Null infinity, Radiation Fields and the BMS Group: Some Surprises

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Organization

1. Maxwell Theory
2. Non-linear GR: Classical Aspects
   - Radiative Modes & the enlargement of the Poincaré group to the BMS
   - Symplectic Geometry of Radiative modes and BMS fluxes across $\mathcal{I}$
3. Non-linear GR: Quantum Aspects
   - Emergence of gravitons in asymptotic states
   - Infrared sectors, recent developments and open Issues: Illustrations
4. Summary

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Preamble

• Radiation fields at null infinity were analyzed extensively in the literature starting from the GR 3 conference in Warsaw, already in the 1960s and 70s. But surprising results have emerged periodically since then both at the classical and quantum level. For example:

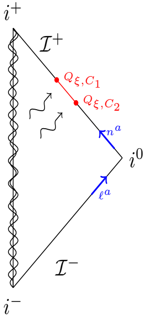
(i) the enlargement of the Poincaré group $\mathcal{P}$ to the BMS $\mathcal{B}$ was traced directly to the fact that, in presence of gravitational waves, the *classical* vacuum degeneracy cannot be ignored at $\mathcal{I}$;
(ii) Quantum asymptotic states admit a precise interpretation in terms of gravitons without having to linearize the theory or carry out perturbative expansions;
(iii) Perturbative S-matrix theory suggests new conservation laws in gravitational scattering for non-linear classical general relativity; and,
(iv) Even in the Maxwell theory in Minkowski space-time, there are subtleties in the notion of angular momentum carried by electromagnetic waves across $\mathcal{I}$!

• Because diversity of the audience I will present a broad overview, without entering into detailed proofs. At the end, I will provide a list of references where these details can be found. Because larger part of the audience focuses on classical GR, I will spend more time on classical aspects and be briefer on quantum aspects.

• Perhaps the most striking aspect of these developments is the rich interplay between geometry and different aspects of physics.

1. Maxwell Fields & Null infinity $\mathcal{I}$

• To analyze radiation, it is convenient to use the $u := t - r$, r, θ, φ coordinates and go to infinity $(r \to \infty)$ along $u = u_0$. Limiting fields evaluated at the null boundary $\mathcal{I}$ coordinatized by u, θ, φ (and $r = \infty$, or, $\Omega = 1/r = 0$).



• Newman-Penrose Components the Maxwell field $\mathbf{F}_{ab}$ in Minkowski space make the asymptotic behavior transparent:

$\mathbf{F}_{ab} n^a \bar{m}^b =: \Phi_2 \equiv \frac{\Phi_2^\circ(u,\theta,\varphi)}{r} + (\frac{1}{r^2});$

$\frac{1}{2}\mathbf{F}_{ab}(n^a \ell^b + m^a \bar{m}^b) =: \Phi_1 \equiv \frac{\Phi_1^\circ(u,\theta,\varphi)}{r^2} + (\frac{1}{r^3})$

$\mathbf{F}_{ab} m^a \ell^b =: \Phi_0 \equiv \frac{\Phi_0^\circ(u,\theta,\varphi)}{r^3} + (\frac{1}{r^4})$

• Natural home for $\Phi_2^\circ, \Phi_1^\circ, \Phi_0^\circ$ is $\mathcal{I}$. Maxwell's equations $\Rightarrow$ Φ_2° is freely specifiable. But $\dot{\Phi}_1^\circ(u,\theta,\varphi) = \eth\Phi_2^\circ(u,\theta,\varphi)$; and $\dot{\Phi}_0^\circ(u,\theta,\varphi) = \eth\Phi_1^\circ(u,\theta,\varphi)$. So, given $\Phi_2^\circ(u,\theta,\varphi)$ specifying intitial values $\Phi_1^\circ(u_0,\theta,\varphi)$, $\Phi_0^\circ(u_0,\theta,\varphi)$ at any $u = u_0$ determine them everywhere. Here $\eth$ is an angular derivative.

• Φ_2° is the radiation field. For example, energy-flux across the 2-sphere the cross-section C_1 of $\mathcal{I}$ $(u = u_1, r = \infty)$ is given by $\oint_{C_1} |\Phi_2^\circ|^2 \, \mathrm{d}^2 S$.
Φ_1° captures Coulombic aspects: $Q + iQ^\star = -\frac{1}{2\pi} \oint \Phi_1^\circ \mathrm{d}^2 S$.

