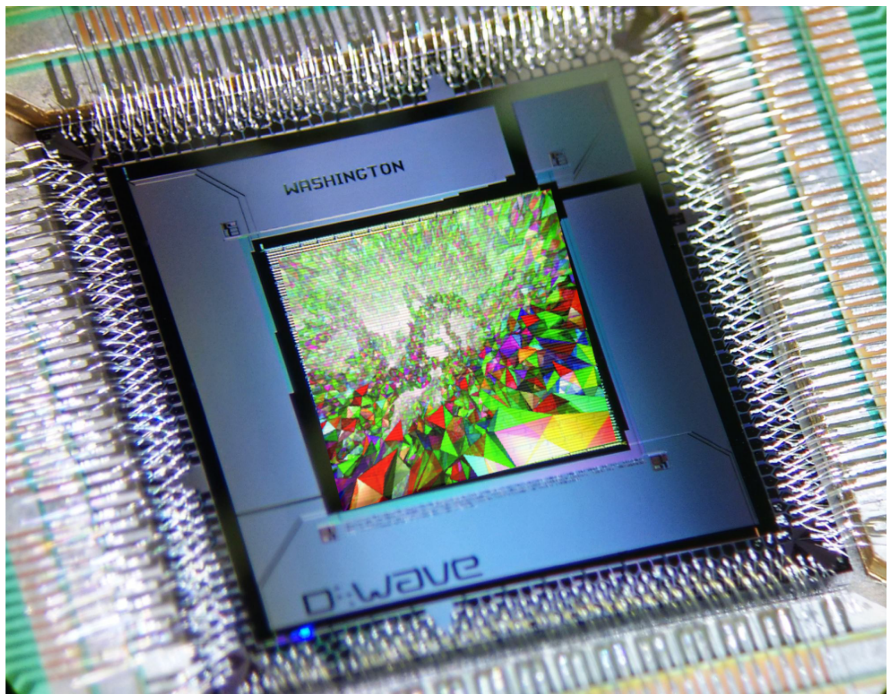


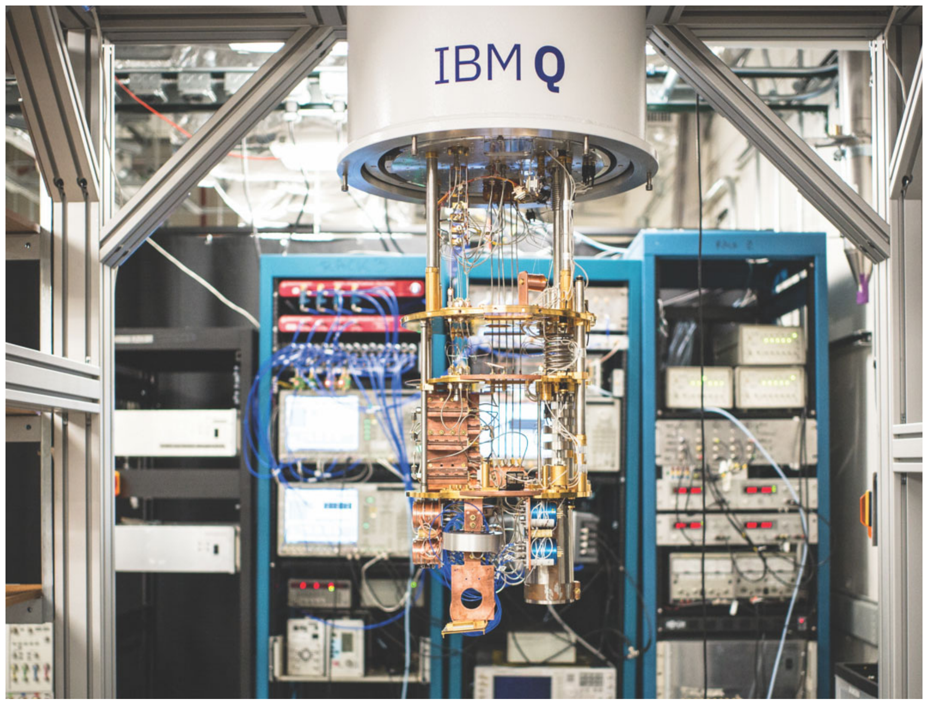

# Quantum Gravity on a Quantum Chip

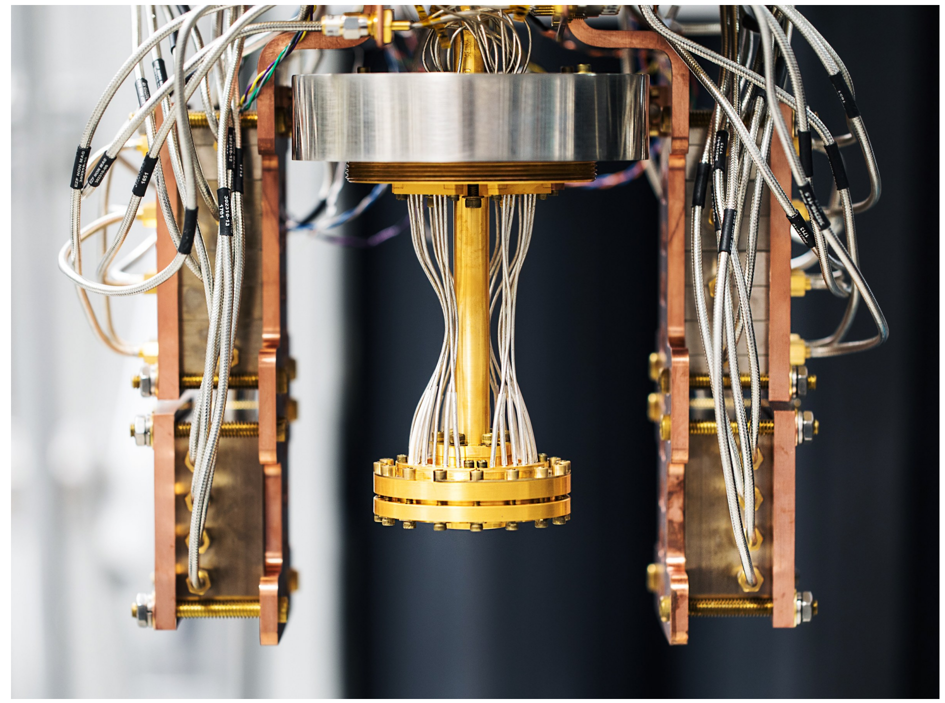
Jakub Mielczarek

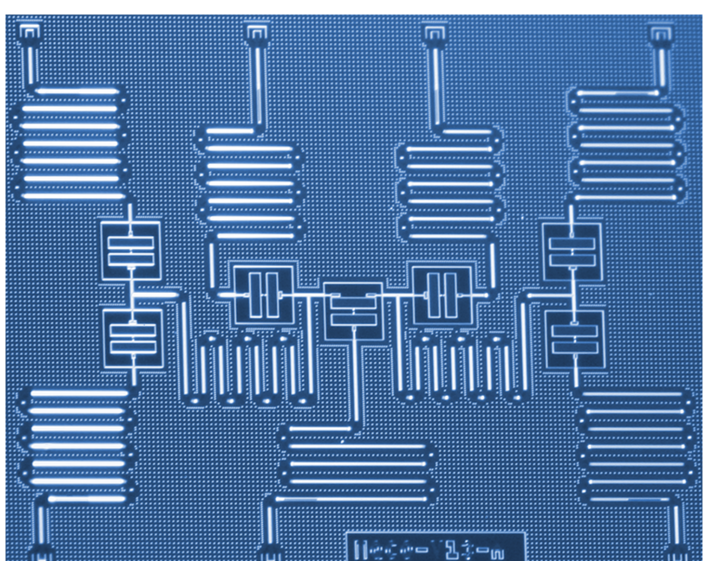
**Jagiellonian University, Cracow**
**Centre de Physique Théorique, Marseille**

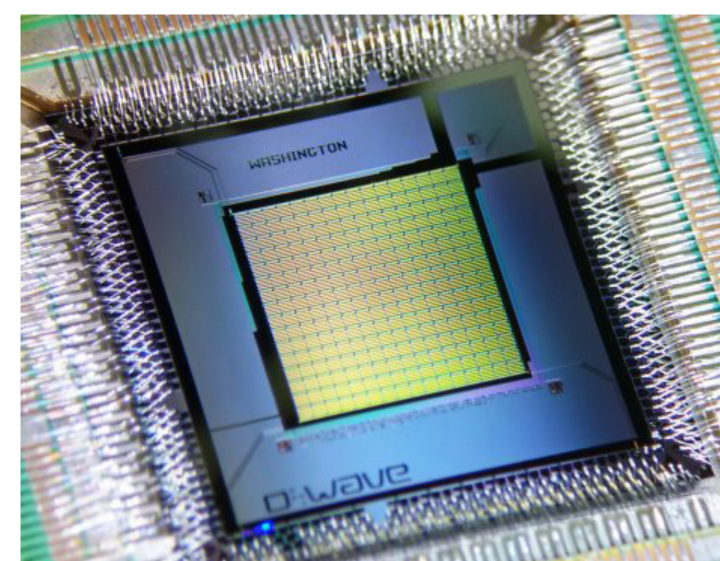
**5th POTOR Conference, Wojanów, 26.09.2018**

