Blueshifted light from the last-scattering epoch

Andrzej Krasiński,
N. Copernicus Astronomical Center, Warsaw, Poland

Contents

1. Motivation and background

The Lemaitre [1] – Tolman [2] (L-T) and Szekeres [3] (Sz) models are the simplest exact generalizations of the Friedmann-class metrics.

The Big Bang (BB) in them is in general non-simultaneous in comoving coordinates.

Some light rays emitted at the BB in these models display ***infinite blueshift*** rather than infinite redshift (see further).

This happens when the BB function $t_B(r)$ has $dt_B/dr \neq 0$ at the emission point
and the ray propagates radially (in L-T) or along one of two preferred directions (in Sz).

→ Rays emitted during the ***last scattering period*** (≈380 000 years after the BB) may have ***finite blueshift***.

Can such rays be now observed?

Yes! – in principle. I will argue that they might be among the gamma-ray bursts.

[1] G. Lemaître, L'Univers en expansion [The expanding Universe], *Ann. Soc. Sci. Bruxelles* **A53**, 51 (1933); *Gen. Rel. Grav.* **29**, 641 (1997).
[2] R. C. Tolman, Effect of inhomogeneity on cosmological models, *Proc. Nat. Acad. Sci. USA* **20**, 169 (1934); *Gen. Rel. Grav.* **29**, 935 (1997).
[3] P. Szekeres, A class of inhomogeneous cosmological models. *Commun. Math. Phys.* **41**, 55 (1975).

2. The Lemaître – Tolman (L-T) models

This class of solutions of Einstein's equations follows when we assume that

1. ***The spacetime is spherically symmetric.***

2. ***The source in the Einstein equations is dust*** (pressure = 0).

→ The metric and velocity field of the dust , in comoving coordinates, have the form:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{A(t,r)}\mathrm{d}r^2 - R^2(t,r)(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2), \tag{2.1}$$

$$u^\alpha = \delta^\alpha{}_0. \tag{2.2}$$

A(t, r) and R(t,r) are to be found from Einstein's equations.

$$ds^2 = dt^2 - e^{A(t,r)}dr^2 - R^2(t,r)(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2),$$
$$u^\alpha = \delta^\alpha{}_0.$$

The solution of Einstein's equations with $R_{,r} \neq 0$ is

$$ds^2 = dt^2 - \frac{R_{,r}{}^2}{1+2E(r)}dr^2 - R^2(t,r)\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \qquad (2.3)$$

where E(r) is arbitrary and R(t,r) is determined by

$$R_{,t}{}^2 = 2E(r) + \frac{2M(r)}{R} - \frac{1}{3}\Lambda R^2, \qquad \rightarrow \text{solution depends on } t - t_B(r)\,. \qquad (2.4)$$

M(r) and $t_B(r)$ are arbitrary, and the mass density is

$$\frac{8\pi G}{c^2}\rho = \frac{2M_{,r}}{R^2 R_{,r}}\ . \qquad (2.5)$$

This solution was found by Lemaître [1] in 1933, then interpreted by Tolman [2] in 1934 and Bondi [4] in 1947

(and investigated by > 100 other authors later. The number is still growing.)

[1] G. Lemaître, L'Univers en expansion [The expanding Universe], *Ann. Soc. Sci. Bruxelles* **A53**, 51 (1933); *Gen. Rel. Grav.* **29**, 641 (1997).
[2] R. C. Tolman, Effect of inhomogeneity on cosmological models, *Proc. Nat. Acad. Sci. USA* **20**, 169 (1934); *Gen. Rel. Grav.* **29**, 935 (1997).
[4] H. Bondi, Spherically symmetrical models in general relativity. *Mon. Not. Roy. Astr. Soc.* **107**, 410 (1947; *Gen. Rel. Grav.* **31**, 1783 (1999).

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{{R_{,r}}^2}{1+2E(r)}\mathrm{d}r^2 - R^2(t,r)\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right) \qquad (2.3)$$

3. Light rays in L–T models

The partly integrated null geodesic equations in this model are:

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = k^t, \qquad (3.1)$$

$$\frac{\mathrm{d}k^t}{\mathrm{d}\lambda} = \left[\frac{C^2}{R^2} - \left(k^t\right)^2\right]\frac{R_{,tr}}{R_{,r}} - \frac{C^2 R_{,t}}{R^3} \qquad (3.2)$$

$$\frac{\mathrm{d}r}{\mathrm{d}\lambda} = k^r, \qquad (3.3)$$

$$\left(k^t\right)^2 = \frac{{R_{,r}}^2\left(k^r\right)^2}{1+2E} + \frac{C^2}{R^2} \qquad (3.4)$$

$$R^2\sin^2\vartheta k^\varphi = J_0, \qquad (3.5)$$

$$R^4\left(k^\vartheta\right)^2\sin^2\vartheta + {J_0}^2 = C^2\sin^2\vartheta, \qquad (3.6)$$

J_0 and C are constants. When C = 0, the geodesic is radial.

The following can be achieved at any t_o by rescaling λ:

$k^t(t_o) = \pm 1,$ (3.7)

(+ for future-directed, - for past-directed rays).