Vector Potential A_a

- In non-Abelian gauge theory (and general relativity) equations satisfied by asymptotic curvature fields depend on the connection. So radiative modes are best isolated using the connection A_a (with, e.g., $A_a n^a \hat{=} 0$ and $A_a \to 0$ at i°).

- A_a is thus a transverse 1-form defined intrinsically on $\mathcal{I}$, whence it has only 2-components, captured in the complex function $A_2 = A_a m^a$. These are the two radiative modes of the Maxwell field. They determine three of the six components of the asymptotic Maxwell field:

$$\Phi_2^0 = \sqrt{2}\dot{A}_2, \qquad \mathrm{Im}\Phi_1^0 = \sqrt{2}\mathrm{Im}\eth A_2$$

But they do *not* capture $\mathrm{Re}\Phi_1^0$, i.e. the Coulombic aspect, just as one would expect of radiative modes. In particular, $\mathrm{Re}\Phi_1^0 - \sqrt{2}\mathrm{Re}\eth A_2^0 = G(\theta, \varphi)$ where $G(\theta, \varphi)$ is determined by sub-leading part of A_a: To get the electric charge we need G: $Q = -(1/2\pi) \oint_C G(\theta, \varphi) \mathrm{d}^2 S$.

- Phase space of radiative modes $\mathbf{\Gamma}$: A_a (or A_2) at $\mathcal{I}$ determine a source-free solution of Maxwell's equation in space-time. So, the symplectic structure on the covariant phase space Γ_{cov} of source-free solutions, induces a symplectic structure on the space Γ of radiative modes: $\Omega(A, \tilde{A}) = \int_{\mathcal{I}} \mathrm{d}^3\mathcal{I}\, (A_a \mathcal{L}_n \tilde{A}_b - \tilde{A}_a \mathcal{L}_n A_b)\, q^{ab}$. Provides fluxes of Poincaré momentum carried by electromagnetic waves as well as a spring board for constructing asymptotic states for quantum S-matrix.

• If ξ^a is the restriction to $\mathcal{I}$ of any Minkowski Killing field , $A_a \to \mathcal{L}_\xi A_a$ is an infinitesimal canonical transformation on $\mathbf{\Gamma}$, generated by the Hamiltonian $H_\xi(A) = (1/2)\, \Omega(\mathcal{L}_\xi A, A)$. For source-free solutions in Minkowski space, we have $H_\xi = F_\xi := \int_{\mathcal{I}} T_{ab}\xi^a \, \mathrm{d}S^b$, where F_ξ is the flux of that component of the Poincaré momentum across $\mathcal{I}$ (i.e. energy-momentum & angular momentum). Note: H_ξ involves *only* the radiative modes $A_a \sim A_2$ on $\mathcal{I}$.

• Quantization: One photon Hilbert space can be constructed directly at $\mathcal{I}$ using the positive and negative frequency decomposition (or, the complex structure) selected by the translation subgroup $\mathcal{T}$ of $\mathcal{P}$ on $\mathcal{I}$. Again 1-photon states refer only the radiative modes $A_a \sim A_2$ on $\mathcal{I}$. Fluxes now represented by operators on the photon Fock space: $\hat{F}_\xi = \hat{H}_\xi = (1/2)\, \Omega(\mathcal{L}_\xi \hat{A}, \hat{A})$, with standard normal ordering. Description equivalent to Fock quantization in Minkowski space. But no space-time Fourier transform involved –works in any asymptotically Minkowskian space.

• Standard assumption: This framework correctly captures physics even in presence of sources (that stay away from $\mathcal{I}$): The Poincaré momenta carried by electromagnetic waves at $\mathcal{I}$ depends only on the radiative modes at $\mathcal{I}$ and not on what sources produced them.

Surprise in the Maxwell Theory!