# Quantum computing industry is booming!



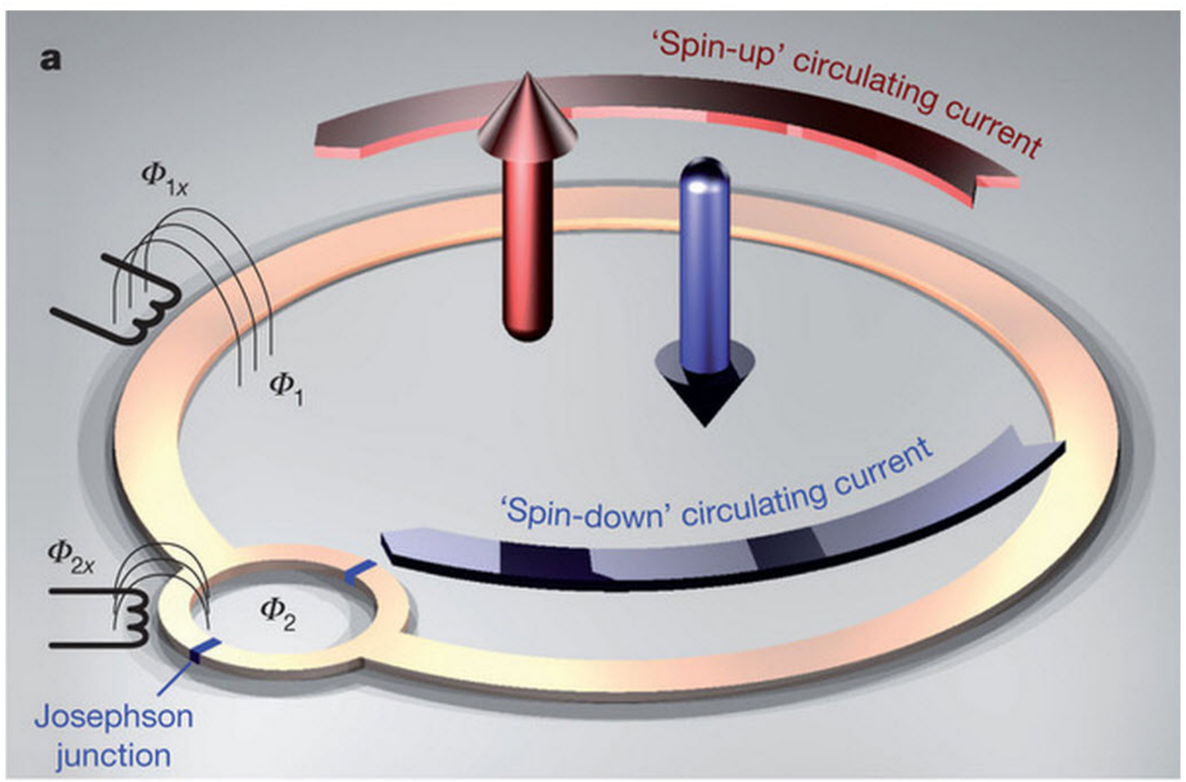



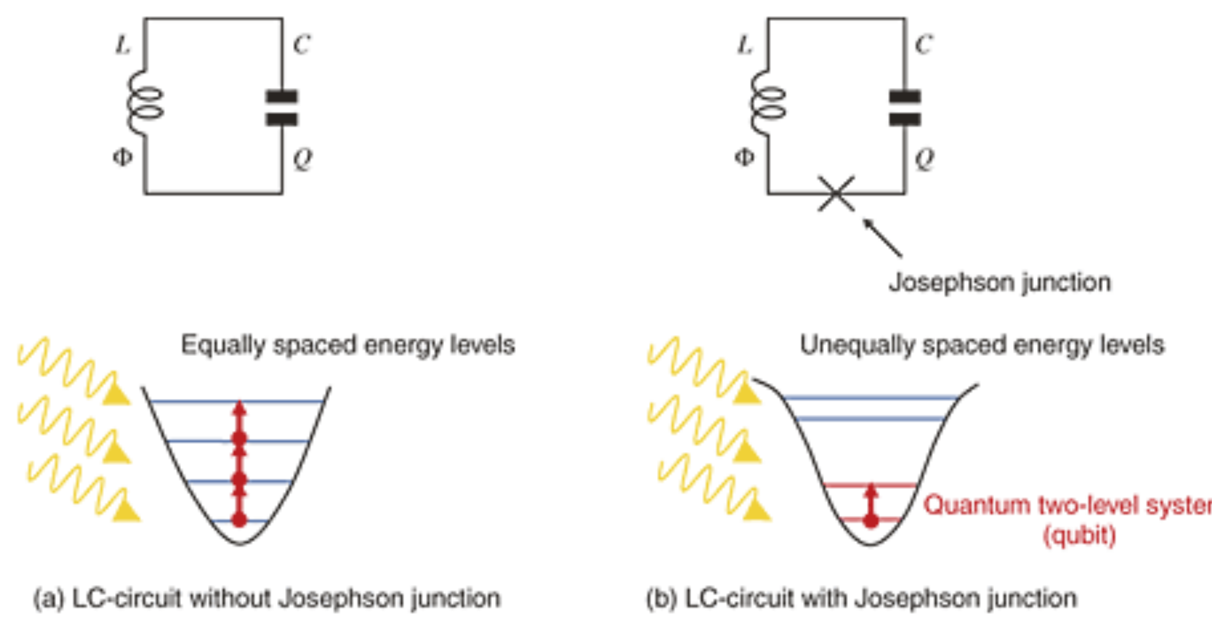





# Superconducting qubits

The recent progress has been made mainly thanks to the technology of manufacturing superconducting qubits.



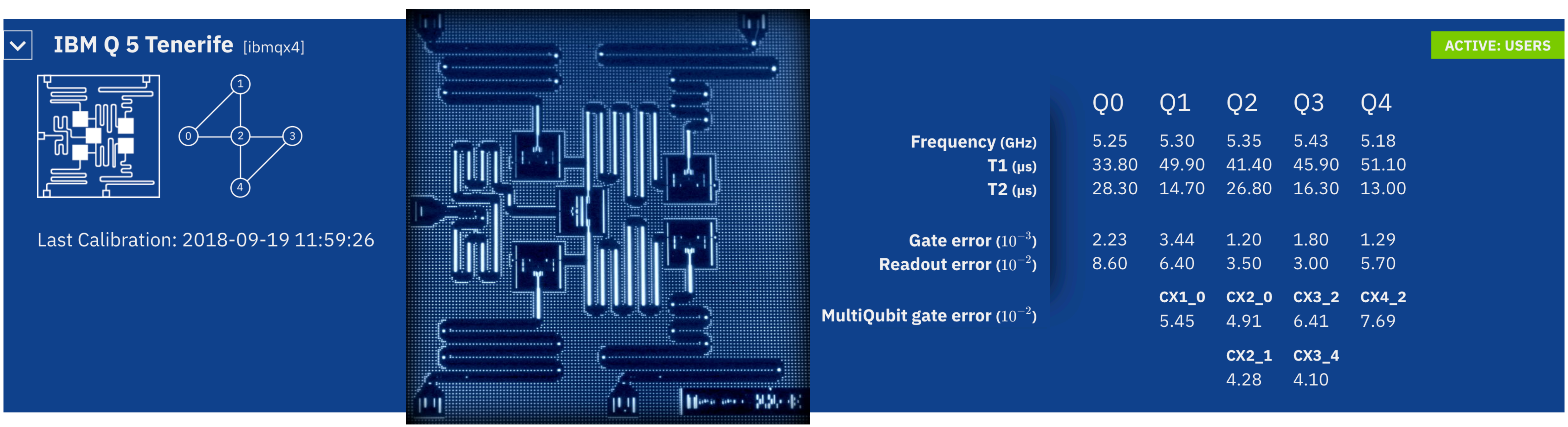




Semba, 2008

# Quantum computers are becoming accessible

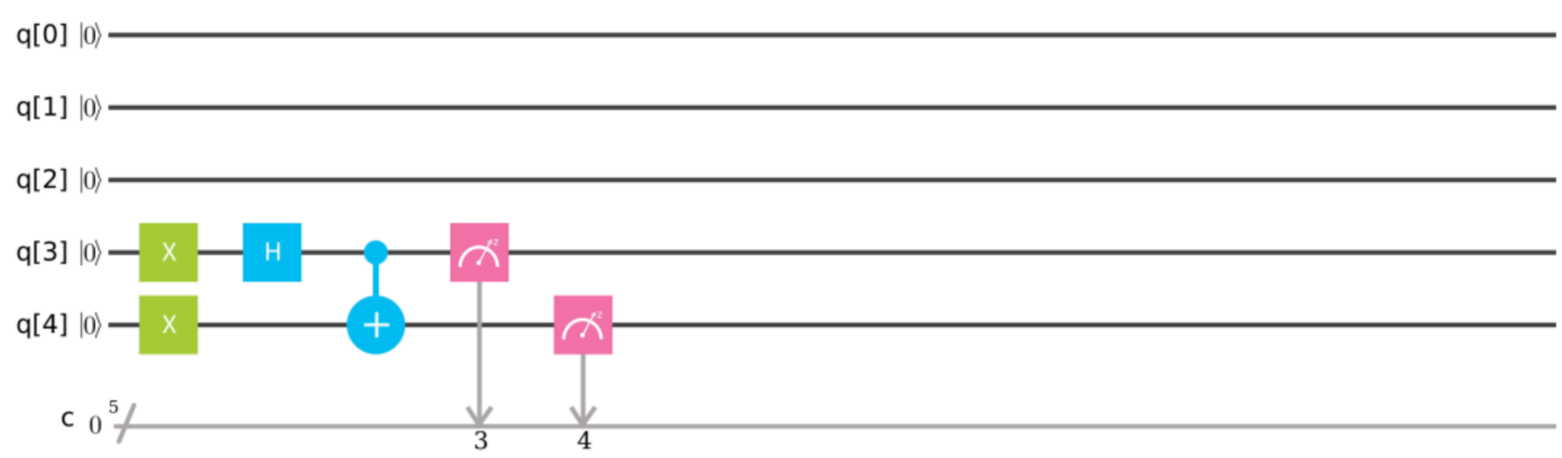
You can program quantum computer this evening…

Visit **www.quantumexperience.ng.bluemix.net** and get access to the IMB Q 5-qubit quantum computer.



