$$[P_\mu, P_\nu] = 0 \ ,$$

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 \ , \quad \Delta P_j = P_j \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_j \ ,$$

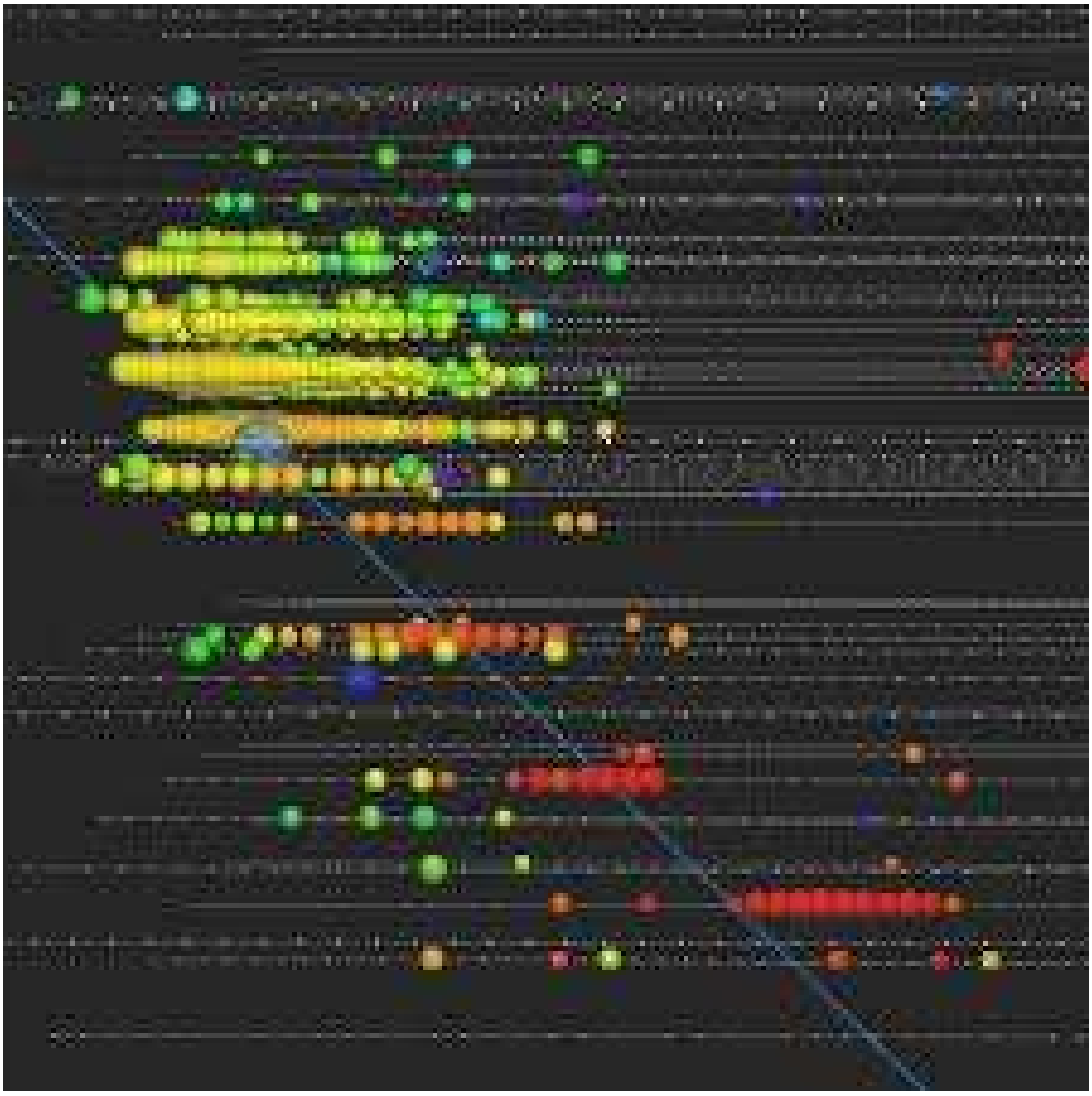
$$\varepsilon(P_\mu) = 0 \ , \qquad S(P_0) = -P_0 \ , \qquad S(P_j) = -e^{P_0/\kappa} P_j \ ,$$

