\frac{i}{\hbar}\hat{p}R}$

$\Downarrow$

TRANSLATION (SHIFT) OPERATOR

(ALSO, $U(\hat{q}) = e^{i2\pi\hat{q}/R}$!)

CHECK: $[q,p] = i\hbar \Rightarrow UV = VU \Rightarrow [U,V] = 0$!!

COMMUTING QUANTUM OBSERVABLES (NO $\hbar \to 0$ LIMIT)

(NOTE: NO CLASSICAL ANALOG OF U & V !!)

AHARONOV'S INSIGHT: (U, V) OR MODULAR VARIABLES

$$(p)_R \equiv \hat{p} \bmod \left(\frac{2\pi\hbar}{R}\right) \; \& \; (q)_R \equiv \hat{q} \bmod (R)$$

CAPTURE THE ESSENCE OF QUANTUM THEORY: NON-LOCALITY THAT IS CONSISTENT WITH CAUSALITY !!

(SO "LOCAL" QFT IS POSSIBLE :))

MODULAR VARIABLES $(p)_R$, $(q)_R$ $\left([(q)_R, (p)_R] = 0 ! \right)$

APPEAR IN THE <u>KINEMATIC</u> NON-LOCALITY OF THE EPR-BELL TYPE

USE q & p (INSTEAD OF SPIN), AS IN THE **ORIGINAL** EPR PAPER,

$\Rightarrow$ $(p)_R$, $(q)_R$ NATURALLY APPEAR (ALSO GHZ)

(SEE ARXIV: QUANT-ph/0103048

OR ARXIV: 1212.5340)

BUT $(q)_R$ & $(p)_R$ CAPTURE DYNAMIC NON-LOCALITY:

LET $\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{q})$

"NON-LOCALITY IN YOUR FACE!"

$$\frac{d(p)_R}{dt} = -\left(\frac{V(q + \frac{R}{2}) - V(q - \frac{R}{2})}{R} \right) \rightarrow \text{BORN-HEISENBERG-JORDAN } \underline{\text{EOM}}$$

(ESSENTIALLY <u>Q. EOM.</u> FOR $e^{\frac{i}{\hbar}\hat{p}R}$!)

$$\Downarrow$$

WHERE $$\frac{d(p)_R}{dt} \equiv \frac{\hbar}{iR} e^{-\frac{i}{2\hbar}\hat{p}R} \left(\frac{d e^{\frac{i}{\hbar}\hat{p}R}}{dt} \right) e^{-\frac{i}{2\hbar}\hat{p}R}$$

EXAMPLES OF MODULAR VARIABLES:

1) AHARONOV-BOHM PHASE (CHARGED PROBE IN $\vec{B}$ FIELD)

2) AHARONOV-CASHER PHASE (SPIN PROBE IN $\vec{E}$ FIELD)

"FINAL REMARKS: MODULAR VARIABLES ARE COMMUTING

(5)

THUS WE NEED TO LOOK AT COMMUTATIVE SUBALGEBRAS OF "$[\hat{q},\hat{p}] = i\hbar$" BORN-HEISENBERG-JORDAN

COMMUTATIVITY $\Rightarrow$ LATTICE IN PHASE SPACE

MODULAR SPACE

UNCERTAINTY PRINCIPLE: $\Leftarrow$ "BRILLOUIN" CELL

$\to$ CAN SPECIFY A POINT IN MODULAR CELL,
BUT IF SO, CAN'T SAY WHICH CELL YOU ARE IN!!

FROM MODULAR POINT OF VIEW, THE SCHRÖDINGER PICTURE IS SINGULAR.

HOWEVER, ONE CAN UNITARILY MAP FROM MODULAR TO SCHRÖDINGER.

$\to$ THE MAP IS KNOWN IN SOLID STATE PHYSICS: ZAK TRANSFORM!

(USE $x = \frac{\hat{z}}{\lambda}$, $\tilde{x} = \frac{\hat{p}}{\epsilon}$, $[x,\tilde{x}] = \frac{1}{2\pi}$ $(\hbar = 2\pi\lambda\epsilon)$)

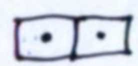
MODULAR FUNCTIONS: $\Phi(x+n, \tilde{x}+\tilde{n}) = e^{2i\pi n\tilde{x}}\,\Phi(x,\tilde{x})$

MODULAR SCHRÖDINGER

ZAK : $\boxed{(Z_\lambda\psi)(x,\tilde{x}) \equiv \sqrt{\lambda}\sum_{n\in\mathbb{Z}} e^{-2\pi i n\tilde{x}}\,\psi(\lambda(x+n))}$

$\downarrow$ INVERSE ZAK: $\left[(Z_\lambda^{-1}\Phi)(x+n) = \frac{1}{\sqrt{\lambda}}\int_0^1 d\tilde{x}\, e^{2\pi i n\tilde{x}}\,\Phi(\lambda^{-1}x,\tilde{x})\right]$, $x\in[0,1)$

$\left(\hat{\tilde{x}} \to -\frac{i}{2\pi}\partial_x\,,\ \hat{x} \to \left(\frac{i}{2\pi}\partial_{\tilde{x}} + x\right)\right) \Rightarrow$ UNIT FLUX THROUGH MODULAR CELL!