# Let's create the Bell state…

$$\widehat{CNOT}(\hat{H} \otimes \hat{I})(\hat{X} \otimes \hat{X})(|0\rangle \otimes |0\rangle) = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle)$$

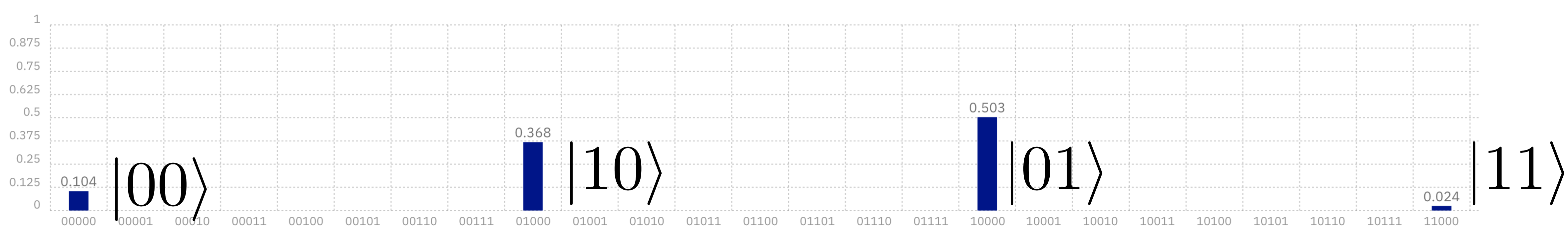



Device: Simulator

Quantum State: Computation Basis


Device: ibmqx4

Quantum State: Computation Basis



# Why it can better than using classical supercomputers?

For a sigle qubit $\dim(\mathcal{H}) = 2$

Dimension of the Hilbert space for N qubits grows exponentially with N:

$$\boxed{\dim(\underbrace{\mathcal{H} \otimes \mathcal{H} \otimes \cdots \otimes \mathcal{H}}_{N}) = 2^N}$$

With the present most powerful classical supercomputers you can simulate quantum systems with **N=56** at most.

IBM and Rigetti Computing are developing quantum chips with **N>100** and certain topologies of couplings between the qubits > *quantum supremacy*

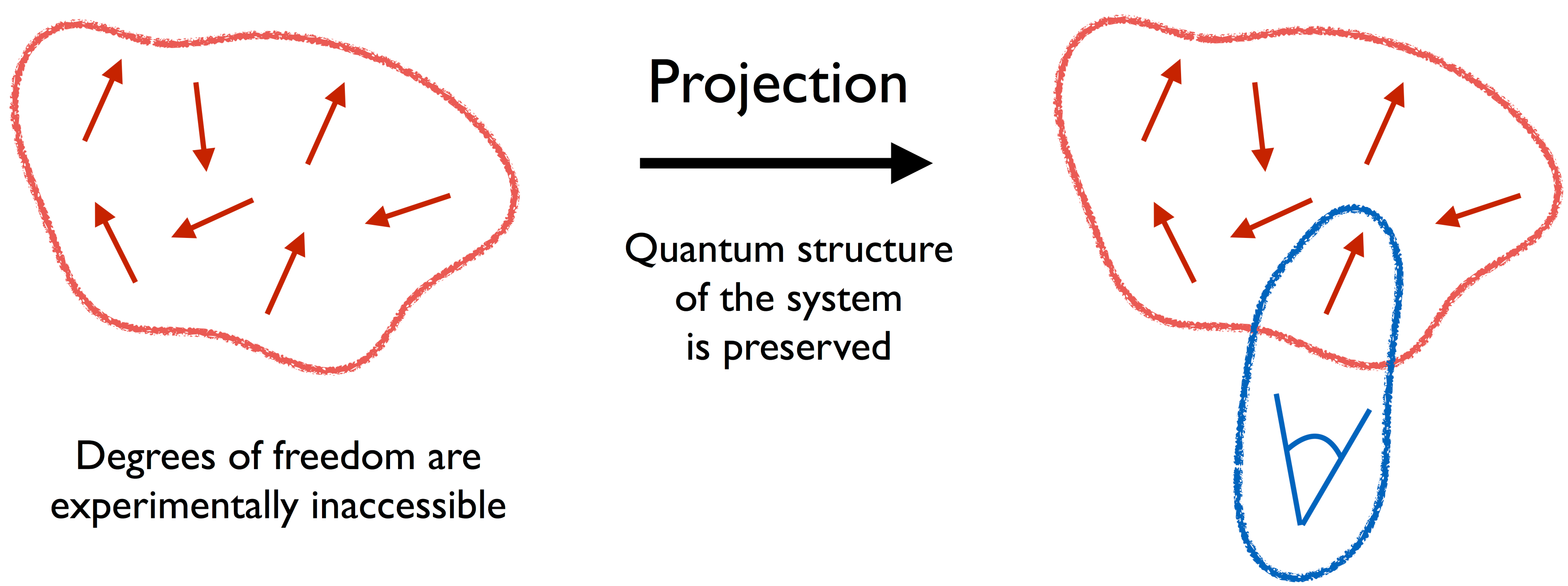
*Quantum parallelism* > reduction of computational complexity of some classical problems (Deutsch, Grover, Shor, … ) > *quantum speed-up*

# Quantum computers will allow to perform *exact simulations* of quantum systems with large N

In particular, we may think about simulating Planck scale physics…

Original system at the Planck scale

Exact simulation (e.g. using superconducting circuits)




Degrees of freedom are experimentally inaccessible

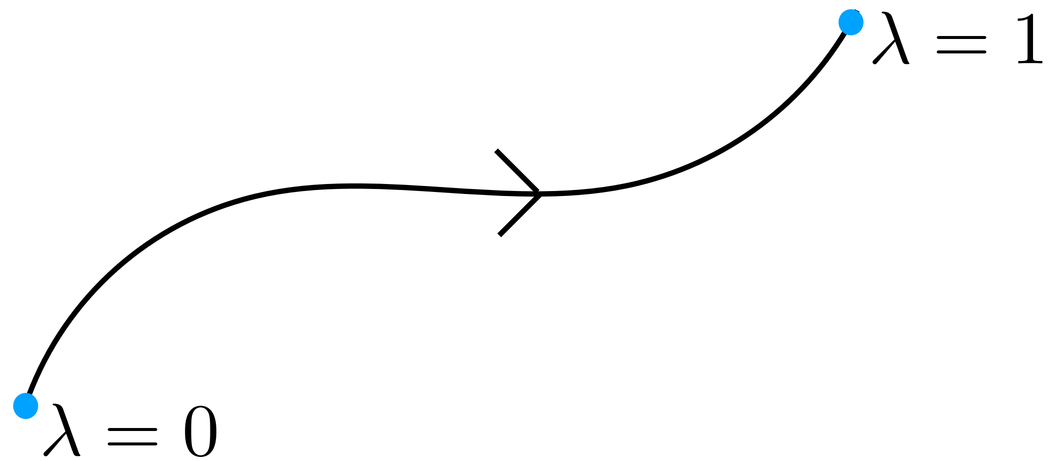
Degrees of freedom are experimentally accessible

But, can we express QG DOF with qubits?

# Loop Quantum Gravity

Holonomies of Ashtekar connection $A \in su(2)$ along some curve $e$

$$\boxed{h[A,e] = \mathcal{P}e^{\int_e A}}$$



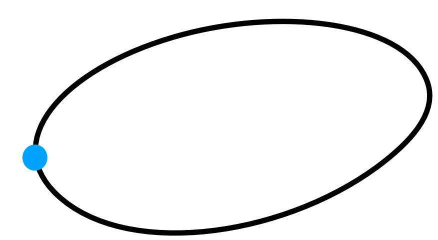

Under the gauge transformations

$$A \rightarrow A_g = g^{-1}dg + g^{-1}Ag$$

$$h[A,e] \rightarrow h[A_g,e] = g(e(0))h[A,e]g(e(1))^{-1}$$

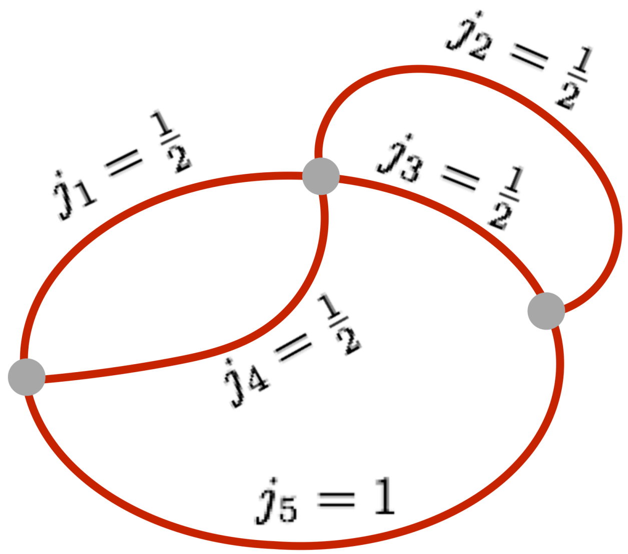
Gauge invariant objects > Wilson loops

$$\boxed{W[A,e] = \mathrm{tr}\left(h[A,e]\right)}$$



The key idea behind LQG is to built a Hilbert space of the theory out of the Wilson loops. However, such basis is in general over-complete and the solution comes with the constructions of **spin networks** which are certain linear combination of products of the Wilson loops.