$$[N_j, P_k] = i\,\delta_{jk} \left(\tfrac{\kappa}{2}(1 - e^{-2P_0/\kappa}) + \tfrac{1}{2\kappa}|\vec{P}|^2 \right) - \tfrac{i}{\kappa} P_j P_k \ ,$$

$$[N_j, P_0] = i\,P_j \ , \quad [R_j, P_k] = i\,\epsilon_{jkl} P_l$$

$$\Delta N_k = N_k \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes N_k + \tfrac{i}{\kappa}\epsilon_{klm} P_l \otimes R_m \ , \quad \Delta R_j = R_j \otimes \mathbb{1} + \mathbb{1} \otimes R_j \ ,$$

$$\varepsilon(N_j) = 0 \ , \quad \varepsilon(R_k) = 0 \ , \quad S(N_j) = -e^{P_0/\kappa} N_j + \tfrac{i}{\kappa}\epsilon_{jkl} e^{\lambda P_0} P_k R_l \ , \quad S(R_k) = -R_k$$



24.09.2018

pleasure to be here…kappaMINKOWSKI noncommutative spacetime a key source of “novel intuition” for quantum-gravity phenomenology…

$$[x_j, t] = i\lambda x_j \qquad [x_j, x_m] = 0$$

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlini,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

kappaPOINCARE'
HOPF ALGEBRA

?

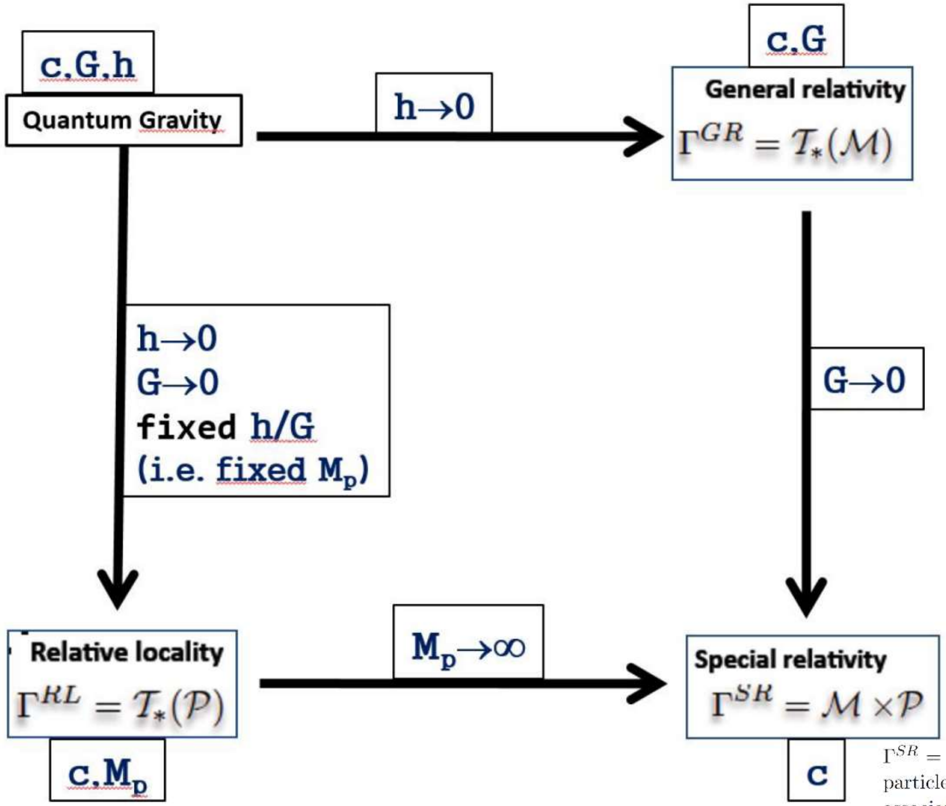
$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

