Unruh effect without spacetime

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BH thermodynamics

- **Bekenstein*** suggested that BHs have entropy

$$S \sim \frac{A}{l_{Pl}^2}$$

- Spectacularly confirmed by Hawking** BH radiate at temperature

$$T_H = \frac{1}{8\pi GM}$$

* Phys. Rev. D7, 2333 (1973); ** Nature 248, 30 (1974):

Puzzles of BH thermodynamics

- To date at least two major issues remain puzzling:
  - The enigmatic nature of degrees of freedom that BH entropy is counting;
  - The fate of unitarity in BH quantum evaporation: do BHs evolve pure states into mixed states?

Puzzles of BH thermodynamics

- To date at least two major issues remain puzzling:
  - The enigmatic nature of the degrees of freedom that BH entropy is counting;
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Crucial to the BH thermodynamics puzzles is the quantum temperature perceived by accelerated observer, but not by inertial one.

Horizon temperature

- Unruh* motivated by Hawking discovery that black holes radiate thermally at T_H, associates temperature to the Rindler horizon

$$T_U = \frac{a}{2\pi}$$

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Horizon temperature

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- **What is the universal, basic structure behind Unruh temperature?**

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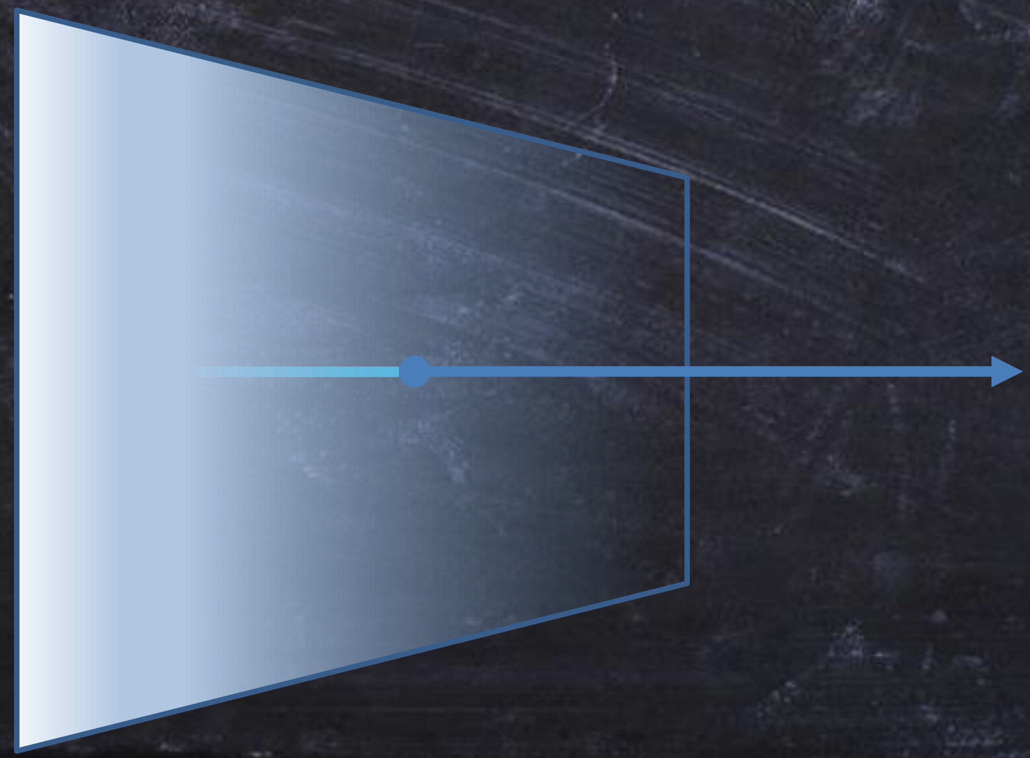
- In this talk I will describe **connection between the geometric notion of a null boundary and thermal quantum states** in the setup stripped down to the bone.
- Minimal setting: only **group theoretic ingredients** associated with symmetries of space-time
    - No space-time
    - No metric
    - No quantum fields (almost)

Null surface

- Consider a generic null surface (codimension 1) in Minkowski space in t-x plane. This surface is the basic ingredient of Unruh effect setup.



Poincaré-Weyl algebra

- Take a subalgebra of Poincaré-Weyl algebra associated with the surface, consisting of translation operators P_t and P_x , boost N and dilation D

$$[P_t, P_x] = 0, \; [D, N] = 0$$

$$[N, P_t] = iP_x, \; [N, P_x] = iP_t$$

$$[D, P_t] = iP_t, \; [D, P_x] = iP_x$$

ax+b algebra

- We can form light-cone generators

$$P \equiv P_t - P_x\,, \quad R \equiv \frac{1}{2}(N - D)$$

- Which satisfy a simple algebra (actually, simplest nontrivial)

$$\boxed{[P,R] = iP}$$

- Unruh effect can be directly derived from representation theory of this algebra.