# Spin Networks



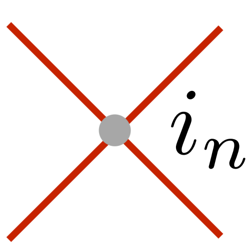

Graph: $\Gamma = \{\text{links } (l), \text{nodes } (n)\}$

Spin network state: $|\Psi\rangle = |\Gamma, j_l, i_n\rangle = \otimes_n |i_n\rangle$

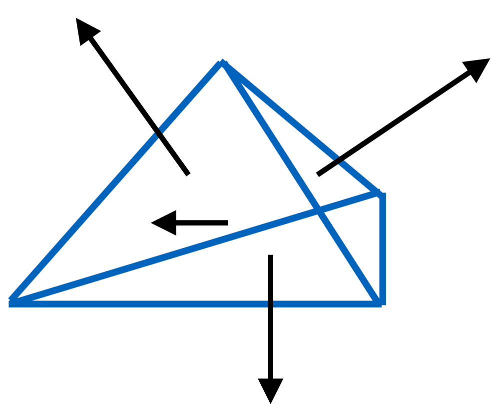
$j_l$ - spin labels $\qquad$ $i_n$ - intertwiners

**„Quantum of space"**

**Gauss constraint:**




=



$$(\hat{E}^1 + \hat{E}^2 + \hat{E}^3 + \hat{E}^4)|i_n\rangle = 0$$

# Qubit

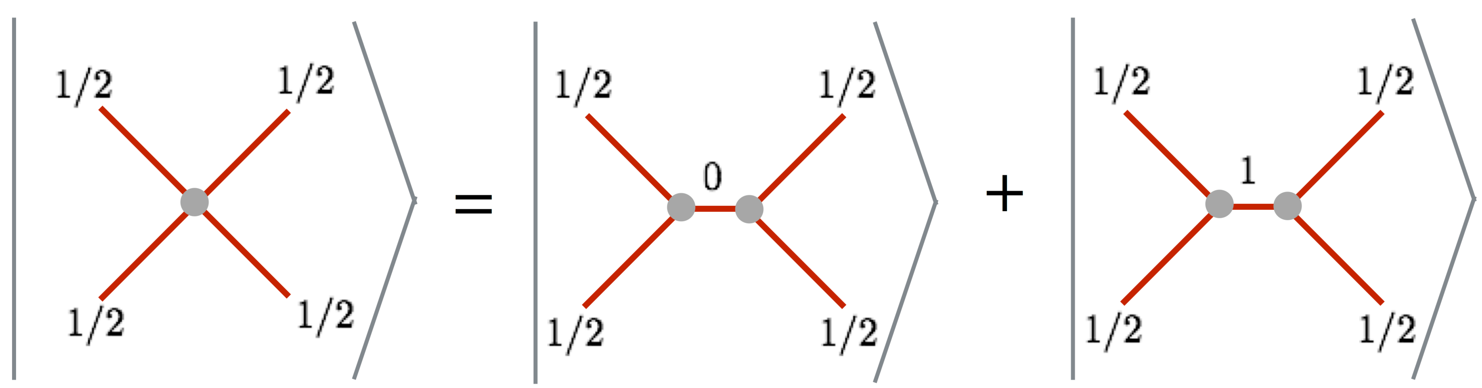
We consider spin networks composed of 4-valent vertices and spin labels corresponding to fundamental representations of the SU(2) group i.e. j=1/2. The Hilbert space at each vertex is:

$$H_{1/2} \otimes H_{1/2} \otimes H_{1/2} \otimes H_{1/2} = \underbrace{H_0 \oplus H_0} \oplus 3H_1 \oplus H_2$$

such that the invariant subspace is two dimensional:

$$\dim \operatorname{Inv}(H_{1/2} \otimes H_{1/2} \otimes H_{1/2} \otimes H_{1/2}) = 2$$

We associate the two dimensional invariant space with the **qubit space**:



# Qubit base states

s-channel basis:

$$|0_s\rangle = \frac{1}{2}\left(|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle\right)$$

$$|1_s\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle - \frac{|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle}{2}\right)$$

Eigenbasis of the volume operator $\hat{V}$:

$$|1\rangle = \frac{1}{\sqrt{2}}\left(|0_s\rangle - i|1_s\rangle\right) \qquad |0\rangle = \frac{1}{\sqrt{2}}\left(|0_s\rangle + i|1_s\rangle\right)$$

$\hat{V}|1\rangle = +V_0|1\rangle \quad \hat{V}|0\rangle = -V_0|0\rangle$ where $V_0 = \frac{\sqrt{3}}{4}l_{Pl}^3$

# Transition amplitude

In both classical and quantum gravity physical states (physical solutions) are obtained by solving the constraints. The physical states belong to the kernel of the constraints:

$$\boxed{\hat{C}|\Psi\rangle = 0}$$

such that $|\Psi\rangle \in \mathcal{H}_{phys} \subset \mathcal{H}_{kin}$

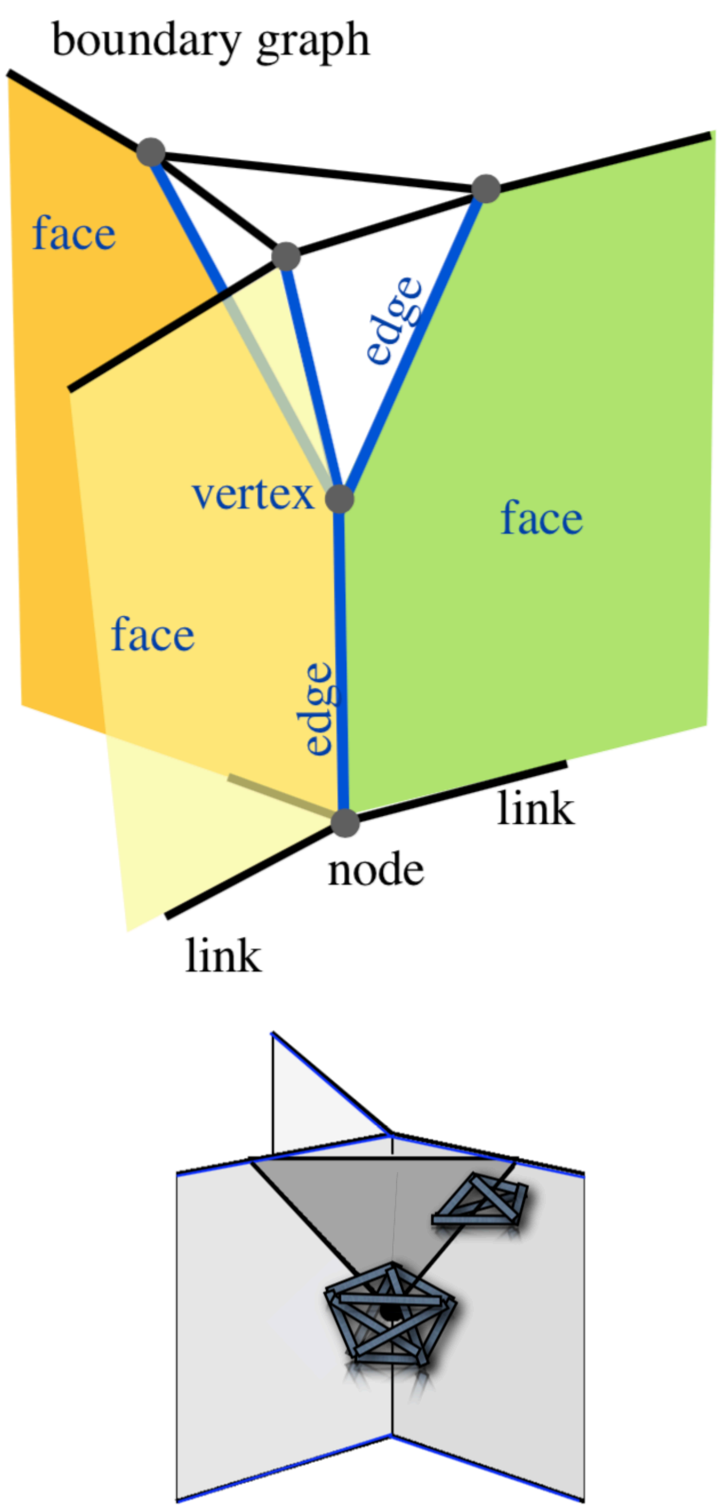
Therefore, transition amplitude between some boundary states is