• Expectation borne out for energy-momentum. A Minkowski translational Killing field ξ^a has the form $\xi^a = \alpha n^a$ at $\mathcal{I}$ (for a specific 4-parameter family of α.) The Hamiltonian H_ξ on the radiative phase space captures the flux of 4-momentum carried by electromagnetic fields correctly even when sources are present:

$$F_{\alpha n}[C] := \oint_C T_{ab}\alpha n^a\, n^b \mathrm{d}^2 S = \oint_C \alpha\, |\mathcal{L}_n A_a|^2$$

Energy-momentum carried by electromagnetic waves across any cross-section C of $\mathcal{I}$ is fully captured in the radiative modes, even in presence of sources.

• Surprise: This is not generally true for angular momentum!
On $\mathcal{I}$, each rotational Killing field is of the type $R^a = R(\theta,\varphi)m^a + \mathrm{CC}$, with $\bar{\eth} R(\theta,\varphi) = 0$. Angular momentum flux:

$$F_R[C] = \sqrt{2}\oint_C \mathrm{Re}(\bar{\Phi}_2^0 \Phi_1^0\, R)\ \mathrm{d}^2 S = 2\oint_C \mathrm{Re}[(\dot{A}_2^0)(\sqrt{2}\eth A_2^0 + G)\ R\ \mathrm{d}^2 S]\,.$$

In presence of sources, $\mathcal{F}_R[C]$ cannot be expressed using just the radiative modes $A_a \sim A_2$! One also needs the extra 'Coulmbic' information contained in $G(\theta,\varphi)$. There is an interesting but subtle interplay between radiative and Coulombic aspects of the asymptotic Maxwell field. (AA, Bonga; Bonga,Poisson, Yang)

The difference between energy-momentum and angular momentum occurs can be traced back directly to the difference in the asymptotic behavior of translations and rotations. Translations $\sim r^0$ while rotations$\sim r$. So while energy momentum can 'feel' only $\Phi_2^\circ \sim 1/r$, angular momentum can also 'feel' $\Phi_1^\circ \sim 1/r^2$.

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2. Non-linear GR: Classical aspects

• Let us now consider on Asymptotically Minkowski (AM) space-times that admit a complete $\mathcal{I}$. Universal structure: Structure shared by all AM space-times.
$\mathcal{I}$ is a 3-manifold, topologically $\mathbb{S}^2 \times \mathbb{R}$, equipped with equivalence classes of pairs (q_{ab}, n^a) such that:
(i) n^a is the null normal to $\mathcal{I}$, and q_{ab} the intrinsic metric q_{ab} a (0,+,+) metric;
(ii) $q_{ab}n^a = 0$, and $\mathcal{L}_n\, q_{ab} \hat{=} 0$; and
(iii) $(q_{ab}, n^a) \approx (\omega^2\, q_{ab},\ \omega^{-1} n^a)$, where ω is nowhere zero on $\mathcal{I}$ and $\mathcal{L}_n\, \omega = 0$.

• BMS group $\mathcal{B}$: Diffeos on $\mathcal{I}$ that preserve the universal structure.
Infinitesimal level: a VF ξ^a on $\mathcal{I}$ is a BMS symmetry iff

$$\mathcal{L}_\xi q_{ab} \hat{=} 2f q_{ab},\ \mathcal{L}_\xi n^a \hat{=} -f n^a, \quad \text{where } \mathcal{L}_n f \hat{=} 0.$$

Lie ideal of Supertranslations: $\xi^a = \alpha n^a$. $[\alpha_1 n, \xi]^a = \alpha_2\, n^a$. So, we have a normal subgroup $\mathcal{S}$ of $\mathcal{B}$. Next, the 'base space' of $\mathcal{I}$ is $\mathbb{S}^2$ and $\mathbb{S}^2$ has a unique conformal structure. The conformal isometry group the Lorentz group $\mathcal{L}$. Hence $\mathcal{B} = \mathcal{S} \rtimes \mathcal{L}$. $\mathcal{B}$ is an infinite dimensional generalization of the Poincaré group $\mathcal{P}$.

• Finally, $\mathcal{B}$ admits an **unique** 4-d normal subgroup $\mathcal{T} \subset \mathcal{S}$ of Translations (Sachs). Used to define Bondi 4-momentum.