$k^t(t_o) = \pm 1$, (3.7) $\qquad (k^t)^2 = \frac{R_{,r}{}^2 (k^r)^2}{1+2E} + \frac{C^2}{R^2}$ (3.4)

4. The redshift

Let event e be the emission of an electromagnetic wave, event o – its observation.

The quantity *z* defined by:

$$1 + z \stackrel{\text{def}}{=} \frac{\lambda_o}{\lambda_e} \equiv \frac{\nu_e}{\nu_o} = \frac{(k_\alpha u^\alpha)_e}{(k_\alpha u^\alpha)_o} \qquad (4.1)$$

is called ***redshift*** (u^α is 4-velocity of the emitter/observer).

In comoving coordinates ($u^\alpha = \delta^\alpha{}_0$) using (3.7), we have

$$1 + z = \pm k^t{}_e \qquad (4.2)$$

(+ for future-directed, - for past-directed rays).

In cosmology, usually $\lambda_o > \lambda_e$ (so $z > 0$), hence ***red***shift, but see further.

On nonradial rays ($C \neq 0$), from (3.4) and (4.2)

$$\lim_{R\to 0} |k^t| \equiv \lim_{R\to 0} z = \infty \,. \qquad (4.3)$$

→ On nonradial rays from the BB $z \to \infty$ for all observers,

→ $z < 0$ at the present observer is possible only on radial rays.

$$1 + z = \frac{\nu_e}{\nu_o} \qquad (4.1)$$

Let a ***radial*** ray go to the past from P_o and hit the BB at P_e.

Perturbative [5] and numerical [6] calculations both show that on such rays

$z \xrightarrow[R\to 0]{} -1$ when $dt_B/dr \neq 0$ at P_e

$z \xrightarrow[R\to 0]{} \infty$ when $dt_B/dr = 0$ at P_e

$z \to -1 \quad \to \quad \nu_o \to \infty$ when $\nu_e < \infty$.

The property $z < 0$ is called ***blueshift*** [7]; when $z = -1$ the blueshift is called infinite.

→ On radial rays emitted just after the BB ν_o can be larger than ν_e .

[5] C. Hellaby and K. Lake, The redshift structure of the Big Bang in inhomogeneous cosmological models. I. Spherical dust solutions. *Astrophys. J.* **282**, 1 (1984) + erratum *Astrophys. J.* **294**, 702 (1985).
[6] A. Krasiński, Blueshifts in the Lemaître – Tolman models. *Phys. Rev.* **D90**, 103525 (2014).
[7] P. Szekeres, Naked singularities. In: *Gravitational Radiation, Collapsed Objects and Exact Solutions*. Edited by C. Edwards. Springer (Lecture Notes in Physics, vol. 124), New York, pp. 477 -- 487 (1980).

$$\frac{\mathrm{d}k^t}{\mathrm{d}\lambda} = \left[\frac{C^2}{R^2} - (k^t)^2\right]\frac{R_{,tr}}{R_{,r}} - \frac{C^2 R_{,t}}{R^3} \qquad (3.2)$$

$$k^r = \pm\frac{\sqrt{1+2E}}{R_{,r}}\sqrt{(k^t)^2 - \frac{C^2}{R^2}} \qquad (3.4a)$$

$$1 + z = \pm\, k^t{}_e \qquad (4.2)$$

5. The extremum-redshift hypersurface

Along a radial ray, $dr/d\lambda \equiv k^r \neq 0$ and $C = 0$.

Then, using (4.2), Eqs. (3.2) and (3.4a) imply

$$\frac{1}{1+z}\,\frac{\mathrm{d}z}{\mathrm{d}r} = \pm\frac{R_{,tr}}{\sqrt{1+2E}}\ . \qquad (5.1)$$

→ $R_{,tr} = 0$ is the locus of local extrema of z along rays.

This locus is the ***extremum-redshift hypersurface*** (ERH).

It is observer-independent and can be calculated numerically.

In the Friedmann limit, the ERH does not exist, but formally may be thought of as coinciding with the BB.

Consider a ray proceeding ***to the past*** from a point that lies later than the ERH.

z on it increases from 0 at observer to a local maximum at the ERH.

Further to the past, *z* ***initially*** decreases.

The ERH may have a nontrivial geometry → a ray may intersect it several times and have several local maxima and minima. Examples will follow.

At the BB, z would either decrease to -1 or increase to ∞.

But L–T models do not apply at times before the last-scattering hypersurface (LSH) because of p = 0.

Can z from the LSH be sufficiently negative to shift the optical frequencies to the gamma-range?

Yes – when the function $t_B(r)$ is suitably chosen.

Note: the blueshift is acquired only below the ERH.

Wherever the ray flies above the ERH, it gets only redshifted.

6. The quasi-spherical Szekeres (QSS) models

Szekeres [3,8] in 1975 took the following Ansatz for the metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{2\alpha}\mathrm{d}z^2 - \mathrm{e}^{2\beta}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right) , \qquad (6.1)$$

α(t,x,y,z) and β(t,x,y,z) to be determined from Einstein's equations with dust source.

Then he found all such solutions.

One sub-family of his metrics generalizes L–T, these are called ***quasi-spherical***.

General Szekeres models have ***no symmetry***.

[3] P. Szekeres, A class of inhomogeneous cosmological models. *Commun. Math. Phys.* **41**, 55 (1975).
[8] P. Szekeres, Quasispherical gravitational collapse. *Phys. Rev*. **D12**, 2941 (1975).