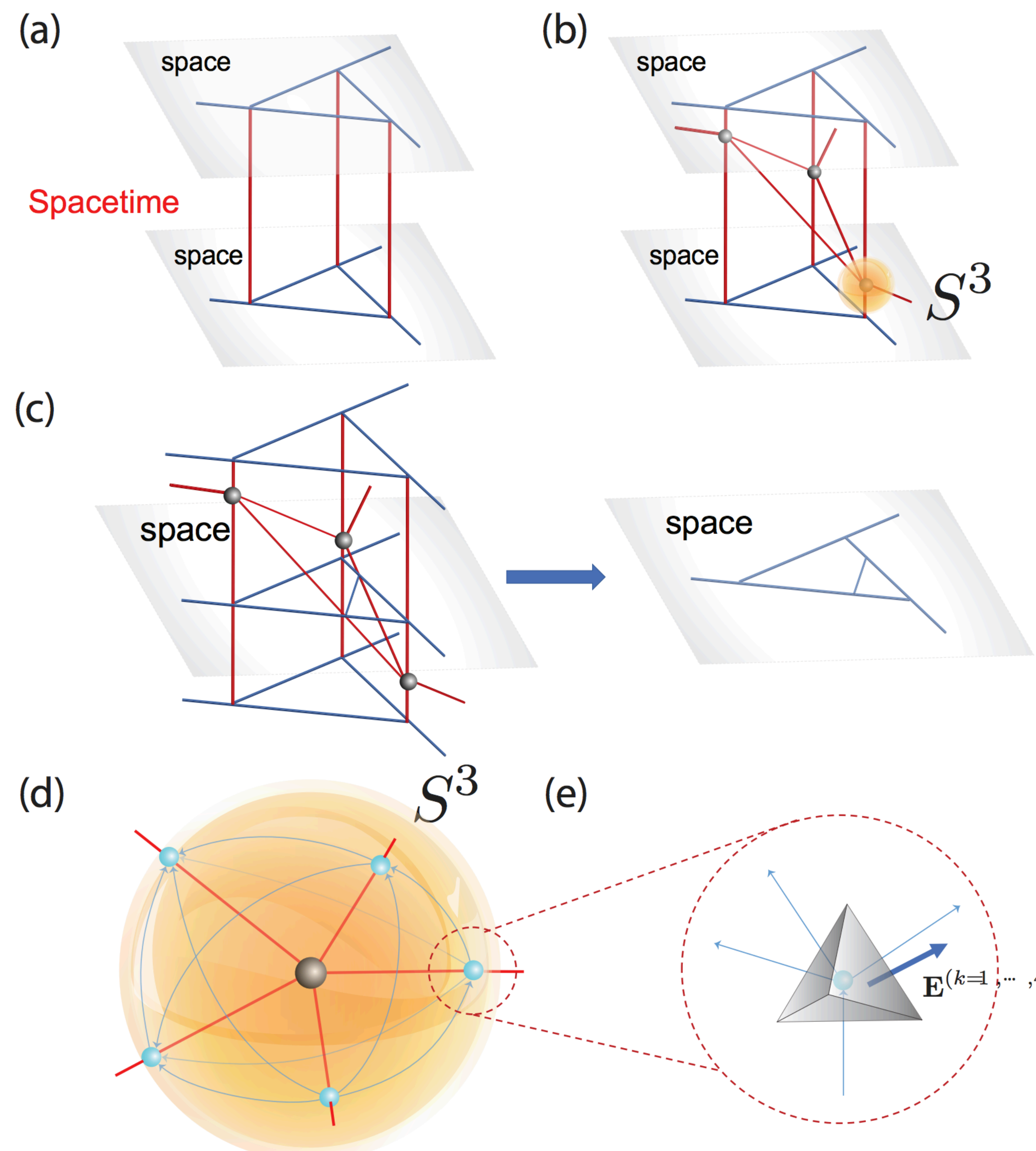
$$W(x, x') = \langle x'|\hat{P}|x\rangle$$

where the projection operator

$$\hat{P} = \delta(\hat{C}) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} d\tau e^{i\tau\hat{C}}$$

# Spin Foams



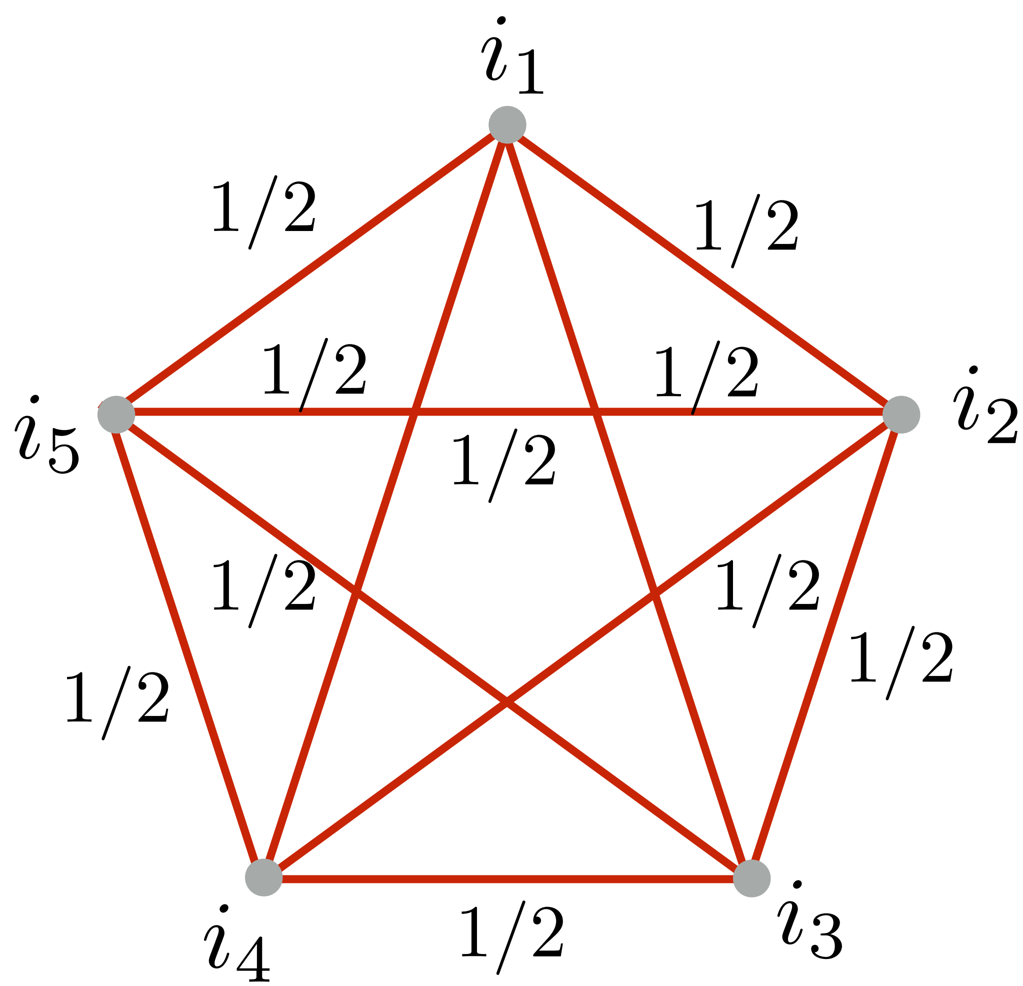

Rovelli & Vidotto, 2015

Keren Li, et al., arXiv: 1712.08711

# Vertex amplitude

**„Quantum of spacetime"**

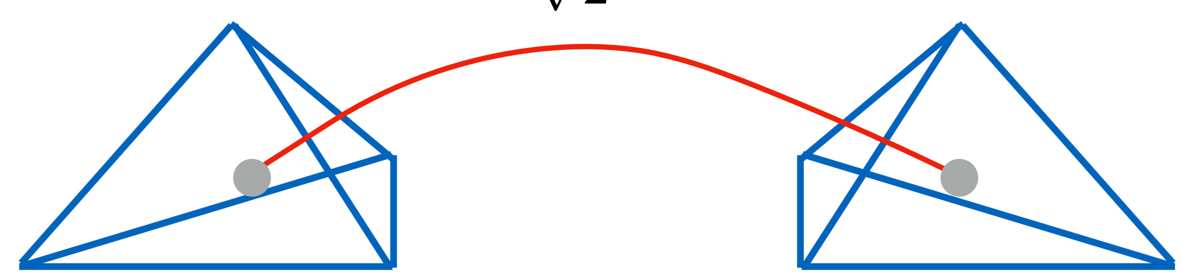
**- boundary state of a single vertex**




Spin network state: $|\Psi_1\rangle = \bigotimes_{n=1}^{5} |i_n\rangle$

Representation: $|\Psi_2\rangle = \bigotimes_{l=1}^{10} |\epsilon_l\rangle$

$|\epsilon_l\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ (Bell state)



Vertex amplitude: $\boxed{\langle\Psi_2|\Psi_1\rangle = A(i_1, i_2, i_3, i_4, i_5)}$

Can be computed with the use of quantum computer (in progress).

see: Keren Li, et al., arXiv: 1712.08711

But, we have to identify the states which satisfy the constraint $C \approx 0$ first…

How to compute $\left|\langle\Psi_2|\Psi_1\rangle\right|^2$ on quantum computer?

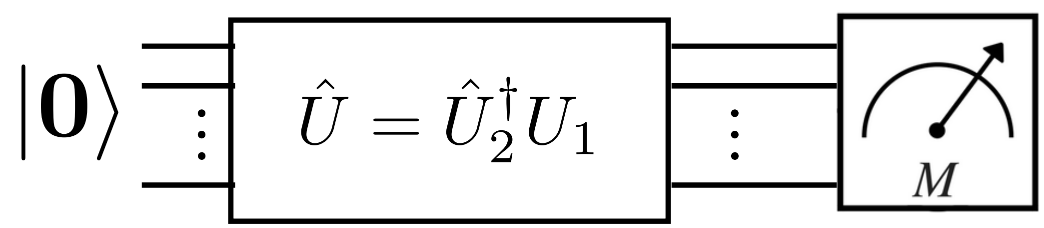
For N nodes of the spin network we need 4N qubits (20 qubits for the vertex amplitude).