(Trautman, Bondi et al, Sachs, Newman & Penrose)

Radiative Modes

• How is the information of gravitational waves in full GR encoded at $\mathcal{I}$? The universal structure is common to all space-times, stationary as well as radiative. So, it serves as a 'kinematic background' (rather like the Minkowski metric for Maxwell fields). Radiative information is encoded in the "next order structure" which is not universal:

• From a geometric viewpoint, the 'next order structure' is a connection. Indeed, the connection ∇ on (M, g_{ab}) induces a natural intrinsic connection D on the 3-manifold $\mathcal{I}$, satisfying $D_a q_{bc} \hat{=} 0$ and $D_a n^b \hat{=} 0$. Because q_{ab} is degenerate, there are infinitely many operators satisfying this condition. Each conformal completion of a physical space-time of interest selects one D on $\mathcal{I}$.

• However, for a physical space-time there is conformal freedom in choice of g_{ab}. under $g_{ab} \to g'_{ab} = \omega^2 g_{ab}$, *even with* $\omega \hat{=} 1$, we have $D \to D' \neq D$. Thus, (for any given conformal fame (q_{ab}, n^a) at $\mathcal{I}$) physical space-time of interest provides us with a conformal class of connections $\{D\}$.

• $\{D\}$ represents the radiative modes of the full, non-linear gravitational field represented by the physical space-time. $\{D\} - \{D\}' \sim s_{ab}$, a Transverse, Traceless (TT) tensor field on $\mathcal{I}$: 2 radiative DOF.

connections, just as in gauge theories: $(s_{ab} \sim A_a; \;\; s_{22} = s_{ab} m^a m^b \sim A_2 = A_a m^a.)$

Classical 'Vacua'

• Curvature of D: Radiation fields at $\mathcal{I}$. $\{D\}$ determines $^\star K^{ac} = \lim \Omega^{-1}\, {}^\star C^{abcd} n_b n_d$ at $\mathcal{I}$ which encodes 5 of the 10 components of asymptotic Weyl curvature, namely $\Psi_4^\circ, \Psi_3^\circ, \mathrm{Im}\Psi_2^\circ$. It *does not* determine $\mathrm{Re}\Psi_2^\circ$ that captures the Coulombic aspects. (Recall the radiative modes in A_a determines Φ_2°, $\mathrm{Im}\Phi_1^\circ$ but not $\mathrm{Re}\Phi_1^\circ$.)

• Following the terminology commonly used in Yang-Mills theory, $\{D\}$ is said to a represent a classical vacuum, denoted $\{\mathring{D}\}$, if $^\star K^{ab} = 0$. (Note: this implies that the Bondi news tensor $N_{ab} = 0$ but not vice versa.) $^\star K^{ab} = 0 \Leftrightarrow$ there is no gravitational radiation. How many vacua are there?

• Surprisingly, there is an infinite vacuum degeneracy! Each vacuum $\{\mathring{D}\}$ is left invariant by the 4-d translation subgroup $\mathcal{T}$ of $\mathcal{B}$ but not by any of the rest of 'pure' supertranslations. In fact the quotient $\mathcal{S}/\mathcal{T}$ acts simply and transitively on the space $\mathbf{V}$ of of classical vacua. Furthermore $\mathbf{V}$ is naturally isomorphic to the space $\mathbf{P}$ of all Poincaré subgroups of B: Each $\{\mathring{D}\}$ is left invariant by precisely one Poincaré subgroup $\mathcal{P}$ of the BMS group $\mathcal{B}$. In presence of radiation, a general $\{D\}$ tends to distinct $\{\mathring{D}\}^\pm$ as $u \to \pm\infty$: The enlargement of $\mathcal{P}$ to $\mathcal{B}$ can be directly traced back to the presence of gravitational waves!

2.B. Phase space of radiative modes

- The phase space Γ of radiative modes of the gravitational field is the affine space of all gravitational connections $\{D\}$ on $\mathcal{I}$ whose curvature falls off appropriately at the two ends (i^+ and i^o for $\mathcal{I}^+$, and i^o and i^- on $\mathcal{I}^-$).