$\boxed{\ell \equiv \lambda}$

GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

relative-locality regime:

$$\hbar \to 0\ , \quad G_N \to 0\ , \quad \text{but with fixed} \sqrt{\frac{c^3\,\hbar}{G_N}} = E_p$$



$\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$. In general relativity, the spacetime manifold $\mathcal{M}$ has a curved geometry, and the particle phase space is no longer a product. Instead, there is a separate momentum space, $\mathcal{P}_x$ associated to each spacetime point $x \in \mathcal{M}$. This is identified with the cotangent space of $\mathcal{M}$ at x, so that $\mathcal{P}_x = \mathcal{T}^*_x(\mathcal{M})$. The whole phase space is the cotangent bundle of $\mathcal{M}$, *i.e.* $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$. In a regime dual to the general-relativity regime one would have a momentum space $\mathcal{P}$ that is curved. Then there must be a separate spacetime $\mathcal{M}_p$ (flat since, in this regime we envision, one has sent $G \to 0$) for each value of momentum, $\mathcal{M}_p = \mathcal{T}^*_p(\mathcal{P})$, and the whole phase space is the cotangent bundle over momentum space, *i.e.* $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$. From this perspective the quantum-gravity problem is the problem of finding a formalization of the more general theory, admitting as limiting cases the regime symbolized by $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$ and the regime symbolized by $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$.

GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

mass of a particle with four-momentum p_μ is determined by the metric geodesic distance on momentum space from p_μ to the origin of momentum space

$$m^2 = d_\ell^2(p,0) = \int dt \sqrt{g^{\mu\nu}(\gamma^{[A;p]}(t))\dot{\gamma}_\mu^{[A;p]}(t)\dot{\gamma}_\nu^{[A;p]}(t)}$$

where $\gamma^{[A;p]}_\mu$ is the metric geodesic connecting the point p_μ to the origin of momentum space with $A^{\mu\nu}{}_\lambda$ the Levi-Civita connection

$$\frac{d^2\gamma_\lambda^{[A]}(t)}{dt^2} + A^{\mu\nu}{}_\lambda \frac{d\gamma_\mu^{[A]}(t)}{dt}\frac{d\gamma_\nu^{[A]}(t)}{dt} = 0$$

the affine connection on momentum space determines the law of composition of momenta,