The QSS solutions have the metric [9]

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{d}t^2 - \frac{\mathcal{E}^2(\Phi/\mathcal{E})_{,r}{}^2}{1+2E(r)}\mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2}\left(\mathrm{d}x^2+\mathrm{d}y^2\right), \\ \mathcal{E} &\stackrel{\text{def}}{=} \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2}, \end{aligned} \tag{6.2}$$

where E(r), M(r), P(r), Q(r) and S(r) are arbitrary functions, and Φ (t,r) obeys

$$\Phi_{,t}{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \tag{6.3}$$

The mass density is

$$\kappa\rho = \frac{2\left(M/\mathcal{E}^3\right)_{,r}}{(\Phi/\mathcal{E})^2\left(\Phi/\mathcal{E}\right)_{,r}}, \qquad \kappa = \frac{8\pi G}{c^2}. \tag{6.4}$$

The solution of (6.3) is

$$\int_0^{\Phi} \frac{\mathrm{d}\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{6.5}$$

[9] C. Hellaby, The nonsimultaneous nature of the Schwarzschild $R = 0$ singularity. *J. Math. Phys.* **37**, 2892 (1996).

$$ds^2 = dt^2 - \frac{\mathcal{E}^2(\Phi/\mathcal{E})_{,r}{}^2}{1+2E(r)}dr^2 - \frac{\Phi^2}{\mathcal{E}^2}\left(dx^2+dy^2\right), \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \tag{6.2}$$

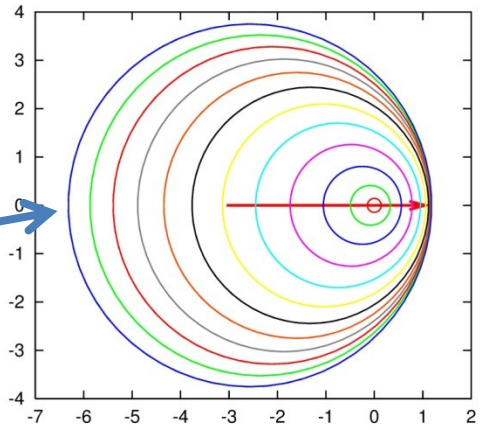
$$\Phi_{,t}{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \tag{6.3}$$

$$\int_0^{\Phi} \frac{d\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{6.5}$$

The surfaces of constant t and r

$ds^2 = \dfrac{\Phi^2}{\mathcal{E}^2}\left(dx^2+dy^2\right)$

are ***nonconcentric spheres***,
x and y are stereographic coordinates on them.



The L-T models are the limit of constant (P, Q, S) – then the spheres become concentric and the spacetime becomes spherically symmetric.

7. Blueshifts in QSS metrics

In QSS metrics, strong blueshifts exist only along two opposite directions.
(This was determined numerically in exemplary models [10]. General proof still missing.)

In axially symmetric QSS, these directions coincide with the symmetry axis [10].

Can blueshifted rays of QSS models be observed as ***gamma-ray bursts*** (GRBs)?

Models of GRB sources must account for [11]:

(1) The observed frequency range of the GRBs ($0.24 \times 10^{19}\text{Hz} \leq \nu \leq 1.25 \times 10^{23}\text{Hz}$);

(2) Their limited duration (up to 30 hours, mostly around 2 minutes [12]);

(3) The existence and duration of afterglows (typically a few days, max. $n \times 100$ [13]);

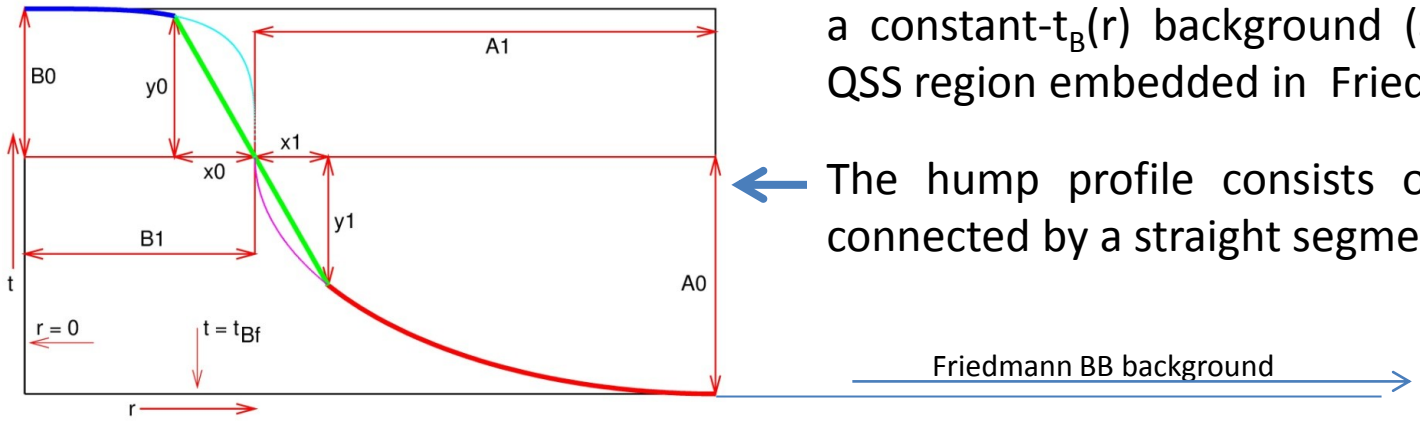
(4) (Hypothetical) collimation of the GRBs into narrow jets.