- The symplectic structure Ω on Γ can be derived *formally* from the covariant phase space of vacuum solutions $\tilde{g}_{ab}$ to Einstein's equations on $\tilde{M}$ (AA & Magnon). Tangent vectors at any $\{D\} \in \Gamma$ are represented by $\delta\{D\} \equiv \sigma_{ab}$, which are symmetric, transverse, traceless tensor fields $\sigma_{ab} \hat{=} \sigma_{(ab)}; \quad \sigma_{ab} n^a \hat{=} 0; \quad \text{and} \quad \sigma_{ab} q^{ab} \hat{=} 0.$ The symplectic structure is then given by

$$\Omega\,|_{\{D\}}\ (\delta\{D\}, \delta'\{D\}) = \frac{1}{8\pi G} \int_{\mathcal{I}} \left(\sigma_{ab}(\mathcal{L}_n \sigma'_{cd}) q^{ac} q^{bd} - \sigma \leftrightarrow \sigma'\right) \mathrm{d}^3\mathcal{I}.$$

- Action of the BMS group on $\mathcal{I}$ preserves Ω. So, associated with each BMS generator ξ^a, we acquire a Hamiltonian function on Γ (momentum map):

$$H_\xi(\{D\}) = \frac{1}{16\pi G} \int_{\mathcal{I}} \left(N_{ab}\, [\mathcal{L}_\xi D_a - D_a \mathcal{L}_\xi] \ell_d \ q^{ac} q^{bd}\right) \mathrm{d}^3\mathcal{I},$$

where ℓ_d is any 1-form with $\ell_d n^d \hat{=} 1$ (and for simplicity we restricted ourselves to ξ^a s.t. $\mathcal{L}_\xi q_{ab} \hat{=} 0$.) Interpretation: H_ξ is the flux of the 'ξ^a component' of the BMS momentum across $\mathcal{I}$. (AA, AA & Streubel) Symplectic methods enable us to arrive at the flux formulas in full non-linear GR where there is no stress-energy tensor for the gravitational field.

Fluxes of supermomenta across $\mathcal{I}$

- For a supertranslation $\xi^a = \alpha\, n^a$, $(\alpha \equiv \alpha(\theta,\phi))$ the flux is given by (AA & Streubel):

$$H_\alpha(\{D\}) = \frac{1}{32\pi G}\int_{\mathcal{I}} \big(\alpha N_{ab} + D_a D_b \alpha\big) N^{ab}\, \mathrm{d}^3\mathcal{I} \equiv H_\alpha^{\rm hard} + H_\alpha^{\rm soft}$$

(in a Bondi conformal frame). The quadratic term in N_{ab}: 'hard part'; linear term: 'soft part' of the supermomentum flux. The soft part vanishes for translations, i.e. for the flux of Bondi-Sachs 4-momentum. The early literature incorrectly assumed that the general supermomentum flux H_α only has 'hard part'! The 'soft part' arises because: (i) $\mathbf{\Gamma}$ is an affine –rather than a vector– space; and, (ii) Classical vacua fail to be invariant under "pure" supertranslations.

- The 'soft part' plays a key role in the gravitational infrared properties. If $\{D\} \to \{\mathring{D}\}^{\pm}$, then $H_\alpha^{\rm soft}(\{D\}) \neq 0$ iff $\{\mathring{D}\}^+ \neq \{\mathring{D}\}^-$, or, iff the difference in the asymptotic shears of $\{D\}$ is non-zero: $[\sigma] := \sigma_{ab}(i^+) - \sigma_{ab}(i^\circ) \neq 0$. Thus, in the classical theory, it captures gravitational memory. In full non-linear GR and generic asymptotically flat situations (pure gravitational waves; sources that scatter; compact binary mergers) one expects that $[\sigma] \neq 0 \Leftrightarrow H_\alpha^{\rm soft} \neq 0$.

Energy-momentum

- For BMS translations, $H_\xi(\{D\})$ only contains the quadratic hard piece. For a BMS time translation, $\alpha > 0$ and the flux of energy is manifestly positive. Historically this property played a key role in establishing the reality of gravitational waves by Bondi (in GR3 at Warsaw!).