ax+b group

- … is a group of transformations g=(a,b)

$$x \to ax+b,\ \ a \in \mathbb{R}^+, b \in \mathbb{R}$$

- consisting of two transformations, translations and dilations

$$T(\alpha)=(1,\alpha),\quad S(\lambda)=(e^{-\lambda},0)$$

- with representation on a Hilbert space (positive energy states)

$$T(\alpha)|k\rangle_+ = e^{-i\alpha P}|k\rangle_+ = e^{-i\alpha k}|k\rangle_+,\quad S(\lambda)|k\rangle_+ = e^{-i\lambda R}|k\rangle_+ = |e^{-\lambda}k\rangle_+$$

$$\boxed{P|k\rangle_+ = k|k\rangle_+,\quad R|k\rangle_+ = -ik\frac{d}{dk}|k\rangle_+,\quad k \in \mathbb{R}^+}$$

Fourier transform

- For "functions on momentum space"

$$T(\alpha)\psi(k)=e^{i\alpha k}\psi(k),\quad S(\lambda)\psi(k)=\psi(e^{\lambda}k),\quad \psi(k)={}_{+}\langle k|\psi\rangle$$

- One can take Fourier transform to get the position space picture

$$\boxed{\begin{aligned}\psi(x)\equiv\langle x|\psi\rangle&=\frac{1}{\sqrt{2\pi}}\int_0^\infty\frac{dk}{k}\left(e^{ikx}\,{}_{+}\langle k|\psi\rangle+e^{-ikx}\left({}_{+}\langle k|\psi\rangle\right)^*\right)\\&=\frac{1}{\sqrt{2\pi}}\int_0^\infty\frac{dk}{k}\left(e^{-ikx}a(k)+e^{ikx}a^*(k)\right)\end{aligned}}$$

$$T(\alpha)\psi(x)=\psi(x+\alpha),\quad S(\lambda)\psi(x)=\psi(e^{-\lambda}x)$$

$$P\psi(x)=-i\frac{d}{dx}\psi(x),\quad R\psi(x)=ix\frac{d}{dx}\psi(x)$$

P-momenta vs. R-momenta

$$\boxed{[P,R]=iP}$$

- So far we discussed the representation in which P was diagonal. Let us now take another representation, in which R is diagonal.

$$|k\rangle_+ = \int_{-\infty}^{\infty} d\omega \langle \omega | k\rangle_+ |\omega\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega k^{-i\omega} |\omega\rangle, \quad k \in \mathbb{R}^+,$$

$$|\omega\rangle = \int_0^{\infty} \frac{dk}{k} k^{i\omega} |k\rangle_+$$

$$\boxed{P|k\rangle_+ = k|k\rangle_+, \quad R|k\rangle_+ = -ik\frac{d}{dk}|k\rangle_+,}$$

$$\boxed{P|\omega\rangle = |\omega - i\rangle, \quad R|\omega\rangle = -\omega|\omega\rangle}$$

ω-representation

- The "field" is now

$$\psi(x)=\frac{1}{2\pi}\int_0^\infty \frac{dk}{k}\int_{-\infty}^\infty d\omega\left(e^{ikx}k^{i\omega}\langle\omega|\psi\rangle+e^{-ikx}k^{-i\omega}\langle\omega|\psi\rangle^*\right)$$

- And can be rewritten as

$$\psi(x)=\frac{1}{2\pi}\int_{-\infty}^\infty d\omega\left(x^{-i\omega}e^{-\frac{\pi\omega}{2}}\Gamma(i\omega)\langle\omega|\psi\rangle+x^{i\omega}e^{-\frac{\pi\omega}{2}}\Gamma(-i\omega)\langle\omega|\psi\rangle^*\right)$$

$$=\frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{d\omega}{\omega}\left(x^{-i\omega}b(\omega)+x^{i\omega}b^*(\omega)\right)$$

k- vs. ω-representation

- $\psi(x)$ is the same function in k- and ω-representation, so that

$$\boxed{b(\omega) = \frac{\omega}{\sqrt{2\pi}}\Gamma(i\omega)\int_0^\infty \frac{dk}{k} k^{-i\omega}\left[a(k)e^{\pi\omega/2} + a^*(k)e^{-\pi\omega/2}\right]}$$

- From which we can read-off the Bogolyubov coefficients.

Quantum (field) theory

- We upgrade the functions a and b to quantum operators:

$$\left[\hat{a}(k),\hat{a}^{\dagger}(k')\right]=k\,\delta(k-k')\,,\quad \left[\hat{b}(\omega),\hat{b}^{\dagger}(\omega')\right]=2\pi\omega\,\delta(\omega-\omega')$$

$$\hat{a}(k)\,|0\rangle_a=0,\qquad \hat{b}(\omega)\,|0\rangle_b=0$$

- And then we find that $N_b(\omega)=\dfrac{1}{2\pi\omega}\hat{b}^{\dagger}(\omega)\hat{b}(\omega)$

$$n_b=\frac{N_b}{\delta(0)}=\frac{1}{\omega}\frac{1}{\delta(0)}\ {}_a\!<0|\,\hat{b}^{\dagger}(\omega)\hat{b}(\omega)\,|0>_a=\frac{1}{e^{2\pi\omega}-1}$$

Back to physics

- We obtained thermal spectrum comparing ax+b algebra

$$\boxed{[P,R]=iP}$$

- in two bases: in one P and in another R was the translation (acted diagonally on momenta).
- But then there is something wrong with dimensions, R is dimensionless while it should have the dimension of momentum. We have to rescale

$$\boxed{[P,aR]=iaP}$$

- where a has dimension of inverse length. But then what is a?