The Hilbert space is spanned by $2^{4N}$ base states $|\mathbf{i}\rangle$ where $i \in \{0, 2^{4N}-1\}$

The initial state for quantum algorithm: $|\mathbf{0}\rangle = \underbrace{|\uparrow\rangle \otimes \cdots \otimes |\uparrow\rangle}_{4N}$

We have to find $\hat{U}_1$ and $\hat{U}_2$ such that: $|\Psi_1\rangle = \hat{U}_1|\mathbf{0}\rangle$ and $|\Psi_2\rangle = \hat{U}_2|\mathbf{0}\rangle$

Our quantum algorithm is defined by the unitary operator (quantum circuit):




$\hat{U} = \hat{U}_2^\dagger U_1$ such that

$$\hat{U}|\mathbf{0}\rangle = \sum_{i=0}^{2^{4N}-1} a_i|i\rangle$$

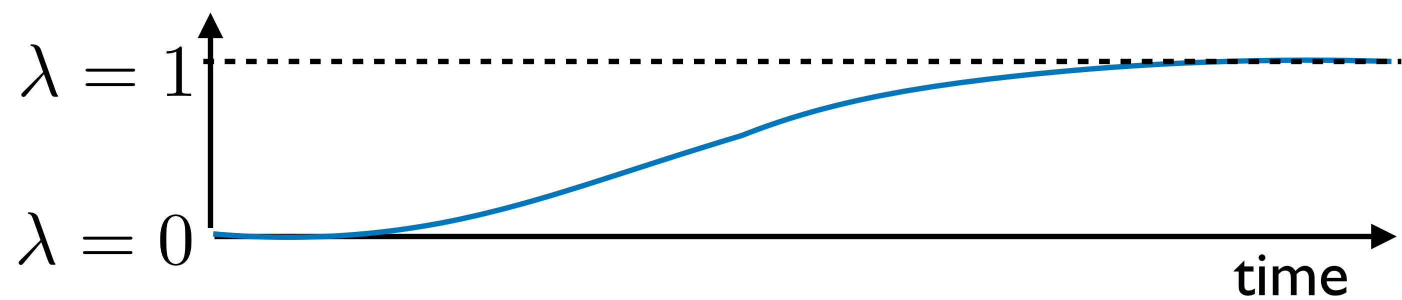
We measure: $P(0) = |a_0|^2$

where $a_0 = \langle\mathbf{0}|\hat{U}|\mathbf{0}\rangle = \langle\Psi_2|\Psi_1\rangle$

# Solving the constraint with the Adiabatic Quantum Computer

The adiabatic quantum computers are designed to solve a specific problem of finding minimum of a Hamiltonian $H_I$ of a coupled system of qubits (spins). In the process of finding the minimum of $H_I$ one employs a time dependent Hamiltonian in the form:

$$H(\lambda) = (1-\lambda)H_B + \lambda H_I$$




where $H_B$ is the so-called base Hamiltonian characterized by a simple and easy to prepare ground state. In practice, the base Hamiltonian is often equal

$$H_B = \sum_I \sigma_i^x$$

such that the ground state corresponds to the alignment of spins in the x direction. Then, the value of $\lambda$ is changed adiabatically from $\lambda = 0$ to $\lambda = 1$, such that while the system is initially in non-degenerate ground state it will remind in the ground state during the process.

Let us observe that, the problem finding solution to the constraint:

$$C \approx 0$$

can be mapped into the problem of finding minimum of the Hamiltonian:

$$H_I \propto C^2$$

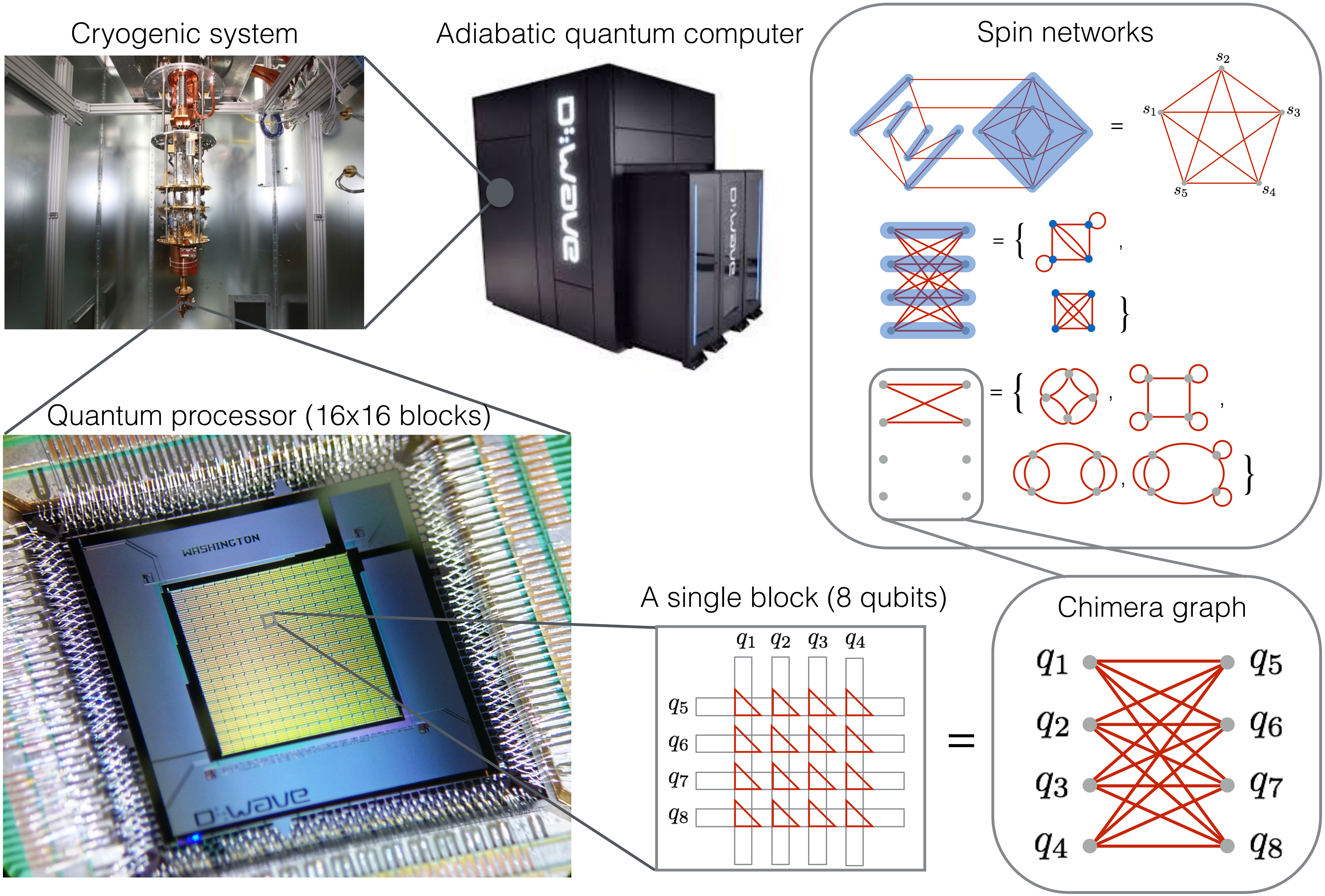
In such case the Hamiltonian is bounded from below and at the ground state the constrain is satisfied.

In physical implementations the most considered form of the base Hamiltonian corresponds to the Ising problem:

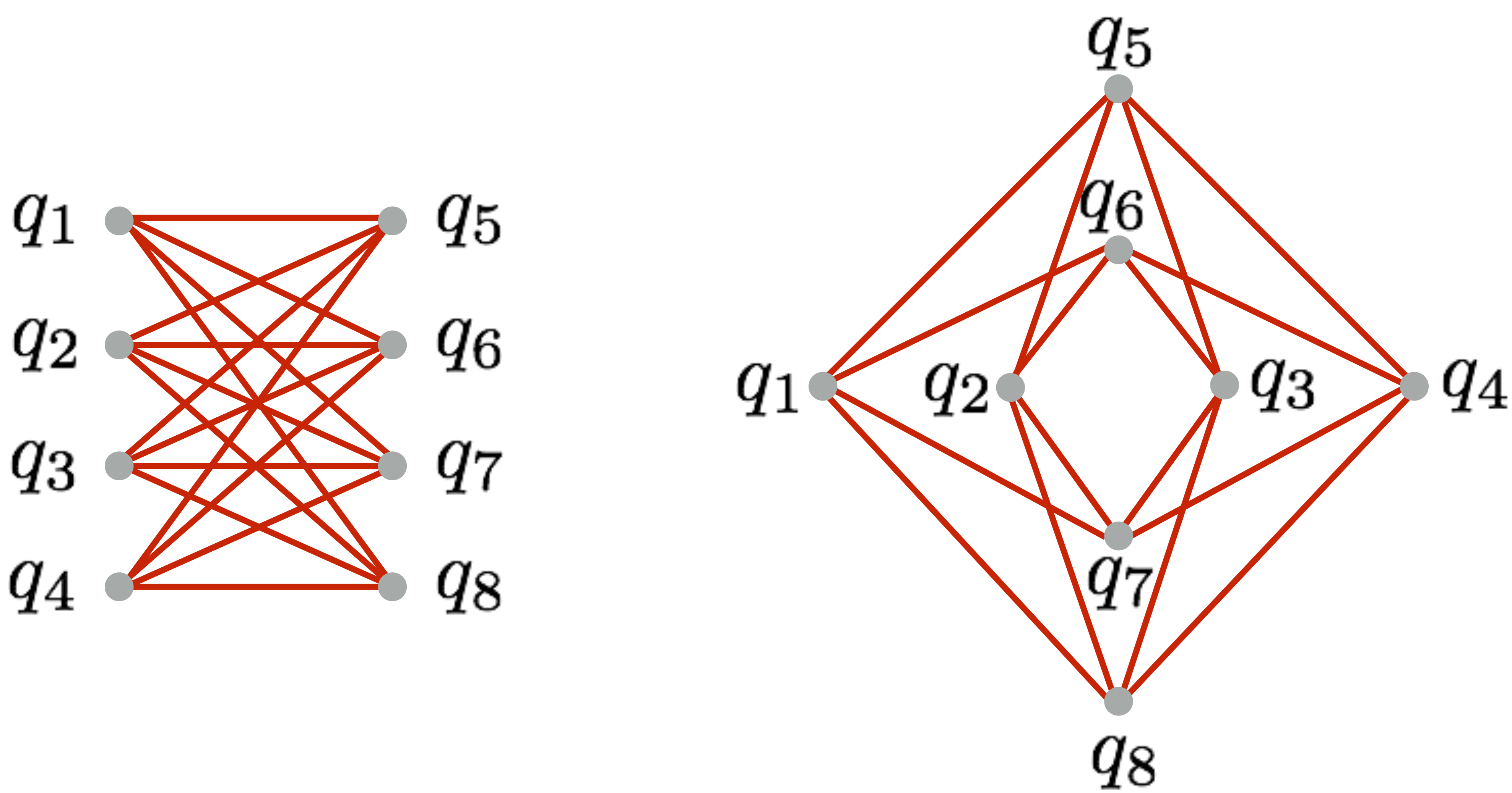
$$H_I = \sum_{<i,j>} b_{ij} \sigma_i^z \sigma_i^z + \sum_i h_i \sigma_i^z$$