$q \oplus k$

and it might not be the Levi-Civita connection of the metric on momentum space

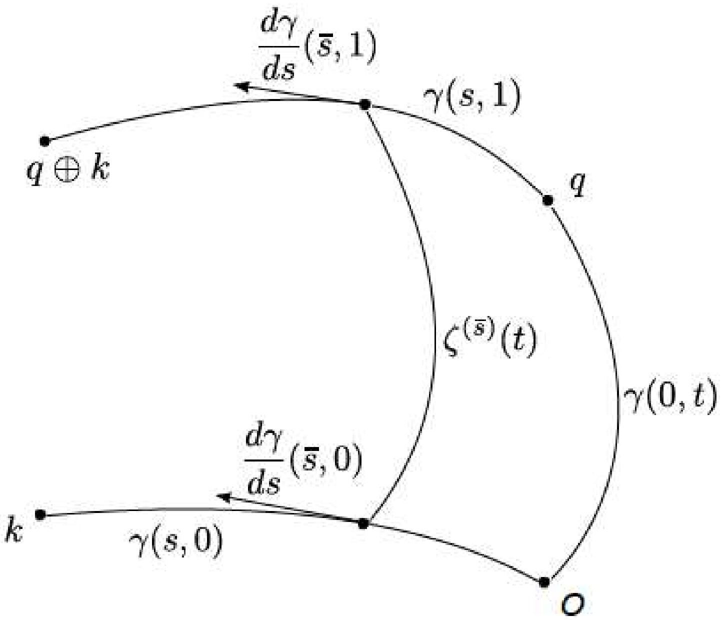


Figure 1. We determine the law of composition of momenta from the affine connection by associating to the points q and k of momentum space the connection geodesics $\gamma^{(q)}$ and $\gamma^{(k)}$ which connect them to the origin of momentum space. We then introduce a third curve $\bar{\gamma}(s)$, which we call the parallel transport of $\gamma^{(k)}(s)$ along $\gamma^{(q)}(t)$, such that for any given value $\bar{s}$ of the parameter s one has that the tangent vector $\frac{d}{ds}\bar{\gamma}(\bar{s})$ is the parallel transport of the tangent vector $\frac{d}{ds}\gamma^{(k)}(\bar{s})$ along the geodesic connecting $\gamma^{(k)}(\bar{s})$ to $\bar{\gamma}(\bar{s})$. Then the composition law is defined as the extremal point of $\bar{\gamma}$, that is $q \oplus_\ell k = \bar{\gamma}(1)$.

notably if momentum space has dS or AdS geometry then the theory can be formulated as "DSR relativistic", i.e. it is a relativistic theory with two non-trivial relativistic invariants, a high speed scale (speed-of-light scale "c") and a high momentum scale (Planck scale)

GAC, grqc0012051, IntJournModPhysD11,35
hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
LectNotesPhys669,131
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

Hopf-algebra symmetries (such as kappa-Poincarè) arise by suitable choice of non-metric affine connection on such a momentum space

Several other choices of affine connection have been shown to also produce a DSR-relativistic theory, and this in particular occurs if one takes as affine connection the metric connection (an example of DSR-relativistic theory which is not linked to Hopf-algebra mathematics)

GAC+Gubitosi+Palmisano,arXiv13077988,IJMPD25,1650027
Banburski+Freidel,arXiv13080300,PRD90,076010

one of the first papers on the quantum gravity problem was a paper by **Max Born [***Proc.R.Soc.Lond.*A165,29(**1938**)**] centered on the dual role within quantum mechanics between momenta and spacetime coordinates (Born reciprocity)**

Born argued that it might be impossible to unify gravity and quantum theory unless we make room for curvature of momentum space

$$p_\mu \leftrightarrow x^\mu$$

our "quantum-gravity phenomenological models" will turn out to be (<u>at best</u>) like the Bohr-Somerfeld quantization…

even the assumption that the quantum-gravity scale should coincide with the Planck scale should be viewed as just a <u>weak</u> guess:

$$E_{QG} \sim E_{Planck} = 1.2 \bullet 10^{19}\,\mathrm{GeV} = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}}$$

i.e. 10^{-35}meters ("Planck length")

mainly comes from observing that at the Planck scale

$$\lambda_C \sim \lambda_S$$

$$\lambda_c \equiv \frac{1}{M}$$

$$\lambda_s \equiv G_N M$$

Note that this can only be a <u>rough order-of-magnitude estimate</u>

in particular this estimate <u>assumes that G does not run at all!</u>!!!!!!!!

it most likely does run!!!

$$\frac{1}{M} = G_N M \Rightarrow M = E_p$$

$$\frac{1}{M} = G_N(M)\, M \Rightarrow M = ?$$

Notion of curvature of momentum space is also proving valuable for phenomenology

Much studied opportunity for phenomenology comes from fact that several pictures of quantum spacetime predict that the speed of photons is energy dependent.