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QUANTUM THEORY IN TERMS OF MODULAR VARIABLES:

$$\hat{q} \to \underline{\hat{U}} = \exp\left(\frac{2\pi i}{\hbar}\hat{q}\hbar R^{-1}\right) \equiv \underline{\exp\left(2\pi i \frac{\hat{q}}{R}\right)}$$

$$\hat{p} \to \underline{\hat{V}} = \underline{\exp\left(\frac{i}{\hbar}\hat{p}R\right)} \quad \text{"VERTEX OPERATORS"}$$

$$\left(\begin{array}{c} [\hat{q}, \hat{p}] = i\hbar \\ \Downarrow \end{array} \right)$$

$$[\hat{U}, \hat{V}] = 0$$

USE $\hat{\mathbb{X}}^A = \begin{pmatrix} \hat{x}^a \\ \tilde{x}_a \end{pmatrix}$ $\quad \hat{x}^a \equiv \frac{\hat{q}^a}{\lambda}, \quad \hat{\tilde{x}}_a \equiv \frac{\hat{p}_a}{\epsilon} \quad (\hbar = \lambda\epsilon)$

$$\Downarrow$$

$$\underline{W_{\mathbb{K}} \equiv \exp(2\pi i\, \omega(\hat{\mathbb{K}}, \hat{\mathbb{X}}))} \; ; \; \underbrace{\omega(\mathbb{X}, \mathbb{Y}) = \hat{x}\cdot y - k\cdot\tilde{y}}_{Sp(2D)}$$

→ COMMUTATIVE SUBALGEBRA OF HEISENBERG ALGEBRA:

MODULAR SPACETIME $\Leftarrow$ SELF-DUAL LATTICE Λ WRT $\underline{\underline{\omega}}$! (NARAIN LATTICE)

$$\Lambda = \ell \oplus \bar{\ell} \to \underbrace{\eta(\mathbb{P}, \mathbb{Q}) = \hat{p}\cdot q + p\cdot\hat{q}}_{O(D,D)} \; ; \; \boxed{U_\lambda = e^{\frac{i}{2}\pi\eta(\lambda,\lambda)} W_\lambda}$$

FINALLY, VACUUM $\mathbb{P}^A|0\rangle = 0 \Rightarrow \underbrace{\hat{E}_H = H^{AB}\hat{\mathbb{P}}_A\hat{\mathbb{P}}_B}$, $\underline{\hat{E}_H|0\rangle = 0}$

(TRANSLATION GENERATOR, BUT TRANSLATIONS BROKEN)

$\underline{H^{AB} = H^{BA}}$ $\quad \underline{O(2, 2(D-1))}$ $\quad$ ($\underline{\text{RELATIVE}}$ $\underline{\mathbb{P}}$ ($\underline{\text{OBSERVER DEPENDENT}}$) $\underline{\text{LOCALITY}}$

NOTE: $\underline{\text{LORENTZ!}}$ $\quad \boxed{O(D-1,1) = Sp(2D) \cap O(D,D) \cap O(2,2(D-1))}$

$$\underbrace{\omega_{AB} \quad \eta_{AB} \quad H_{AB}}_{\text{BORN GEOMETRY}}$$

FLM

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SUMMARY:

(SCHWINGER; AHARONOV)

MODULAR VARIABLES: FROM QUANTUM THEORY TO STRING THEORY (QUANTUM GRAVITY)

I) QUANTUM THEORY (MECHANICS): NON-LOCALITY ⊕ CAUSALITY

(AHARONOV)

INSTEAD OF $\hat{q}, \hat{p}$ WITH $[\hat{q}, \hat{p}] = i\hbar$ USE

$$[\hat{q}]_R \equiv \hat{q} \bmod (R)$$
$$[\hat{p}]_R \equiv \hat{p} \bmod \left(\frac{2\pi\hbar}{R}\right)$$

(NON-LOCAL MODULAR VARIABLES $[\hat{q}]_R$, $[\hat{p}]_R$)

WITH $[[\hat{q}]_R, [\hat{p}]_R] = 0$!

$\Downarrow$ MODULAR SPACETIME

FULLY COVARIANT!

$\Downarrow$

II) (LOCAL) QUANTUM FIELD THEORY POSSIBLE BECAUSE OF CONSISTENCY BTW QUANTUM NON-LOCALITY & CAUSALITY

(STILL NEW INSIGHTS AND PHYSICS! PROVIDED BY MODULAR VARIABLES)

• METAPARTICLES!