$b_{ij}$ and $h_i$ are couplers. NP-hard problem in general.

# Spin networks on adiabatic quantum computer, JM, arXiv:1801.06017



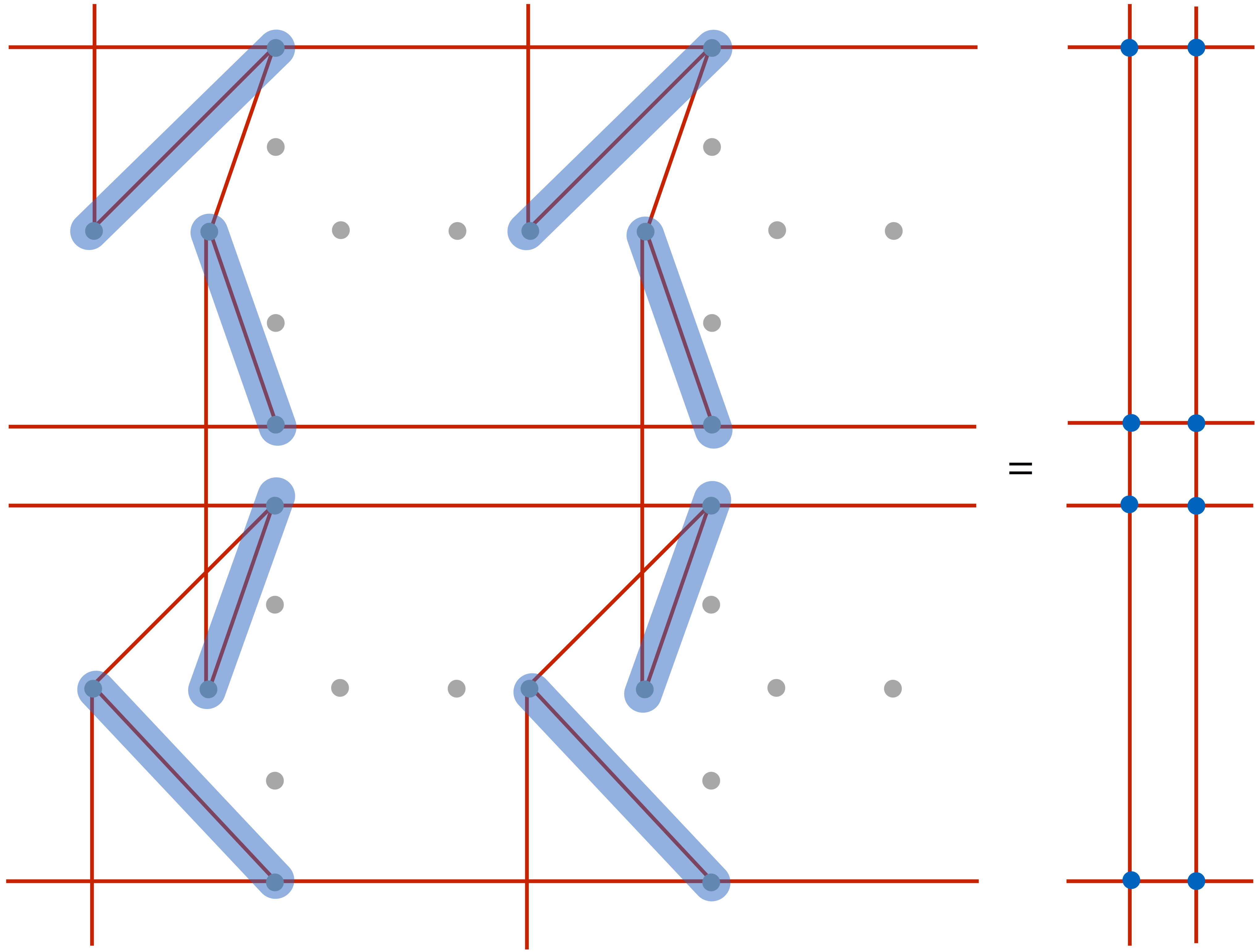
# Chimera graph



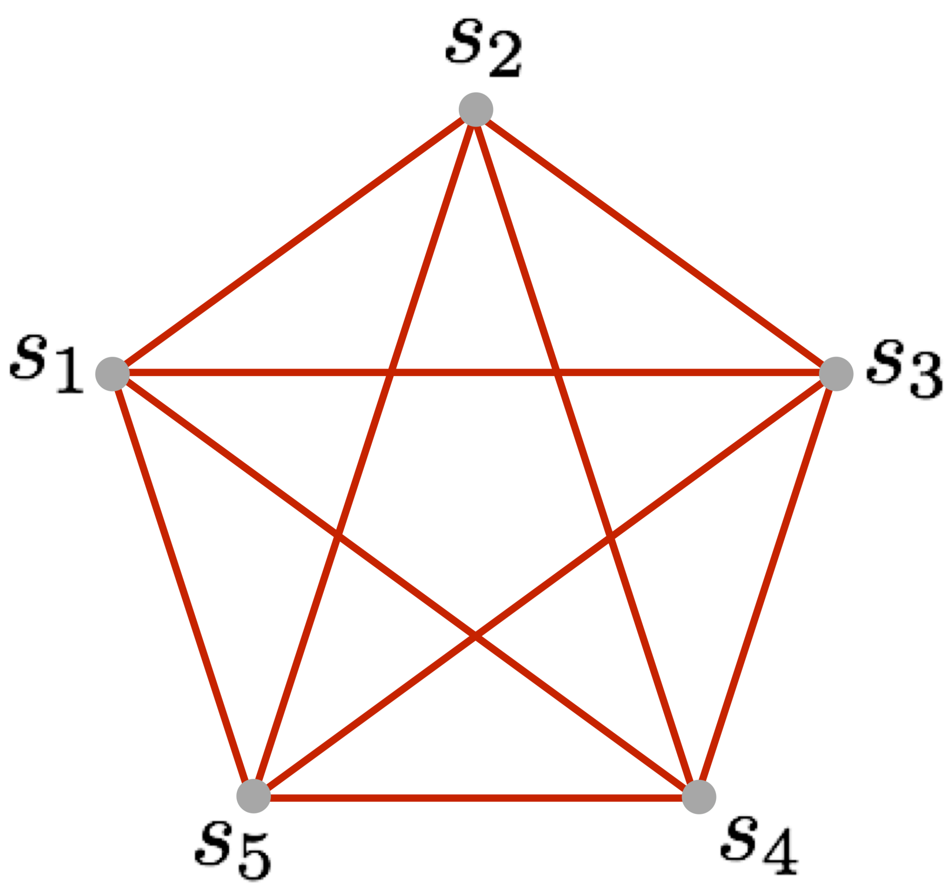

Elementary 8-qubit block of the quantum chip.

D-Wave 2000 - a matrix 16x16=256 of 8-qubit blocks (2048 qubits in total).