(5) The large distances to their sources ($n \times 10^9$ ly);

(6) The multitude of the observed GRBs (nearly 1 per day).

[10] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).
[11] Gamma-Ray Bursts, http://swift.sonoma.edu/about_swift/grbs.html
[12] https://imagine.gsfc.nasa.gov/science/objects/bursts1.html
[13] http://astronomy.swin.edu.au/cosmos/G/gamma+ray+burst+afterglow

8. Adapting the BB profile to generating a sufficiently strong blueshift



A source of a blueshifted ray is a hump on a constant-$t_B(r)$ background (a finite-size QSS region embedded in Friedmann).

The hump profile consists of two arcs connected by a straight segment.

The upper arc is a segment of the curve:

$$\frac{r^n}{{B_1}^n} + \frac{(t - t_{Bf} - A_0)^n}{{B_0}^n} = 1$$

where n = 4 or 6. (8.1)

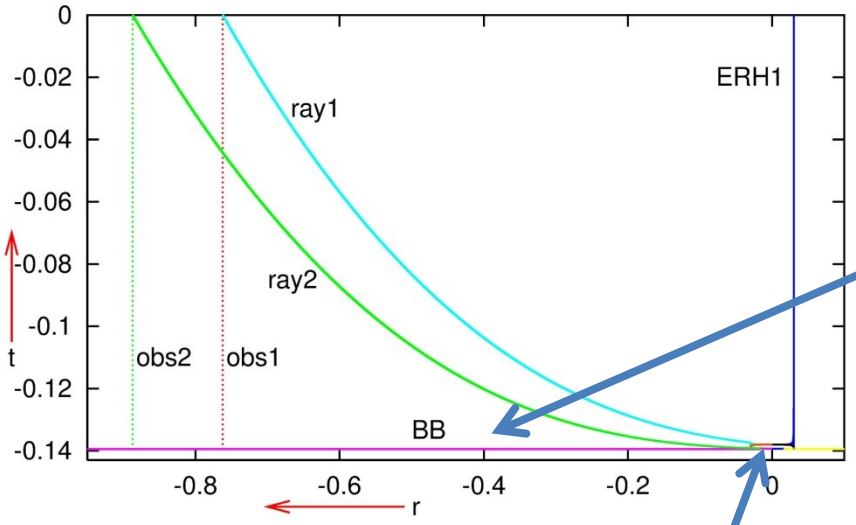
Note: all this material is a proof by example of existence of the blueshifting effect, not a model of a real GRB!

The lower arc is a segment of an ellipse (n = 2).

The straight segment prevents $dt_B/dr \to \infty$ at the junction of full arcs.

The free parameters are A_0, A_1, B_0, B_1 and x_0.

Numerical time unit:
1 NTU =
9.8×10^{10} y =
3×10^{4} Mpc



present time

background (Friedmann) BB

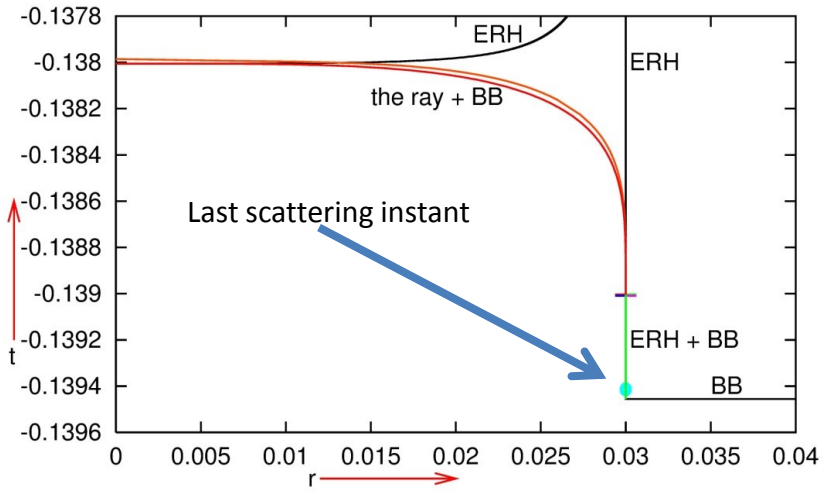
Here two humps are drawn in proportion to the age of the Universe

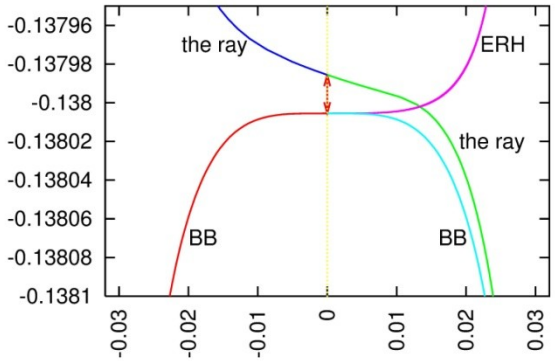
The lower hump (with ray 2) mimics a GRB source of the lowest observed energy.