Calculation of the energy dependence in a given model used to be lengthy and cumbersome. We now understand those results as dual redshift on Planck-scale-curved momentum spaces:

these results so far are fully understood for the case of
[maximally symmetric curved momentum space] ⊗ [flat spacetime]

it turns out that there is a duality between this and the familiar case of
[maximally-symmetric curved spacetime] ⊗ [flat momentum space]

In particular,
ordinary redshift in deSitter spacetime implies that massless particles emitted with same energy but at different times from a distant source reach the detector with different energy

dual redshift in deSitter momentum space implies that massless particles emitted simultaneously but with different energies from a distant source reach the detector at different times

GAC+Barcaroli+Gubitosi+Loret, Classical&QuantumGravity30,235002 (2013)
GAC+Matassa+Mercati+Rosati, PhysicalReviewLetters106,071301 (2011)

dual redshift on Planck-scale-curved momentum spaces **(but with flat spacetime) produces time-of-arrival effects which at leading order are of the form ($n \in \{1,2\}$)**

$$\Delta T = \left(\frac{E}{E_P} \right)^n T$$

and could be described in terms of an energy-dependent "physical velocity" of ultrarelativistic particles

$$\mathrm{v} = c + s_{\pm} \left(\frac{E}{E_P} \right)^n c$$

n=1 for kappaPoincarè
<u>onshellness shown before</u>

<u>these are very small effects but (at least for the case n=1) they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!</u>

GRBs are ideally suited for testing this:
cosmological distances (established in 1997)
photons (and neutrinos) emitted nearly simultaneously
with rather high energies (GeV…..TeV…100 TeV…)

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
GAC, NaturePhysics10,254(2014)

problem:

solid theory is for (curved momentum space and) <u>flat spacetime</u>

phenomenological opportunities are for propagation over cosmological distances, whose analysis requires <u>curved spacetime</u>

study of theories with <u>both curved momentum space and curved spacetime</u> still in its infancy

GAC+Rosati, PhysRevD86,124035(2012)
KowalskiGlikman+Rosati,ModPhysLettA28,135101(2013)
Heckman+Verlinde,arXiv:1401.1810(2014)

Jacob and Piran [JCAP0801,031(2008)] used a compelling heuristic argument for producing a formula of energy-dependent time delay applicable to FRW spacetimes, which has been the only candidate so far tested

$$\Delta T = -s_{\pm}\frac{E}{M_{QG}}\frac{c}{H_0}\int_0^z d\zeta \frac{(1+\zeta)}{\sqrt{\Omega_\Lambda + (1+\zeta)^3\Omega_m}}$$

where as usual H_0 is the Hubble parameter, Ω_Λ is the cosmological constant and Ω_m is the matter fraction.
Jacob-Piran formula is surely <u>not</u> the most general possibility.
It is important for phenomenology to understand this issue, but it requires handling the interplay between curvature of spacetime and curvature of momentum space in subtle ways

GAC+Rosati, PhysRevD92,124042

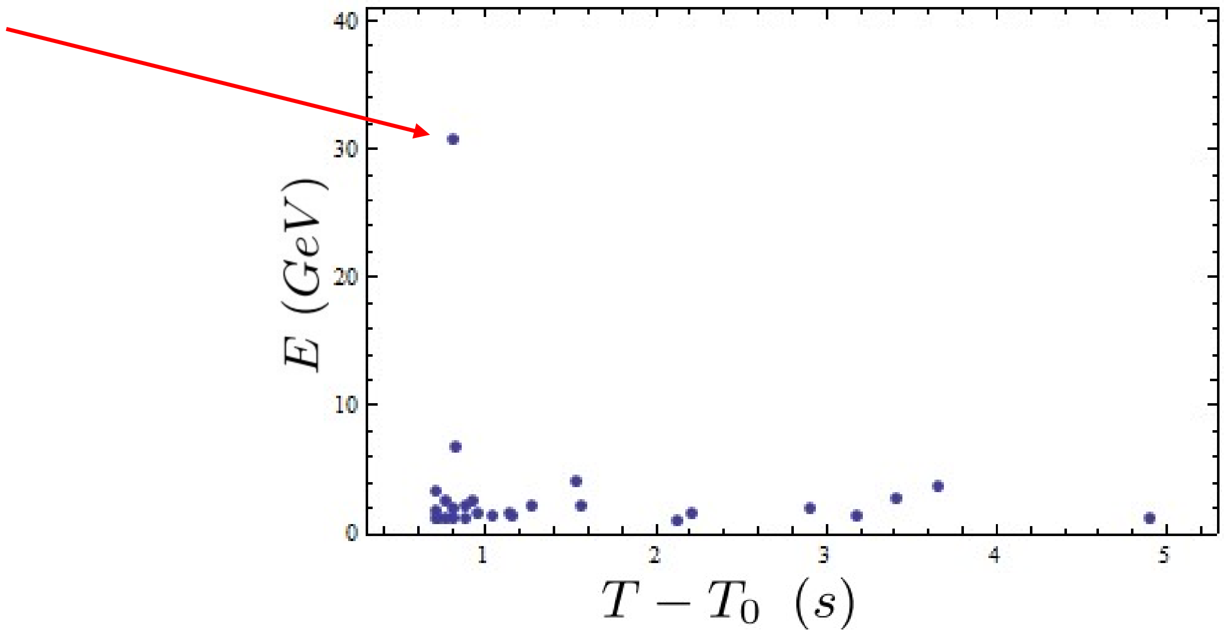
testing Jacob-Piran formula:

focus on n=1 case (sensitivity to the n=2 case still far beyond our reach presently but potentially within reach of future neutrino astrophysics)

first came GRB080916C data providing a limit of $M_{QG}>10^{-1}M_{planck}$ for hard spectral lags and $M_{QG}>10^{-2}M_{planck}$ for soft spectral lags

analogous studies of blazars lead to comparable limits

then came GRB090510 (magnificent short burst) allowing to establish a limit at M_{planck} level on both signs of dispersion (soft and hard spectral lags)



a test with accuracy of about one part in 10^{20}!!!

this Planck-scale sensitivity is illustrative of how we have learned over this past decade that there are ways for achieving in some cases sensitivity to Planck-scale-suppressed effects,
<u>something that was thought to be impossible up to the mid 1990s</u>

Quantum-Gravity Phenomenology exists!!!

a collection of other plausible quantum-gravity effects and of some associated data analyses where <u>Planck-scale sensitivity</u>
was achieved (or is within reach) can be found in my "living review"

GAC, LivingRev.Relativity16,5(2013)

http://www.livingreviews.org/lrr-2013-5

still makes sense to test in-vacuo dispersion statistically…

our "quantum-gravity phenomenological models" will turn out to be (at best) like the Bohr-Somerfeld quantization…

in order to best setup the statistical analysis it is convenient to notice that we are testing a linear relationship between Δt and the product of energy and the redshift-dependent function D(z)

$$\Delta t = \eta \frac{E}{M_P} D(z) \qquad \textbf{with} \qquad D(z) = \int_0^z d\zeta \frac{(1+\zeta)}{H_0\sqrt{\Omega_\Lambda + (1+\zeta)^3\Omega_m}}$$

we can absorbe the redshift dependence into an "accordingly rescaled energy", which we call E*