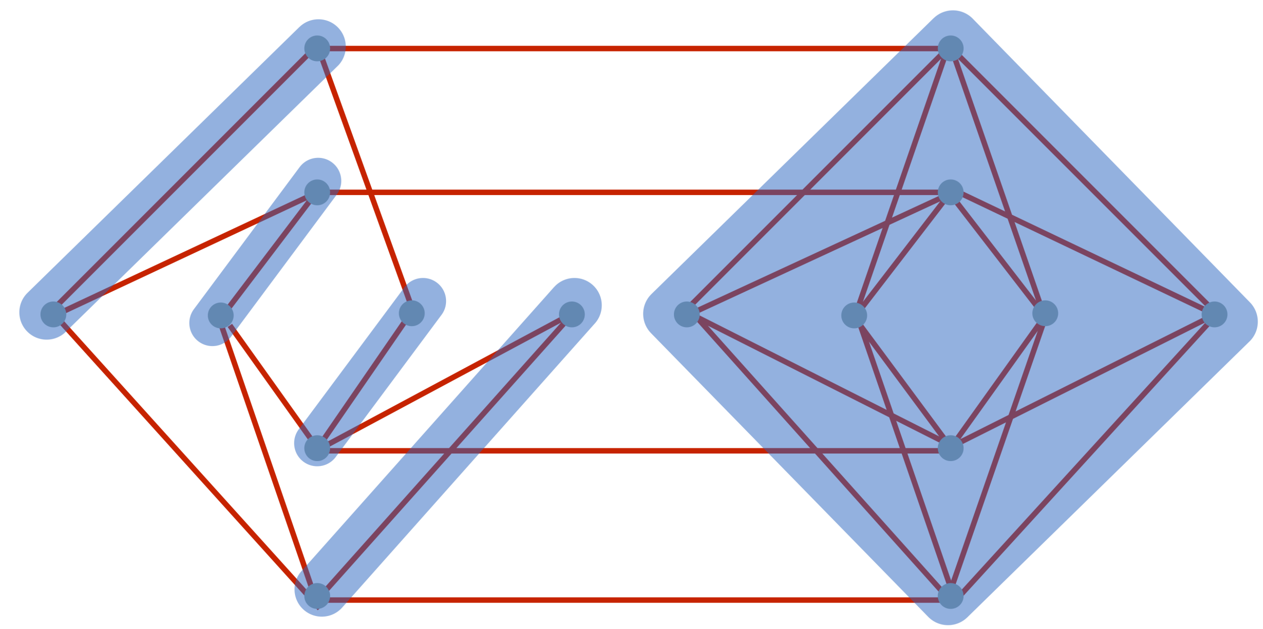
# Regular lattice spin network



# Pentagram spin network




Physical implementation of the spin network:



$$s_i \in \{-1, 1\}$$

Prototype constraint: $C = \sum_{i=1}^{N} s_i - c \approx 0$

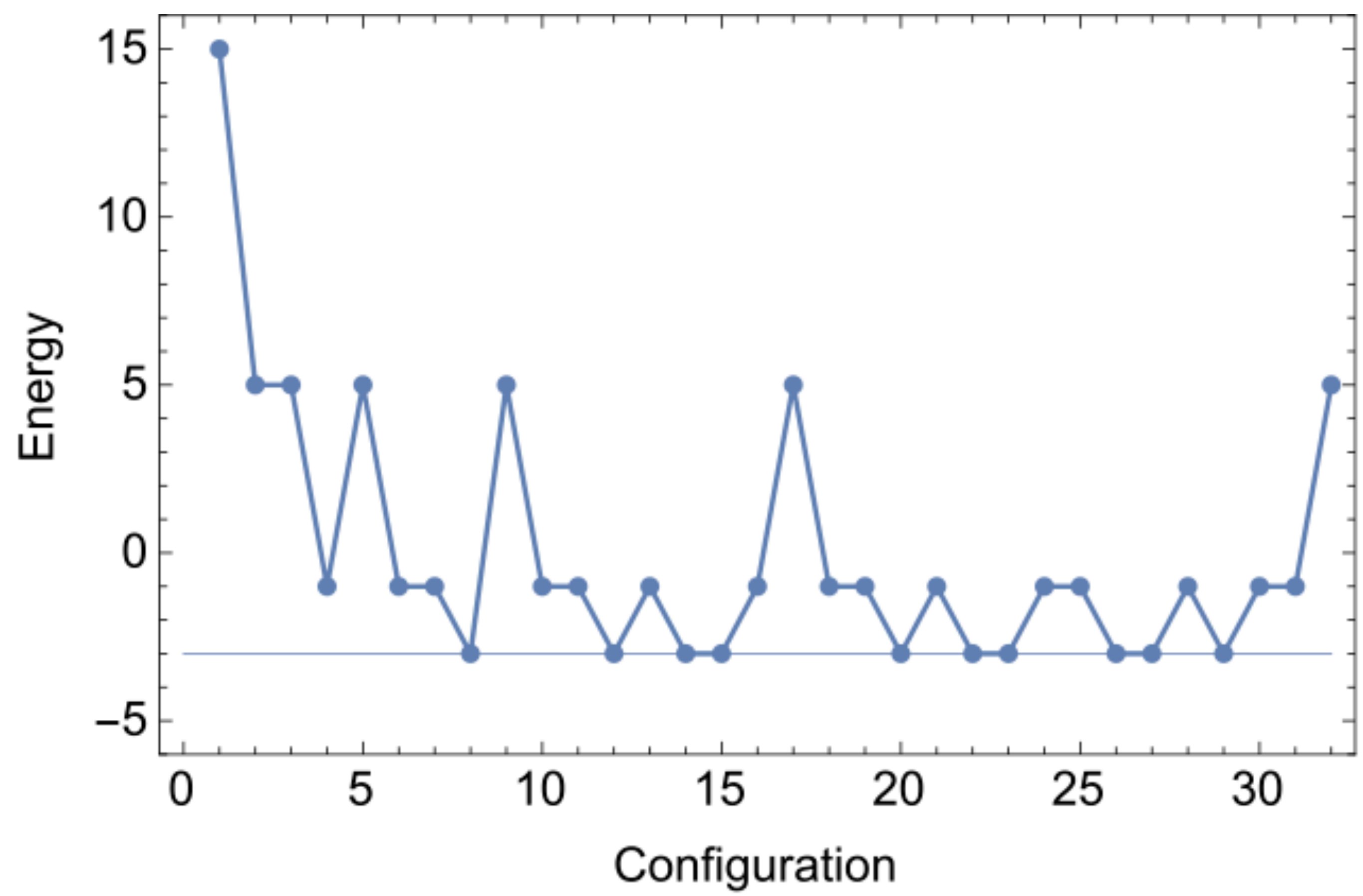
$$N = 5$$

$$\boxed{H = \frac{C^2 - N - c^2}{2} = \sum_{<i,j>}^{N} s_i s_j + h \sum_{i=1}^{N} s_i}$$

where $h := -c$

# Energy landscape

Dimension of the kinematical space: $\dim \Gamma_{\text{kin}} = 2^5 = 32$




Dimension of the physical space: $\dim \Gamma_{\text{phys}} = \begin{pmatrix} 5 \\ \frac{5-1}{2} \end{pmatrix} = 10$

$\Gamma_{\text{phys}} \subset \Gamma_{\text{kin}}$

The ground states of $H$ are the physical states of the theory with $C \approx 0$

# Outlook

Evaluation of the vertex amplitudes utilizing quantum algorithms.

A lot can be done using available simulators of quantum computers.

More general spin networks.

Application to spin foam cosmology.

Analysis of quantum states of BH horizons.

Extraction of physical states adopting Quantum Phase Estimation Algorithm to the projection operator.

Simulating gravity using gauge/gravity duality.

New architectures to be explored.

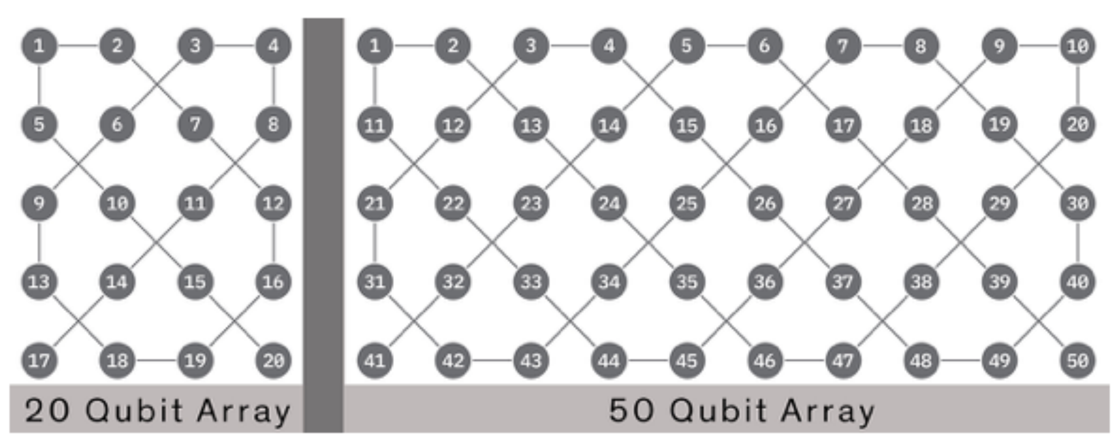
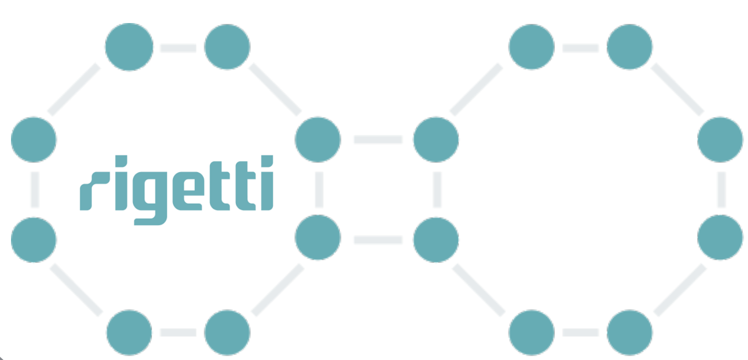



Illustration: IBM




128-qubit

Simulations of mesoscopic Planck scale systems on adiabatic quantum computers.