Its height is 8.9×10^{-4} × (the ΛCDM age of the Universe) ≈ 1.23×10^{7} years,

it encompasses the mass ≈ 3.1×10^{6} masses of our Galaxy.

The other hump is 11.5 times higher and 2 times wider, and mimics a GRB source of the highest observed energy.





Top of the hump, magnified

The real profile of the hump, and the maximally blueshifted ray near the BB

Backward in time along the ray, z increases up to the first intersection with the ERH.

Further down, z decreases until the ray intersects the ERH again or until it hits the BB.

The hump parameters are chosen such that

$$2.5 \times 10^{-8} < 1 + z_{\text{observed now}} < 1.7 \times 10^{-5}$$

which moves the frequencies from the hydrogen emission range to the GRB range:

$$0.24 \times 10^{19} < \nu_{GRB} < 1.25 \times 10^{23} \text{ Hz.}$$

QSS models of this type reproduce [14,15]:

(1) The observed frequency range of the GRBs ($0.24 \times 10^{19}\text{Hz} \leq \nu \leq 1.25 \times 10^{23}\text{Hz}$);

(2) The duration of the GRBs (up to 30 hours, mostly ≈2 min) – see next page;

(4) Collimation of GRBs into narrow jets;

(5) The large distances to their sources ($n \times 10^9$ ly);

(6) The multitude of the GRBs (observed: ≈ 1/day)
by putting many BB humps into a Friedmann background.

At the current resolution of detectors (0.5°) 44,000 GRB sources would fill the whole sky.
In the currently best QSS model this number is $11\,005 < N < 11\,014$.

The remaining property:

(3) The duration of afterglows (observed: up to n × 100 days, typically a few) is still to be dealt with,

but the afterglows necessarily exist.

[14] A. Krasiński, Properties of blueshifted light rays in quasi-spherical Szekeres metrics. *Phys. Rev.* **D97**, 064047 (2018).
[15] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).

9. Explaining the brief durations of the gamma-flashes

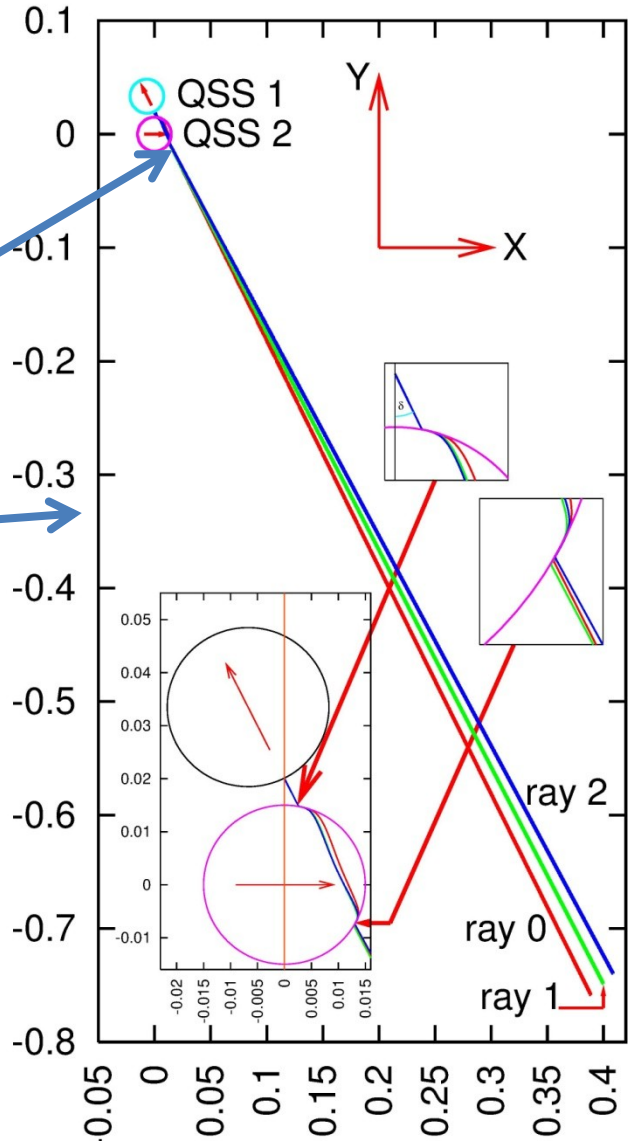
This is explained by means of the drift effect [16,17,18].

The maximally blueshifted ray emitted in QSS1
propagating over a second BB hump (QSS2)
is ***deflected***,
and ***the angle of deflection changes with time***.

→ ***The ray will miss the observer after a while***.

In this configuration the current observer
would not see any gamma ray from QSS1
10 minutes after the arrival of the first flash [19].

Instead, she would see UV rays (the afterglow!)
from the same direction.



[16] A. Krasiński and K. Bolejko, Redshift propagation equations in the β' ≠ 0 Szekeres models. *Phys. Rev*. **D83**, 083503 (2011).
[17] M. Korzyński and J. Kopiński, Optical drift effects in general relativity. *J. Cosm. Astropart. Phys.* **03**, 012 (2018).
[18] C. Quercellini, L. Amendola, A. Balbi, P. Cabella, M. Quartin, Real-time cosmology. *Phys. Rep.* **521**, 95 -- 134 (2012).
[19] A. Krasiński , Short-lived flashes of gamma radiation in a quasi-spherical Szekeres metric. ArXiv 1803.10101, submitted for publication.

