Backreaction of gravitational and scalar waves

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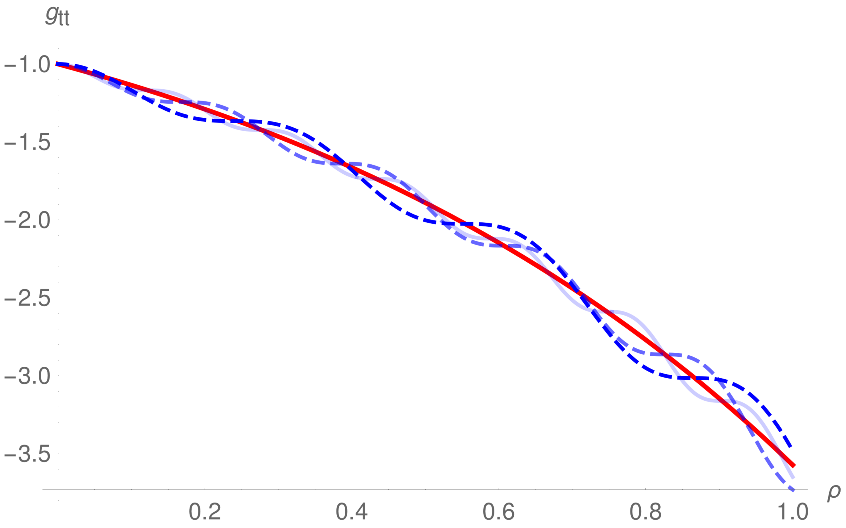
Introduction

- going beyond cosmological context
- radiation and backreaction
- a toy–model: exact spacetimes
- plan:
  1. start with an exact solution
     (gravitational and scalar waves in cylindrical symmetry)
  2. calculate backreaction effect using
     - ⋆ Green—Wald formalism
     - ⋆ Isaacson method (Charach–Malin extension)
     - ⋆ direct interpretation of a background metric
  3. study properties of the background spacetime
- steps 1 and 2 were completed in
  Phys. Rev. D 94, 024059 (2016)
  SJS and Michał Wyrębowski
- I will present recent development related to the step 3



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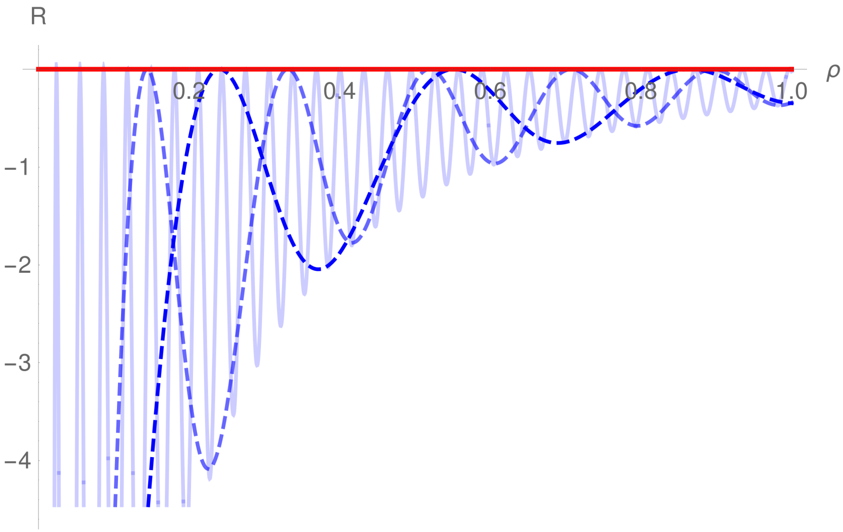

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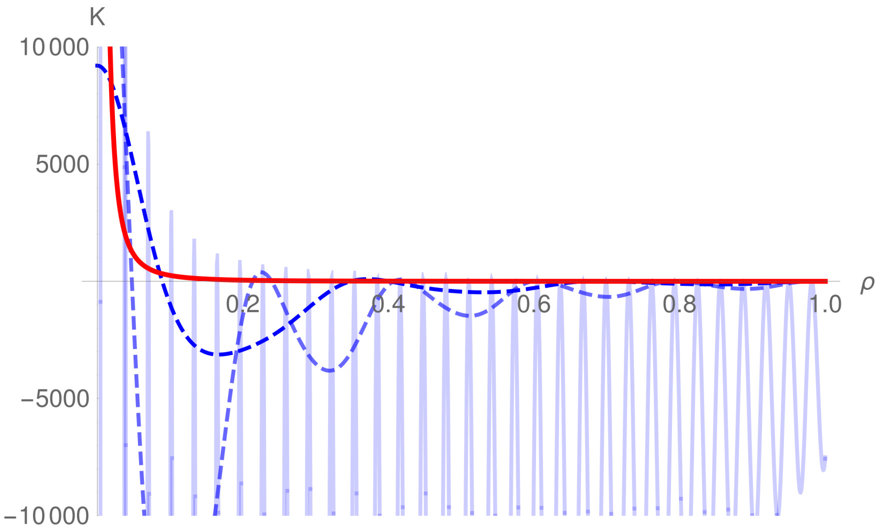