("NON-LOCALITY" ... "WEAK MEASUREMENTS" ETC)

$\Uparrow$ $\Downarrow$

III) (QUANTUM GRAVITY) STRING THEORY IN METASTRING FORMULATION

- LIVES IN MODULAR SPACE TIME
- "GRAVITIZES" THE GEOMETRY OF QUANTUM THEORY IN MODULAR FORMULATION

$$\underbrace{(\omega_{AB}(X),\ \eta_{AB}(X),\ H_{AB}(X))}_{\text{FLM}}$$

- R is λ!

CONTEXTUAL — NOT-CONTEXTUAL IN QUANTUM GRAVITY

(B)

(T-DUALITY) (METASTRING) FLM

COVARIANT FORMULATION (METASTRING) $\mathbb{X}^A = \begin{pmatrix} x^a/\lambda \\ \tilde{x}_a/\epsilon \end{pmatrix}$

"PHASE SPACE"

$$S_{MS} = \frac{1}{4\pi}\int \left[\partial_\tau \mathbb{X}^A (\eta_{AB} + \omega_{AB})(\mathbb{X})\, \partial_\sigma \mathbb{X}^B - \partial_\sigma \mathbb{X}^A H_{AB}(\mathbb{X})\, \partial_\sigma \mathbb{X}^B\right]$$

$\left(\begin{array}{l} \hbar = \lambda\epsilon \\ \alpha' = \frac{\lambda}{\epsilon} \end{array}\right)$

$\begin{pmatrix} 0 & \delta_a{}^b \\ \delta_a{}^b & 0 \end{pmatrix} \leftarrow O(D,D)$ $\quad Sp(2D) \hookrightarrow \begin{pmatrix} 0 & -\delta_a{}^b \\ \delta_a{}^b & 0 \end{pmatrix}$ $\quad O(2, 2(D-1)) \left[\begin{pmatrix} G_{ab} & 0 \\ 0 & G_{ab}^{-1} \end{pmatrix} \right]$

THEN IMMEDIATELY:

$$[\hat{\mathbb{X}}^A, \hat{\mathbb{X}}^B] = 2i\,[\pi\,\omega^{AB} - \eta^{AB}\, \theta(\sigma,\sigma')]$$

"STAIRCASE DISTRIBUTION"

NOTE $J \equiv \eta^{-1} H$, $J^2 = 1$

$\left(\mathbb{X}^A(\sigma + 2\pi) = \mathbb{X}^A(\sigma) + \Delta^A\right)$

GENERAL MONODROMIES

T-DUALITY $\mathbb{X} \rightarrow J(\mathbb{X})$

CHIRAL FORMULATION $\longleftrightarrow$ "FULLY COMPACTIFIED" $\leftarrow$ MONSTER, BORCHERDS.

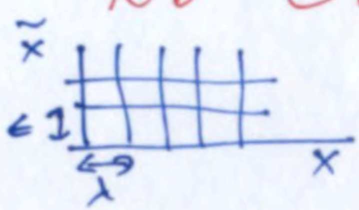
(AND NON-LOCAL)

D = 26 BOSONIC STRING $\leftrightarrow$ (UNIQUE SELF-DUAL LORENTZIAN LATTICE)

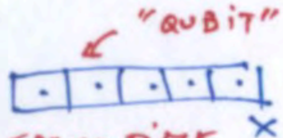
No CTCs $\rightarrow$ "MODULAR SPACETIME" (QUANTUM)

$\Downarrow$

MODULAR VARIABLES OF QUANTUM THEORY



$\Rightarrow$



SPACE TIME "EXTENSIFICATION"

EFFECTIVE PICTURE OF STRINGS:

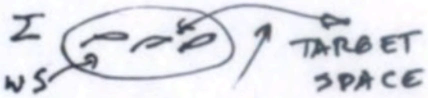
"TARGET SPACE = INDEX SPACE"

• OLD PICTURE:

$\lambda \; \bigcirc \xrightarrow{L \gg \lambda} \bullet$ "PARTICLES"

$\Phi(x)$ – LOCAL FIELD

INDEX SPACE



$X(\sigma,\tau)$

(EFFECTIVE FIELD THEORY / WILSONIAN RG

DERIVATIVE EXPANSION

DECOUPLING OF IR & UV)

...