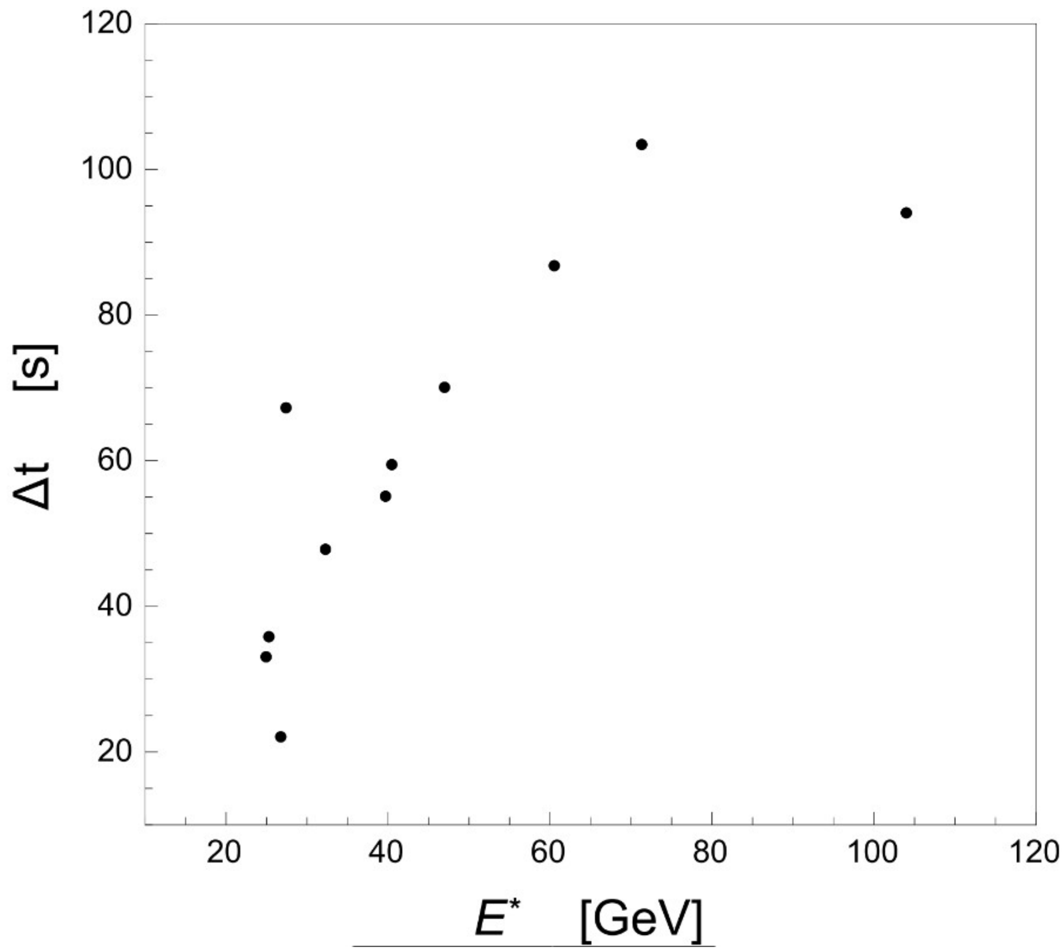
$$E^* \equiv E\frac{D(z)}{D(1)}$$

This then affords us the luxury of analysing data in terms of a linear relationship between Δt and E*

$$\Delta t = \eta\, D(1)\frac{E^*}{M_P}$$

H.Xu+B.Q.Ma, PhysLettB760(2016)602
GAC+G.D'Amico+G.Rosati +N.Loret, arXiv:1612.02765, NatureAstronomy1,0139

criteria:
- focus on photons whose energy at emission was greater than 40 GeV
-take as **Δt** the time-of-observation difference between such high-energy photons and the first peak of the (mostly low-energy) signal



[note that this makes sense only for photons which were emitted in (near) coincidence with the first peak…not all those with >40GeV will …and surely only a rather small percentage of all photons…]

in order to get a sense of how striking this data situation is one can ask how often such high correlation between Δt and E^* would occur if the pairing of values of Δt and E^* was just random: overall having such high correlation would happen in less than 0.1% of cases, and correlation as high as seen for the best 8 out of 11 in 0.0013% of cases

H.Xu+B.Q.Ma, PhysLettB760(2016)602
GAC+G.D'Amico+G.Rosati +N.Loret, arXiv:1612.02765, NatureAstronomy1,0139