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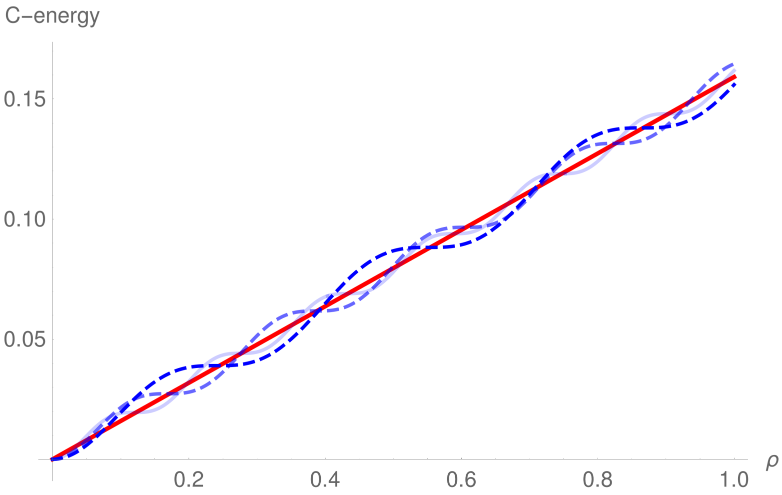

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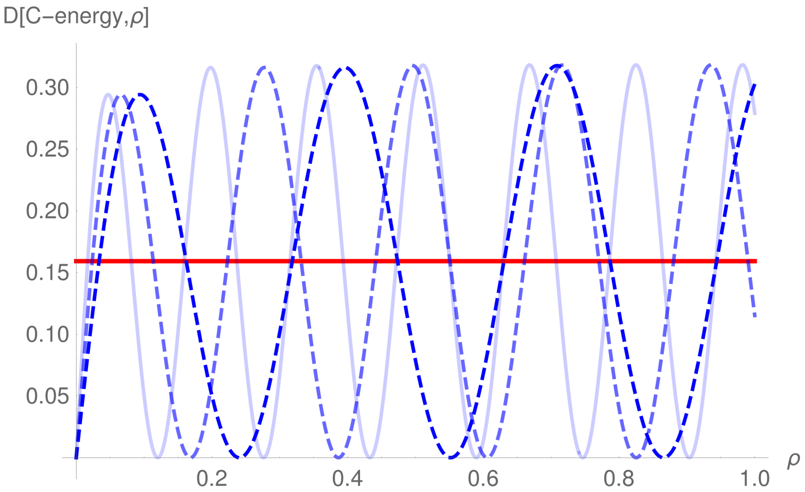

t=0

The Einstein-Rosen metric

The line element has the form (Einstein, Rosen 1937; Rosen 1954)

$$g = \mathrm{e}^{2(\gamma-\psi)} \left(-\mathrm{d}t^2 + \mathrm{d}\rho^2\right) + \rho^2 \,\mathrm{e}^{-2\psi}\, \mathrm{d}\varphi^2 + \mathrm{e}^{2\psi}\, \mathrm{d}z^2 \,,$$

where (cylindrical symmetry)

$$\begin{aligned} &\rho > 0, \quad -\infty < t, z < \infty, \quad 0 \le \varphi < 2\pi, \\ &\psi = \psi(t,\rho), \qquad \gamma = \gamma(t,\rho)\,. \end{aligned}$$

We add massless minimally coupled scalar field ϕ.

$$T_{ab} = \frac{1}{4\pi}\left(\partial_a\phi\partial_b\phi - \frac{1}{2}g_{ab}\partial_c\phi\partial^c\phi\right)$$

The energy density of the scalar field as measured by observers comoving with the coordinate system (with four-velocity $u = e^{\psi-\gamma}\partial_t$)

$$\epsilon = T_{ab}u^a u^b = \frac{1}{8\pi}\,\mathrm{e}^{2(\psi-\gamma)}\left(\dot\phi^2 + \phi'^2\right).$$

One-parameter family of solutions

We choose the following special solutions of the field equations:

$$\phi_\lambda(t,\rho) = \alpha\sqrt{\lambda}\,F_\lambda(t,\rho)\ , \quad \psi_\lambda(t,\rho) = \beta\sqrt{\lambda}\,F_\lambda(t,\rho)\ , \quad \lambda > 0\ ,$$

where:
$F_\lambda(t,\rho) = J_0\left(\frac{\rho}{\lambda}\right)\sin\left(\frac{t}{\lambda}\right)$; λ – parameter; J_0 – Bessel function of the first kind and zero order; constants α, β – real and independent of λ.
Integrating the remaining field equations we get

$$\gamma_\lambda(t,\rho) = \frac{(\alpha^2+\beta^2)}{2\lambda}\rho^2\left[J_0^2(\frac{\rho}{\lambda}) + J_1^2(\frac{\rho}{\lambda}) - 2\frac{\lambda}{\rho}J_0(\frac{\rho}{\lambda})J_1(\frac{\rho}{\lambda})\sin^2(\frac{t}{\lambda})\right].$$

This gives $g(\lambda)$.