Accelerated observers

- Accelerated worldline is the Lorentz orbit of the vector (0; 1/a)

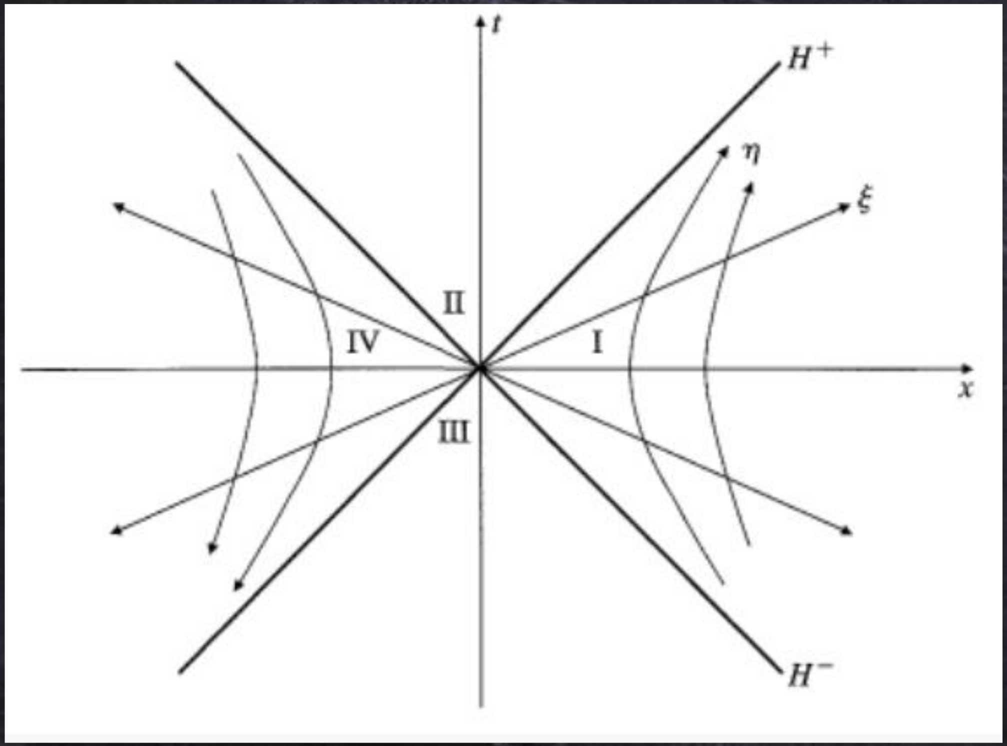
$$t^2(\tau) - x^2(\tau) = \frac{1}{a^2}$$

- Lorentz transformations move points along the orbit of constant acceleration; but what is the transformation that changes the acceleration?
- It is the dilation that does the job.

$$(t, x) \to (t', x') = e^{\delta}(t, x)$$

Rindler space

- We define the Rindler coordinates with the help of the boost and dilation parameters.



$$t = \frac{1}{a} e^{a\xi} \sinh a\eta$$

$$x = \frac{1}{a} e^{a\xi} \cosh a\eta$$

Poincaré–Weyl algebra

- We have two translation operators P_t and P_x generating translation in Minkowski space;
- We have two translation operators aN and aD generating translation in Rindler space.

$$\left[P_t, P_x\right] = 0,\ \left[D, N\right] = 0$$

$$\left[aN, P_t\right] = iaP_x,\ \left[aN, P_x\right] = iaP_t$$

$$\left[aD, P_t\right] = iaP_t,\ \left[aD, P_x\right] = iaP_x$$

k- vs. ω-representation

- So when we compare k and ω bases we are comparing theories as seen by Minkowski and Rindler observers.
- The thermal distribution we found is the number of quanta Rindler observer sees in Minkowski vacuum.

Unruh effect

- Then the thermal spectrum becomes

$$n_b = \frac{1}{e^{2\pi\omega/a} - 1}$$

- which is thermal distribution at temperature T=2π/a, Unruh temperature, physically interpreted as the temperature of the thermal bath the accelerating observer is immersed in.

Big picture

- The Unruh effect is just a particular example of the construction based on representations of ax+b group.
- There might be (should be) other instances where representation theory of ax+b group might be of use.

Open problems

- Deformations are thought to capture some quantum gravity effects. Thus it will be of interest to consider representations of quantum deformed ax+b
- Better understanding of Jacobson's Einstein equations as equations of state.
- And so on …