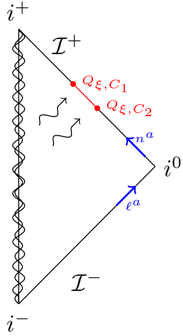
- Fluxes are functions on radiative phase space. As one would expect, BMS momenta themselves require 'Coulombic information' that is not in the radiative modes. But given any one space-time, using Einstein equations we can systematically 'integrate' the flux to obtain a 2-sphere integral representing BMS energy-momentum associated with the BMS translation $\xi^a = \alpha n^a$

$$P^{\rm Bondi}_{(\alpha)}[C] = \frac{1}{8\pi G} \oint_C \left(\alpha {\rm Re}\Psi_2^o + \tfrac{1}{2}\alpha (D_a \ell_b)\, N^{ab}\right)$$

where ℓ_a is the normal to the cross-section [C]. (Ψ_2^o carries the Coulombic information.) If space-time is Asymptotically Flat At Spatial and Null Infinity (AEFANSI), one can also define the ADM 4-momentum at i°. One can then show: (AA, Magnon)

$$P^{\rm Bondi}_a[C] = P^{\rm ADM}_a - H_a[\Delta],$$

where $H_a[\Delta]$ is the energy-momentum flux carried by gravitational waves from i° up to C. (conceptual non-triviality.)



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3. General Relativity: Quantum Aspects

• The Affine space structure of $\mathbf{\Gamma}$ leads us to regard smeared News operators $\hat{N}(\tau) = (-1/8\pi G) \int N_{ab}\tau^{ab}\mathrm{d}^3\mathcal{I}$ as 'elementary quantum observables' satisfying: $[\hat{N}(\tau),\, \hat{N}(\tilde{\tau})] = i\hbar\Omega(\tau, \tilde{\tau})\, \hat{I}$.

There is a natural (cyclic) state on the Weyl algebra generated by these operators that it BMS invariant. The vacuum state $0\rangle$ in the resulting representation is a coherent state peaked on the space $\mathbf{V}$ of all classical vacua, just as one might hope!

The Hilbert space $\mathcal{H}$ of first excitations $\hat{N}(\tau)|0\rangle$ provides a unitary representation of every Poincaré subgroup $\mathcal{P}$ of the BMS group $\mathcal{B}$. There are two irreducible sectors $\mathcal{H}^{\pm}$. The Casimir operators have eigenvalues $m = 0, h = \pm 2$! (AA)

Thus, these asymptotic quantum states of the full, non-linear gravitational field can be naturally interpreted as gravitons, without recourse to linearization or perturbative expansion!

• Second (complementary) surprise: These asymptotic graviton states all correspond to only to the radiative modes $\{D\}$ that tend to the same classical vacuum at i^+ and i^o on $\mathcal{I}^+$! For these $\{D\}$, we have $[\sigma] = 0$: There is no gravitational memory; all soft fluxes vanish: $H_\alpha^{\rm soft} = 0$. This severe restriction arises because in quantum theory, we have a new requirement: The 1-particle norm of the excitation in $\mathcal{H}$ defined by the phase space point $\{D\}$ has to be finite .This condition has no classical analog.

Thus, if we restrict ourselves to asymptotic states of gravitons, we miss out on interesting scattering, radiation produced by binary black holes or neutron stars, ...!

Infrared sectors, S-matrix and conservation laws

• Perturbative quantum considerations also tell us that restriction to the Fock space of asymptotic gravitons is too restrictive from infrared considerations: We have $\langle \mathrm{out}|\mathrm{S}|\mathrm{in}\rangle = 0$ unless the incoming and outgoing momenta in the $|in\rangle$ and $|out\rangle$ of the (so called hard) particles are finely tuned (e.g. if there is no scattering). The amplitude is completely suppressed by loop corrections. (Choi, Kol & Akhoury).

• We need to include new 'infra-red sectors' –i.e., representations of the News algebra that are unitarily inequivalent to the Fock representation, each labelled by $[\sigma](\theta, \phi) \neq 0$. This is a 'displaced Fock representation' in the sense that while Fock states correspond to radiative modes $\{D\}$ with $[\sigma] = 0$, the displaced Fock states correspond to radiative modes $\{\bar{D}\}$ with $[\bar{\sigma}] \neq 0$. These representations are constructed using an automorphism on the News algebra that is not unitarily implemented on the Fock space.

• The S-matrix is now infrared finite and has the following interesting property vis a vis supermomentum fluxes: $\langle \mathrm{out}, [\sigma]_{\mathrm{out}}|\, [\mathrm{S}, \hat{\mathrm{H}}_\alpha]\, |\mathrm{in}, [\sigma]_{\mathrm{in}}\rangle = 0$ for all 'in' and 'out' states. Thus, order by order in perturbation theory (with ultraviolet cut-off), quantum S-matrix theory says that supermomenta are conserved in gravitational scattering!