10. Expression of hope

Most astronomers treat inhomogeneous models as an enemy to kill.

Example [18]: Gaia or E-ELT could distinguish between FLRW and L-T ``***possibly eliminating an exotic alternative explanation to dark energy***''.

(Is a lumpy mass distribution really exotic? This is all there is to L-T and Sz models. On the other hand, you cannot buy dark energy in a local shop. [As Jürgen Ehlers would probably say.])

But L-T and Szekeres models imply interesting processes – and do it within the exact Einstein theory.

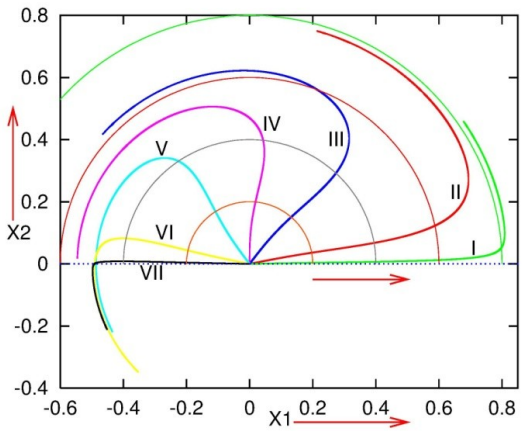
History of science teaches us that if a well-tested theory predicts a phenomenon, then the prediction has to be taken seriously, thoroughly investigated, then put to experimental tests.

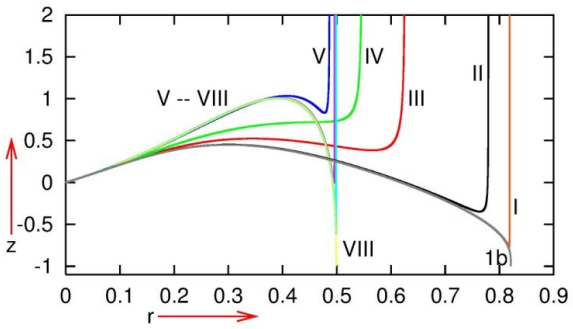
I did my work with the hope that this will happen with the results reported here (but will I live long enough to see it?).

[18] C. Quercellini, L. Amendola, A. Balbi, P. Cabella, M. Quartin, Real-time cosmology. *Phys. Rep.* **521**, 95 -- 134 (2012).

11. Appendix: Blueshifts in exemplary QSS models

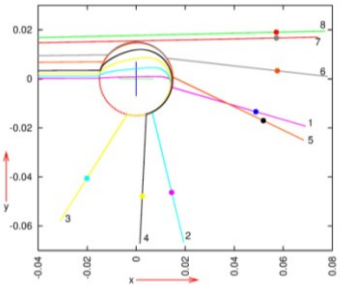
In an *axially symmetric* QSS model, a *necessary condition* for infinite blueshift is that the ray is ***axial*** (intersects every space of constant t on the symmetry axis) [11].





Rays 1b and VIII are axial.

Rays overshooting the hump would be strongly deflected and would hit the BB in the Friedmann region with $z_{obs} = \infty$.



Rays projected on a surface of constant t and φ and z-profiles along them

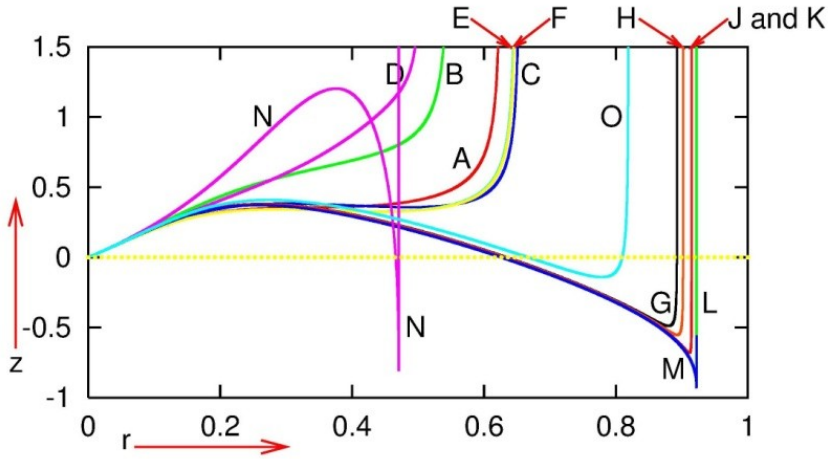
$z_{min} \rightarrow -1$ when the ray approaches axial.

On rays 1b and VIII, $1 + z_{min} < 10^{-5}$.