PARTICLE QUANTA

• NEW PICTURE:

$\lambda \; \bigcirc \xrightarrow[\text{FLM}]{L \gg \lambda}$ "METAPARTICLES"

$[x, \tilde{x}] = i\lambda^2 (2\pi)$

$\Phi(x, \tilde{x})$ – NON-COMMUTATIVE FIELD THEORY

$(\hbar = \lambda\epsilon$

$\alpha' = \frac{\lambda}{\epsilon})$

(VIOLATION OF DECOUPLING

UV/IR MIXING

DOUBLE SCALE RG ; SELF-DUAL FIXED POINT)

"TARGET SPACE $\neq$ INDEX SPACE"

CONSIDER

FLAT (FREE) STRING: $S_P \sim \int {}^*dx^\mu \wedge dx^\nu \, h_{\mu\nu}$

$\delta S_P = 0 \rightarrow \Box X = 0 \rightarrow$ 2 SOLUTIONS

$$\boxed{X(\tau,\sigma) = X_R(\tau+\sigma) + X_L(\tau-\sigma)} \qquad \lambda \equiv \sqrt{\frac{\pi\alpha'}{2}}$$

$$X_L(\tau-\sigma) \equiv x_L + \frac{\alpha'}{2} p_L(\tau-\sigma) + i\lambda \sum_{n=-\infty}^{+\infty} \frac{1}{n} \alpha_n e^{-in(\tau-\sigma)}$$

$$X_R(\tau+\sigma) \equiv x_R + \frac{\alpha'}{2} p_R(\tau+\sigma) + i\lambda \sum_{n=-\infty}^{+\infty} \frac{1}{n} \tilde{\alpha}_n e^{-in(\tau+\sigma)}$$

ALSO:

$$\boxed{\tilde{X}(\tau,\sigma) = X_R(\tau+\sigma) - X_L(\tau-\sigma)}$$

NOTE: $\underline{dX = {}^*d\tilde{X}}$, i.e. $\underline{\partial_\tau X = \partial_\sigma \tilde{X};\ \partial_\sigma X = \partial_\tau \tilde{X}}$

STRINGS ON S^1:

$\tilde{X} \rightarrow$ T-DUALITY $\qquad \underline{\tilde{X}(\tau, \sigma+2\pi) = \tilde{X}(\tau,\sigma) + 2\pi\alpha' p} \qquad \left(p = \frac{p_L + p_R}{2}\right)$

$(R; \bigcirc \rightarrow \bigcirc; \tilde{R} \quad ; \quad R\tilde{R} = 2\lambda^2) \qquad \left(2\pi\alpha' p = \int_0^{2\pi} d\sigma\, \partial_\tau X\right)$

CIRCLE $\qquad$ "DUAL" CIRCLE

USUALLY $[x, \tilde{x}] = 0$ (TEXTBOOK)

"ZERO MODES x_L, x_R COMMUTE"

$\boxed{\text{NOT TRUE!}}$

4 DIFFERENT CALCULATIONS TO SEE THAT $[x, \tilde{x}] = i\lambda^2_{(2\pi)}$

1) PERHAPS THE SIMPLEST: START FROM CANONICAL COMM.

$$[\hat{X}(\tau, \sigma_1), \partial_\tau \hat{X}(\tau, \sigma_2)] = 2\pi i \hbar \alpha' \delta(\sigma_{12})$$

HOWEVER, $\partial_\tau X = \partial_\sigma \tilde{X}$

$$[\hat{X}(\tau, \sigma_1), \partial_\sigma \hat{\tilde{X}}(\tau, \sigma_2)] = 2\pi i \hbar \alpha' \delta(\sigma_{12})$$

(WORLD-SHEET)

INTEGRATE OVER σ; INVOKE WS CAUSALITY $\leftrightarrow$ (MUTUAL LOCALITY)

FLM $\quad [\hat{X}(\tau, \sigma_1), \hat{\tilde{X}}(\tau, \sigma_2)] = 2\pi i \lambda^2 [\pi - \theta(\sigma_{12})]$ $\quad \leftarrow$ STAIRCASE DISTRIBUTION

ZERO MODES DO NOT COMMUTE $\quad [\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b$

($x \equiv x_L + x_R$; $\tilde{x} \equiv x_L - x_R$) $\quad$ HEISENBERG ALGEBRA

USE THE DOUBLE NOTATION:

$$\mathbb{X}^A(\tau,\sigma) \equiv \left(X^\mu(\tau,\sigma),\ \tilde{X}_\mu(\tau,\sigma) \right)$$

$$\Rightarrow \left[\hat{\mathbb{X}}^A(\tau,\sigma_1),\ \hat{\mathbb{X}}^B(\tau,\sigma_2) \right] = 2i\lambda^2 \left[\pi\omega^{AB} - \eta^{AB}\theta(\sigma_{12}) \right]$$

ZERO MODES

OSCILLATORS

$\omega^{AB} \rightarrow Sp(2D)$

↳ dim OF SPACETIME

$\eta^{AB} \rightarrow O(D,D)$

$\theta(\sigma)$ – STAIRCASE DISTRIBUTION

2) WHERE IS $\tilde{X}$ IN THE POLYAKOV ACTION?