• Is there then a new conservation law for classical gravity? Rigorous global existence results available only for vacuum GR in a non-linear neighborhood of Minkowski space Christodoulou & Klainnermann, Chrusciel & Delay, Friedrich, ...). In all cases I know of, the supermomentum is indeed conserved, $H_\alpha[\mathcal{I}^-] = H_\alpha[\mathcal{I}^+]$, but for a trivial reason: both quantities are identically zero for 'pure supertranslations'! A very interesting question in geometric analysis is whether there is a non-trivial conservation law for a sufficiently large set of vacuum solutions.

Summary: Surprises with radiation fields at $\mathcal{I}$

- Maxwell theory:

General expectation: Poincaré momentum carried by electromagnetic fields is imprinted just in radiative modes (which are quantized to obtain asymptotic states of photons). Incorrect for angular momentum. Subtle interplay between Coulombic aspect with radiative modes.

- General Relativity: Classical Aspects

i) There is classical vacuum degeneracy: Infinitely many $\{\mathring{D}\}$ on $\mathcal{I}$ with trivial curvature! Degeneracy directly 'responsible' for the enlargement of the Poincaré group to the BMS. Space $\mathbf{V}$ of classical vacua is naturally isomorphic with the space $\mathbf{P}$ of Poincaré subgroups of the BMS group.
ii) The fact that the phase space $\mathbf{\Gamma}$ of radiative modes is an affine space –as opposed to vector space– is responsible for the 'soft' contribution to the supermomentum flux that encodes gravitational memory and infrared sectors in quantum theory.

- General Relativity: Quantum Aspects:

i) Radiative modes of the full non-linear gravitational field can be quantized at $\mathcal{I}$. There is a vacuum state that is BMS invariant. Poincaré subgroups of the BMS group enable us to interpret the quantum excitations over this vacuum as having $m = 0, h = \pm 2$. These asymptotic states of the in full non-linear gravitational field can be interpreted as gravitons, without having to linearize the theory or carry out a perturbative expansion!
ii) But the Fock space of gravitons is not the full space of asymptotic states. To make the S-matrix infrared finite, we need additional sectors (corresponding to gravitational memory in the classical theory).
iii) Perturbative S-matrix arguments suggest a new conservation law in gravitational scattering. Is there a non-trivial one in classical GR?

Main References for this Talk

Summaries:

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Physical space-time perspective

- Boundary conditions at null infinity imply $\hat{g}_{ab} = \eta_{ab} + O(\frac{1}{r})$ (where the $O(1/r)$ terms have a specific form). Then why is the asymptotic symmetry group of the physical metric $\hat{g}_{ab}$ not just the Poincaré group –the group of isometries of η_{ab} to which $\hat{g}_{ab}$ approaches near $\mathcal{I}$?

- The reason: We can find a distinct Minkowski metric η'_{ab} such that $\hat{g}_{ab} = \eta'_{ab} + O(\frac{1}{r'})$ where the $O(1/r')$ terms again have the required specific form. Asymptotically, the two Minkowski metrics are related by a angle dependent translation, e.g.:

  $t' = t + f(\theta, \phi)$, $x' = x$, $y' = y$ and $z' = z$ i.e. a BMS supertranslation

  Therefore the Poincaré group of η'_{ab} is distinct from that of η_{ab}. Roughly, the BMS group is the union of all the Poincaré groups of Minkowski metrics $\eta_{ab},\ \eta'_{ab},\ \eta''_{ab}\ \ldots$ to which $\hat{g}_{ab}$ approaches in the desired fashion. In particular the Poincaré groups of η_{ab} and η'_{ab} are subgroups of the BMS group $\mathcal{B}$. They share the same 4-dimensional group of translations but their Lorentz subgroups are not shared; they are mapped into each other by the supertranslation.

- Gravitational Memory Effect and Infrared Issues/Soft Gravitons in quantum theory can also be understood in terms of supertranslations and non-trivial classical vacua $\{\mathring{D}\}$.