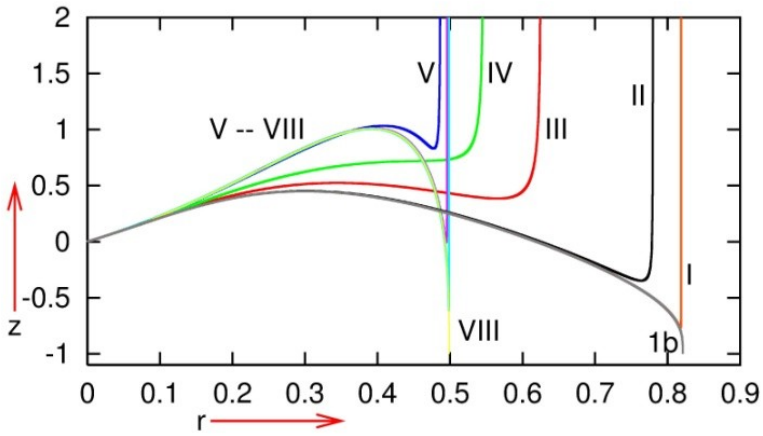
Non-axial rays hit the BB hump tangentially to r = constant surfaces, with $z_{obs} \rightarrow \infty$ (the same happens with nonradial rays in L-T).

[11] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).

In a fully nonsymmetric QSS model two opposite null directions exist along which the blueshift is near to -1 [11].



On approaching a preferred direction in a nonsymmetric QSS model the redshift profiles behave similarly



to the redshift profiles on rays approaching the axial direction in an axially symmetric QSS

But these directions ***do not*** coincide with the two principal null directions of the Weyl tensor, except in the axially symmetric case [11].

[11] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).

In the now-best Szekeres/Friedmann model [11], the angular radius of a GRB source is

$0.9681^\circ < \vartheta < 0.9783^\circ$, depending on the direction of observation.

The current precision in determining the direction to an observed GRB source is a disk in the sky of radius $\approx 0.5^\circ$.

Thus, the whole sky could accommodate

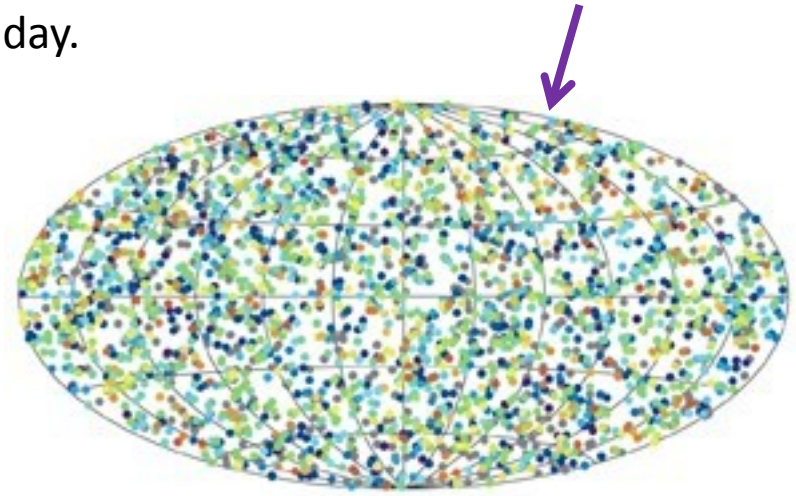
$11\,005 < N < 11\,014$

such objects.

The BATSE detector (Burst And Transient Source Experiment) detected 2704 GRBs between 1991 and 1999 [19] – this is nearly 1 per day.

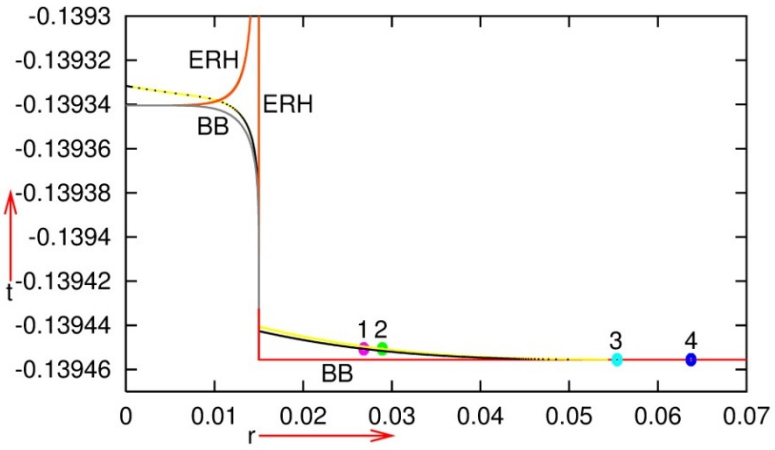
So, during the 27 years from 1991 to now **8112** GRBs should have been discovered .

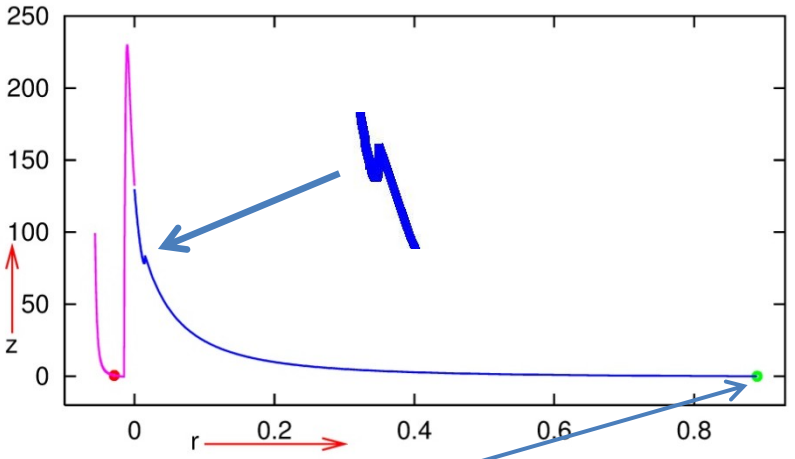
→ The **numbers** in the model and in the observations are not in contradiction.