T-DUALITY $\rightarrow$ $S_P \rightarrow S_P + \hbar \int \omega$ $\Rightarrow$ METASTRING S_{MS}

FLM

$$\omega \equiv \frac{1}{8\pi\lambda^2} \left[d\tilde{X}_a \wedge dX^a \right]$$

WZW-LIKE TERM

$\tilde{X}$ – TOPOLOGICAL DEGREE OF FREEDOM

(ONLY ZERO MODES DYNAMICAL)

3) SYMPLECTIC FORM FROM δS_p

$$\left(S_p \equiv \frac{1}{4\pi\alpha'} \int d^2\sigma \, \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2 \right] \right)$$

BE CAREFUL WITH BOUNDARY TERMS

FOR A CYLINDRICAL WS CUT OPEN AT $\sigma = 0$

$$(\sigma \in [0, 2\pi], \quad \tau \in [\tau_0, \tau_1])$$

USE $\partial_\tau X = \partial_\sigma \tilde{X}$ ON THE BOUNDARY

$\delta S_p \rightarrow$ SYMPLECTIC 1-FORM $\Theta(\tau)$

$\delta \Theta = \Omega$ - SYMPLECTIC 2-FORM

FLM

$$\boxed{\Omega = \delta p \wedge \delta x + \delta \tilde{p} \wedge (\delta \tilde{x} - \pi\alpha' \delta p) + \text{OSCILLATOR TERMS}}$$

BERRY-PHASE-LIKE TERM : $-\pi\alpha' \delta\tilde{p} \wedge \delta p$

(METAPARTICLE)

$$\Rightarrow \underline{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}$$

1 CANONICAL COMMUTATORS

4) COCYCLES OF TEXTBOOK VERTEX OPERATORS:

$$\hat{U}_k = e^{ik_L x_L + ik_R x_R}$$ (COMMUTING WITH ZERO MODES!)

$$\hat{U}_k \hat{U}_{k'} = \epsilon_{k,k'} \hat{U}_{k+k'}$$

DEDUCE (FROM MUTUAL LOCALITY) $\epsilon_{k,k'} = e^{2\pi i \lambda^2 \tilde{k} k'}$

REALIZE INTRINSIC NONCOMMUTATIVITY $[x, \tilde{x}] = 2\pi i \lambda^2$

NOW $\hat{U}_k = e^{ik\hat{x}} \, e^{i\tilde{k}\hat{\tilde{x}}}$ $\quad (x = x_L + x_R)$ $\quad (\tilde{x} = x_L - x_R)$

$\downarrow$

REPS OF HEISENBERG-WEYL

COCYCLES $\rightarrow$ 1 ! FLM

(NEW INSIGHT ON ASYMMETRIC ORBIFOLDS & OTHER NON-GEOMETRIC BACKGROUNDS)

TURN ON $\underset{}{\overset{G_{ba}}{\overset{\parallel}{G_{ab}}}}$ & $\overset{-B_{ba}}{\overset{\parallel}{B_{ab}}}$: FLM

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$$S_{P(G,B)} = \frac{1}{4\pi\alpha'}\int d^2\sigma \left(G_{ab}\left[\partial_\tau X^a \partial_\tau X^b - \partial_\sigma X^a \partial_\sigma X^b\right] + 2B_{ab}\,\partial_\tau X^a \partial_\sigma X^b\right)$$

$$\Rightarrow \quad \boxed{[\hat{x}^a, \hat{x}^b] = 0}\,, \quad \boxed{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a{}_b}\,, \quad \boxed{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = -4\pi i \lambda^2 B_{ab}}$$

INTRODUCE

$$\underset{O(D,D)}{\eta_{AB} = \begin{pmatrix} 0 & \delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix}} \qquad \underset{Sp(2D)}{\omega_{AB} = \begin{pmatrix} 0 & -\delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix}} \qquad \boxed{\begin{matrix} K = \eta^{-1}\omega \\ K^2 = 1 \end{matrix}}$$

IF $B_{ab} = 0$ $\quad \boxed{\Omega = \eta_{AB}\,\delta\mathbb{P}^A \wedge \delta\mathbb{X}^B + \frac{\pi\alpha'}{2}\,\omega_{AB}\,\delta\mathbb{P}^A \wedge \delta\mathbb{P}^B}$

IF $B_{ab} \neq 0$ $\quad \eta^{(B)}_{AB} = \eta_{AB} = \begin{pmatrix} 0 & \delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix}, \quad \omega^{(B)}_{AB} = \begin{pmatrix} -2B_{ab} & -\delta_a{}^b \\ \delta_a{}^b & 0 \end{pmatrix}$

$$\boxed{\Omega = \eta_{AB}\,\delta\mathbb{P}^A \wedge \delta\mathbb{X}^B + \frac{\pi\alpha'}{2}\,\omega^{(B)}_{AB}\,\partial\mathbb{P}^A \wedge \delta\mathbb{P}^B}$$

THEN $\boxed{[\mathbb{X}^A, \mathbb{X}^B] = 2\pi i \lambda^2 \,(\omega^{(B)})^{AB}}$, $\quad (\omega^{(B)})^{AB} = \begin{pmatrix} 0 & \delta^a{}_b \\ -\delta_a{}^b & \underbrace{-2B_{ab}}_{} \end{pmatrix}$

!!