The background metric

$$g^{(0)} = \mathrm{e}^{2(\alpha^2+\beta^2)\rho/\pi}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2\right) + \rho^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2\,.$$

In the high-frequency limit nonzero components of the scalar field energy-momentum tensor are

$$T^{(0)}_{tt} = T^{(0)}_{\rho\rho} = \frac{\alpha^2}{8\pi^2\rho}\,.$$

The nonzero components of an effective energy-momentum tensor

$$t^{(0)}_{tt} = t^{(0)}_{\rho\rho} = \frac{\beta^2}{8\pi^2\rho}\,.$$

(One may rederive the result using $G(g^{(0)})$.)

Scalars

auxilary constant $\kappa = 2\frac{\alpha^2+\beta^2}{\pi}$

- the Ricci scalar

$$R = 0$$

- the Kretschmann scalar blows up at center

$$K = 2\left(\frac{2\kappa}{\rho}\right)^2 \mathrm{e}^{-2\kappa\rho}$$

Morgan solution

- our background spacetime

$$g^{(0)} = \mathrm{e}^{\kappa\rho}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2\right) + \rho^2\mathrm{d}\varphi^2 + \mathrm{d}z^2\,.$$

- T. A. Morgan, GRG 4 (1973), 273
(like Vaidya metric, but in cylindrical symmetry)

$$g^{(0)} = \mathrm{e}^{F(t\pm\rho)}\left(-\mathrm{d}t^2 + \mathrm{d}\rho^2\right) + \rho^2\mathrm{d}\varphi^2 + \mathrm{d}z^2\,,$$

where $F(u)$ is an arbitrary function of a null coordinate u

Standing waves!!!

Gravitational waves in general relativity XVI. Standing waves
Proceedings of The Royal Society A, 460, 2042, 2004
Sir Hermann Bondi

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The background metric

We have

$$G(g^{(0)}) = \frac{\alpha^2 + \beta^2}{\pi\rho}\left(\mathrm{d}t \otimes \mathrm{d}t + \mathrm{d}\rho \otimes \mathrm{d}\rho\right) \ ,$$

but we may also rewrite it in the form of a null fluid

$$G(g^{(0)}) = \frac{1}{2\pi\rho}\big(\alpha^2(k_+ \otimes k_+ + k_- \otimes k_-) + \beta^2(k_+ \otimes k_+ + k_- \otimes k_-)\big) \ ,$$

where $k^\flat{}_\pm = \mathrm{d}t \pm \mathrm{d}\rho$.

- Cécile Huneau, Jonathan Luk
  High-frequency backreaction for the Einstein equations under polarized U(1) symmetry
  arXiv:1706.09501

Petrov classification

- non-holonomic basis, null tetrad, $g(l,n)=-1$, $g(m,\bar{m})=1$ (Newman-Penrose formalism)

$$g = -l\otimes n - n\otimes l + m\otimes\bar{m} + \bar{m}\otimes m\ ,$$

where $\bar{m}$ is complex conjugate of m and

$$l^\flat = \frac{1}{\sqrt{2}}\,\mathrm{e}^{\kappa\rho}\,k^\flat{}_+$$

$$n^\flat = \frac{1}{\sqrt{2}}\,\mathrm{e}^{\kappa\rho}\,k^\flat{}_-$$

$$m^\flat = \frac{1}{\sqrt{2}}(\mathrm{d}z + i\mathrm{d}\varphi)$$

- bivectors

$$U = -l\wedge m\ ,\ \ V = n\wedge m\ ,\ \ W = m\wedge\bar{m} - n\wedge l$$

Petrov classification

- decomposition of the Weyl tensor

$$C = \psi_0 U \otimes U + \psi_1 (U \otimes W + W \otimes U) + \psi_2 (V \otimes U + U \otimes V + W \otimes W) +$$

$$+ \psi_3 (V \otimes W + W \otimes V) + \psi_4 V \otimes V + c.c.$$

- Petrov type D (like Kerr)

$$\psi_0 = \psi_1 = \psi_3 = \psi_4 = 0$$

$$\psi_2 = \frac{1}{12\rho^2}\, \mathrm{e}^{-\kappa\rho}$$

Summary

- consistent description of backreaction (radiation, exact toy-model)
- three different methods to derive the result
- an example of smooth standing waves in general relativity
- the background spacetime — an effective model of standing waves
- interesting structure of geodesics
  (bachelor thesis: Adam Cieślik)
- causal structure — a projection diagram follows immediately from
  Space-time diagrammatics
  Phys. Rev. D 86, 124041, 2012
  Piotr T. Chruściel, Christa R. Ölz, and SJS