[11] A. Krasiński, Properties of blueshifted light rays in quasi-spherical Szekeres metrics. *Phys. Rev.* **D97**, 064047 (2018).
[19] Gamma-Ray Bursts, http://swift.sonoma.edu/about_swift/grbs.html

When local blueshifts are present, ***redshift fails*** to be a distance indicator [16].





z(r) seen by the observer at r ≈ 0.9

calculated along the **yellow ray**.

The redshift first increases toward the past, then decreases under the ERH.

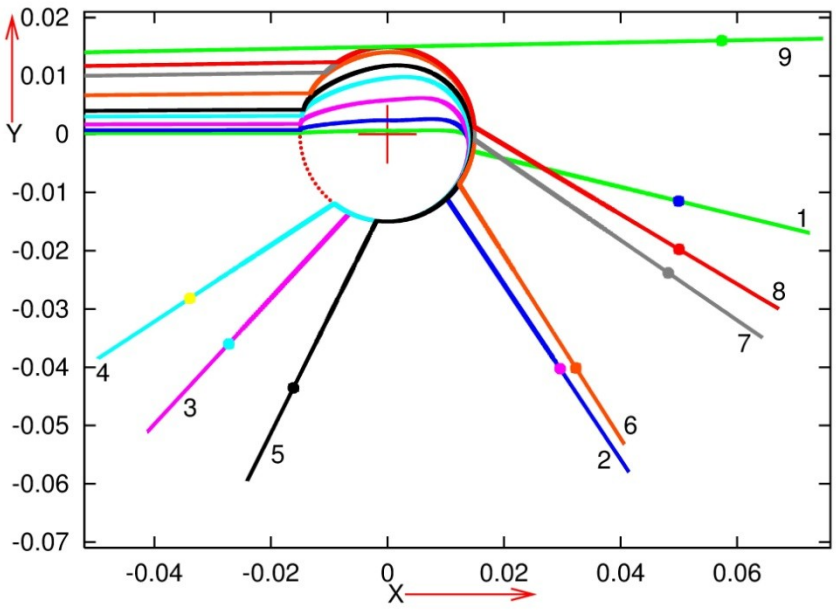
At the **red dot** in the right graph z = 0.598.

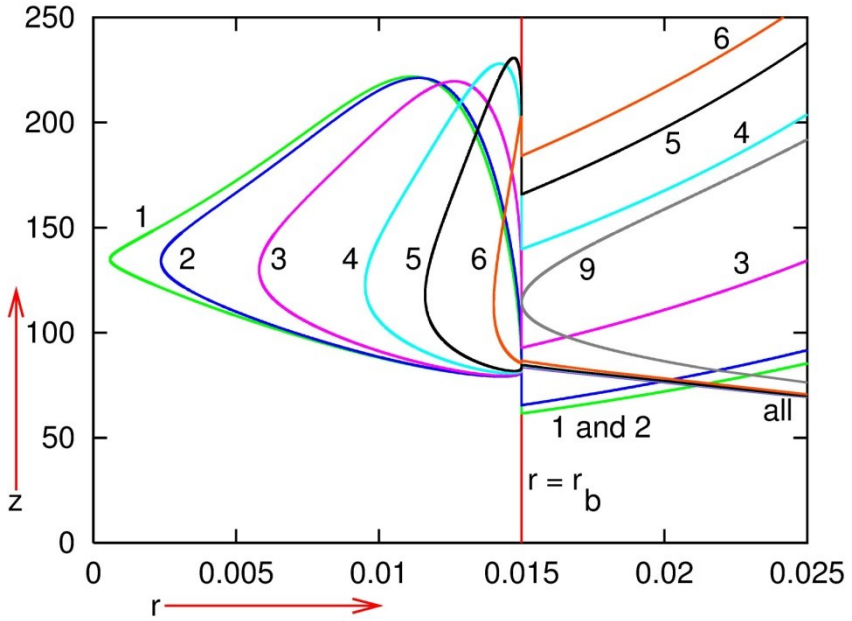
The standard formula implies that source to lie 5.9×10^9 years to the past [17,18].

In this model, the source lies 1.37×10^{10} years to the past.

[16] A. Krasiński, Properties of blueshifted light rays in quasi-spherical Szekeres metrics. *Phys. Rev.* **D97**, 064047 (2018).
[17] E. L. Wright, A Cosmology Calculator for the World Wide Web. *Publ. Astr. Soc. Pac.* **118**, 1711 (2006).
[18] E. L. Wright, http://www.astro.ucla.edu/~wright/ACC.html

Examples of non-monotonic redshift along non-axial rays:





Rays overshooting the BB hump of a QSS region
The observer is at X ≈ -0.9, far at left.

and z(r) profiles along them [16].
The observer is at r ≈ 0.9, far at right.

[16] A. Krasiński, Properties of blueshifted light rays in quasi-spherical Szekeres metrics. *Phys. Rev.* **D97**, 064047 (2018).