B_{ab} is in ω_{AB}

NOTE: $\mathbb{B}$-FIELD TRANSFORMATION

$$\mathbb{X} = (x^a, \tilde{x}_a) \overset{B}{\longmapsto} (x^a, \tilde{x}_a + B_{ab} x^b)$$

WHICH LEADS TO $\underbrace{[\hat{x}^a, \hat{x}^b] = 0}$, $\underbrace{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}$, $\underbrace{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = -4\pi i \lambda^2 B_{ab}}$

HOWEVER, WE CAN ALSO CONSIDER

$$(x^a, \tilde{x}_a) \overset{\beta}{\longmapsto} (x^a + \beta^{ab} \tilde{x}_b, \tilde{x}_a)$$

"STRINGY" (NON-LOCALITY)

THIS LEADS TO NON-COMMUTATIVITY OF x^a!

$\underbrace{[\hat{x}^a, \hat{x}^b] = 4\pi i \lambda^2 \beta^{ab}}$, $\underbrace{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}$, $\underbrace{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = 0}$

FINALLY, NOTE THAT FOR THE B-FIELD BACKGROUND

$$\underbrace{[\hat{\tilde{x}}_a, [\hat{\tilde{x}}_b, \hat{\tilde{x}}_c]] + \text{cyclic} = H_{abc}(x)}$$

$H_{abc} = \partial_a B_{bc} + \text{cyclic}$ $\longrightarrow$ H-FLUX $\longleftarrow$ $\boxed{\text{"MONOPOLE"}}$ 3-COCYCLES

POSSIBILITY OF NON-ASSOCIATIVITY.

ALL FLUXES $(H, \underbrace{F, R}_{\text{DOUBLE FIELD THEORY}} \ldots)$ FROM NON-COMMUTATIVITY & NON-ASSOCIATIVITY OF "PHASE SPACE" $\mathbb{X}$

FLM

EFFECTIVE FIELD DESCRIPTION OF CLOSED STRINGS

$$\boxed{\Phi(x,\tilde{x}) \equiv \sum_{w} \phi_w(x)\, e^{iw\tilde{x}/\tilde{\ell}}}$$

$$[x,\tilde{x}] = 2\pi i \lambda^2$$

THUS

$$\Phi(x,\tilde{x}) = \underbrace{\Phi(x)}_{\text{LOCAL EFFECTIVE FIELD}} + \overbrace{\sum_{w\neq 0} \phi_w(x)\, e^{iw\tilde{x}}}^{\text{NON-LOCAL FIELD}}$$

(DOUBLE RG
UV/IR mixing)
SELF-DUAL FIXED POINTS

NON-PARTICLE QUANTA OF $\Phi(x,\tilde{x})$:

FROM NON-COMMUTATIVE CLOSED STRING ZERO MODES

"METAPARTICLES"

FLM
⊕
Kowalski-Glikman

$$S_{MP} = \int \Big[p\dot{x} + \tilde{p}\dot{\tilde{x}} - \underbrace{\alpha' \pi\, p\dot{\tilde{p}}}_{\omega} + N_1 \big(\overbrace{p^2 + \tilde{p}^2 - M_1^2}^{H}\big) + N_2 \big(\underbrace{p\tilde{p}}_{\eta} - M_2^2\big) \Big]$$

(FK-GLM) CONSTRAINTS $(p^2 + \tilde{p}^2 = M_1^2)$ & $(p\tilde{p} = M_2^2)$

FROM METASTRING CONSTRAINTS

$$\begin{bmatrix} H : \partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^B H_{AB} \approx 0 \\ D : \partial_\tau \mathbb{X}^A \partial_\sigma \mathbb{X}^B \eta_{AB} \simeq 0 \end{bmatrix}$$

(USUALLY WE IGNORE $\tilde{x}$!!
"$(\tilde{x} \to 0)$"
OR $(\tilde{p} \to 0)$ & $(M_2 \to 0)$)

METAPARTICLES:

DOUBLED TARGET PHASE SPACE

$$(x^\mu, \; p_\mu, \; \tilde{x}_\mu, \; \tilde{p}^\mu)$$

$\mu = 0, 1, \ldots, d-1$

HAMILTONIAN CONSTRAINT:

$$\mathcal{H} \equiv \frac{1}{2}\left(p^2 + \tilde{p}^2 + m^2\right)$$

"DIFF." CONSTRAINT:

$$\mathcal{D} \equiv p_\mu \tilde{p}^\mu - \mu$$

COME FROM (STRING) FIXED OSCILLATOR LEVELS

HAMILTONIAN:

$$H ds = \mathcal{H} e + \mathcal{D} \tilde{e}$$

$e, \tilde{e}$ – LAGRANGE MULTIPLIERS

S – WORLD-LINE PARAMETER

SYMPLECTIC STRUCTURE:

"BERRY-PHASE"

$$\boxed{\omega = \delta p_\mu \wedge \delta x^\mu + \delta \tilde{p}^\mu \wedge \delta \tilde{x}_\mu + \pi \alpha' \delta p_\mu \wedge \delta \tilde{p}^\mu}$$

NOTE: $\tilde{p} \to 0$ (PARTICLE LIMIT; SINGULAR!)

SYMMETRIES: $O(1, d-1)_+ \times O(1, d-1)_-$ (2 REPARAMETRIZATIONS OF $\pm$ MODES) (REPS)

(L R) MODES

$$x^\mu_\pm = x^\mu \pm \hat{G}^{\mu\nu} \hat{x}_\nu, \quad p^\pm_\mu = \frac{1}{2}\left(p_\mu \pm G_{\mu\nu} \hat{p}^\nu\right), \quad e_\pm = e \pm \hat{e}$$

→ Z_2 (T-DUALITY): $x \leftrightarrow \tilde{x}$, $e \leftrightarrow \tilde{e}$ ⊕ IDF. REPS.

DIFFERENT POLARIZATIONS:

FK-GLM

- $(x, \tilde{p})$ AFTER ELIMINATING $p, \tilde{x}$:

METAPARTICLES LOOK LIKE PARTICLES MOVING IN THE SPACETIME TRANSVERSE TO $\hat{p}^\mu$, WITH EFFECTIVE MASS $m_{cH}^2 = \hat{p}^2 + m^2 + \frac{\mu^2}{\hat{p}^2}$ $\left(\tilde{p}^2 \to \frac{\mu^2}{\hat{p}^2} \right)$

(EXTENDED, "DIPOLE-LIKE" EXCITATIONS, AS IN NCFT)

- $(x, \hat{x})$ DOUBLE CONFIGURATION SPACE (BUT $\{x, \hat{x}\} \neq 0$)

THE CLASSICAL MOTION FOLLOWS A CURVE IN $t, \hat{t}$ PLANE, WITH NORMAL DIRECTION NON-DYNAMICAL

(ONE TIME; NO TWO TIMES)
NO CTC ETC.

- MIXING OF SCALES:

$$p^2 + \hat{p}^2 = m^2$$
$$p \hat{p} = \mu$$

EXAMPLE:

$$\begin{cases} p \sim 10^{19}\,GeV \\ \hat{p} \sim 10^{-3}\,eV \end{cases} \Rightarrow \mu \sim 1\,TeV$$
$$m \sim 10^{19}\,GeV$$

HIERARCHY

(UV/IR MIXING
↳ DOUBLE RG ; AS IN NCFT
(TWO SCALES) (NON-COMMUTATIVE FIELD THEORY))

- CLASSICAL PHYSICS DOES NOT SEE NON-LOCALITY ($\alpha' p\hat{p}$)

QUANTUM METAPARTICLE:

PROPAGATOR: $(x, \hat{p})$ $\quad K(x_f, \hat{p}_f; s_f; x_i, \hat{p}_i, s_i) = \langle x_f, \hat{p}_f | e^{-i\hat{H}(s_f - s_i)} | x_i, \hat{p}_i \rangle$

SIMPLEST ANSWER IN $p, \hat{p}$ POLARIZATION:

$$\underline{K(p_f, \hat{p}_f, s_f; p_i, \hat{p}_i, s_i)} \sim \underbrace{\delta^{(D)}(p_f - p_i)\, \delta^{D}(\hat{p}_f - \hat{p}_i)\, \frac{\delta(p\cdot\hat{p} - \mu)}{p^2 + \hat{p}^2 + \omega^2 - i\epsilon}}$$

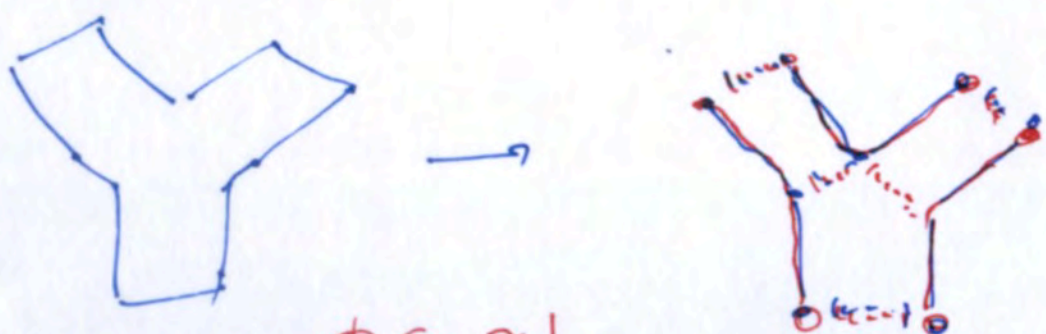
PROPAGATOR IN ANY OTHER POLARIZATION (ZAK TRANSFORM)

$$\underline{K(x_f, \hat{p}_f, s_f; x_i, \hat{p}_i, s)} \sim \underbrace{\delta^{D}(\hat{p}_f - \hat{p}_i) \int d^D p \, \frac{\delta(p\cdot\hat{p} - \mu)}{p^2 + \hat{p}^2 + \omega^2 - i\epsilon}\, e^{ip(x_f - x_i)}}$$

INTERACTIONS: (CONSERVATION OF MOMENTA)

THINK OF METAPARTICLE AS A "PARTICLE" REMNANT OF THE "OPEN" STRING/METASTRING

NAIVELY:



$\underline{\Phi(x, \tilde{x})}$

$\hookrightarrow$ MODULAR FIELD $\to$ METAPARTICLE QUANTA

STRING

INTERACTIONS :

INCORPORATE

$[x, \tilde{x}] = 2\pi i \lambda^2$

FLM

$$\Phi_w(x) = \Phi^{(+)}(x, x+\pi w R)\, \Phi^{(-)}(x+\pi w R, x+2\pi w R)$$

CLOSED STRING ~ PRODUCT OF TWO OPEN STRINGS

$$(\Phi \circ \psi)^{(\pm)}_{w'}(x, x+\pi w R) \equiv \Phi^{(\pm)}(x, x+\pi w' R)\, \psi^{(\pm)}(x+\pi w' R, x+\pi w R)$$

REALIZATION OF THE HEISENBERG GROUP $[x, \tilde{x}] = 2\pi i \lambda^2$

?! NON-PERTURBATIVE FORMULATION $\partial_\sigma \hat{\mathbb{X}}^A \to \frac{1}{\lambda}[\hat{\mathbb{X}}^{26}, \hat{\mathbb{X}}^A]$

INVOLVING $\hat{\mathbb{X}}^A(\sigma, \tau) \to \hat{\mathbb{X}}^a(\tau)$ $A = 0, 1 \dots 25 \longrightarrow a = 0, 1, \dots, 25, 26$

$$S_{NP} \sim \frac{1}{\lambda} \int \mathrm{Tr} \left(\partial_\tau \hat{\mathbb{X}}^a [\hat{\mathbb{X}}^b, \hat{\mathbb{X}}^c]\, \eta_{abc}^{(*)} - \frac{1}{\lambda} H_{ac}^{(*)} [\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b][\hat{\mathbb{X}}^c, \hat{\mathbb{X}}^d] H_{bd}^{(*)} \right)$$

"27 = 11+16" ↳ SUSY, EMERGENT

↓

M-THEORY

11D – M-THEORY

COVARIANT MATRIX THEORY

$(\eta_{abc}^{(*)} \to \omega_{AB}^{(*)} + \eta_{AB}^{(*)})$

APPLICATIONS:

QUANTUM GRAVITY

1) GENERIC "LOW ENERGY" PREDICTION OF (STRING TH.)

$\Phi(x, \hat{x})$ $[x, \hat{x}] = 2\pi i \lambda^2$

Λ_{cc} SENSITIVE

$\rightsquigarrow$ (NON-PARTICLE DARK MATTER WITH MILGROM'S SCALING)

Λ_{cc} - COSMOLOGICAL CONSTANT

D.M. WITH FRIENDS (EDMONDS, MG, TAKEUCHI, FARRAH, HO)

2) NON-DECOUPLING ("MIXING") BTW UV & IR

HIERARCHY PROBLEM; NATURALNESS

3) VACUUM ENERGY λ_{cc} & T-DUALITY

(TSEYTLIN $\rightsquigarrow$ FLM) (RELATED TO SEQUESTER OF KALOPER, PADILLA?)

$\int \sqrt{g\hat{g}}\,(R(g) + \hat{R}(\hat{g})) + \ldots \to$ DFT

⋮

(BH INFORMATION PUZZLE)

⋮

METAPARTICLES, RELATIVISTIC & NON-RELATIVISTIC

GENERIC PREDICTION OF MODULAR QUANTIZATION