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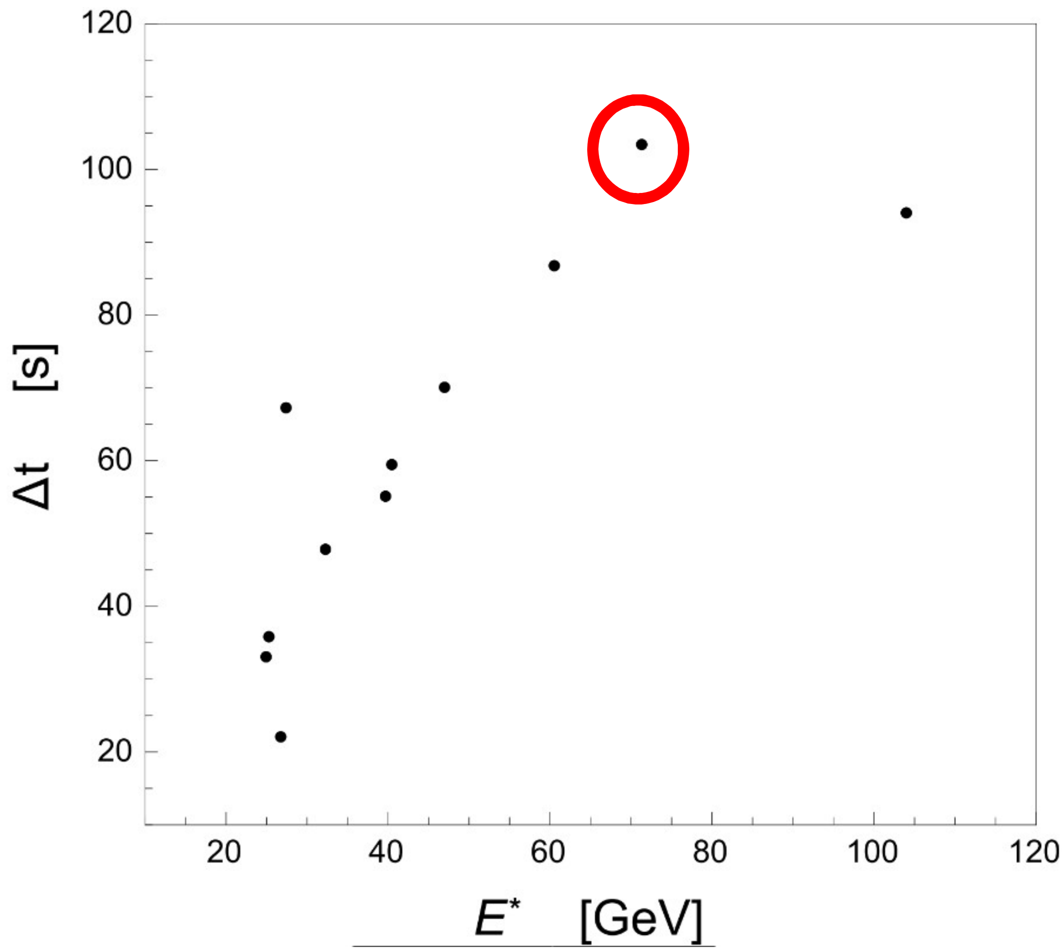

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GAC+Barcaroli+D’Amico+Loret+Rosati, arXiv1605.00496, PhysicsLettersB761,318
GAC+D’Amico+Rosati +Loret, arXiv:1612.02765, NatureAstronomy1,0139

[also see previous exploratory analysis on 2008-2010 IceCube data
GAC+Guetta+Piran, Astrophys.J.806,269]

IceCube still found no GRB neutrinos (expected at least a dozen at this point)

If effect is of seconds for GeV photons it can be very large for 300TeV neutrinos…the time window adopted by IceCube would never catch such GRB neutrinos…

IceCube has reported so far 21 shower neutrinos with energy between 60 and 500 TeV

we found that 9 of them could be “GRB-neutrino candidates” (direction compatible with the GRB direction and time of observation within 3 days of the GRB)
so let’s see if they provided some support for the linear dependence between Δt and E*

GAC+Barcaroli+D'Amico+Loret+Rosati, arXiv1605.00496, PhysicsLettersB761,318
GAC+D'Amico+Rosati +Loret, arXiv:1612.02765, NatureAstronomy1,0139

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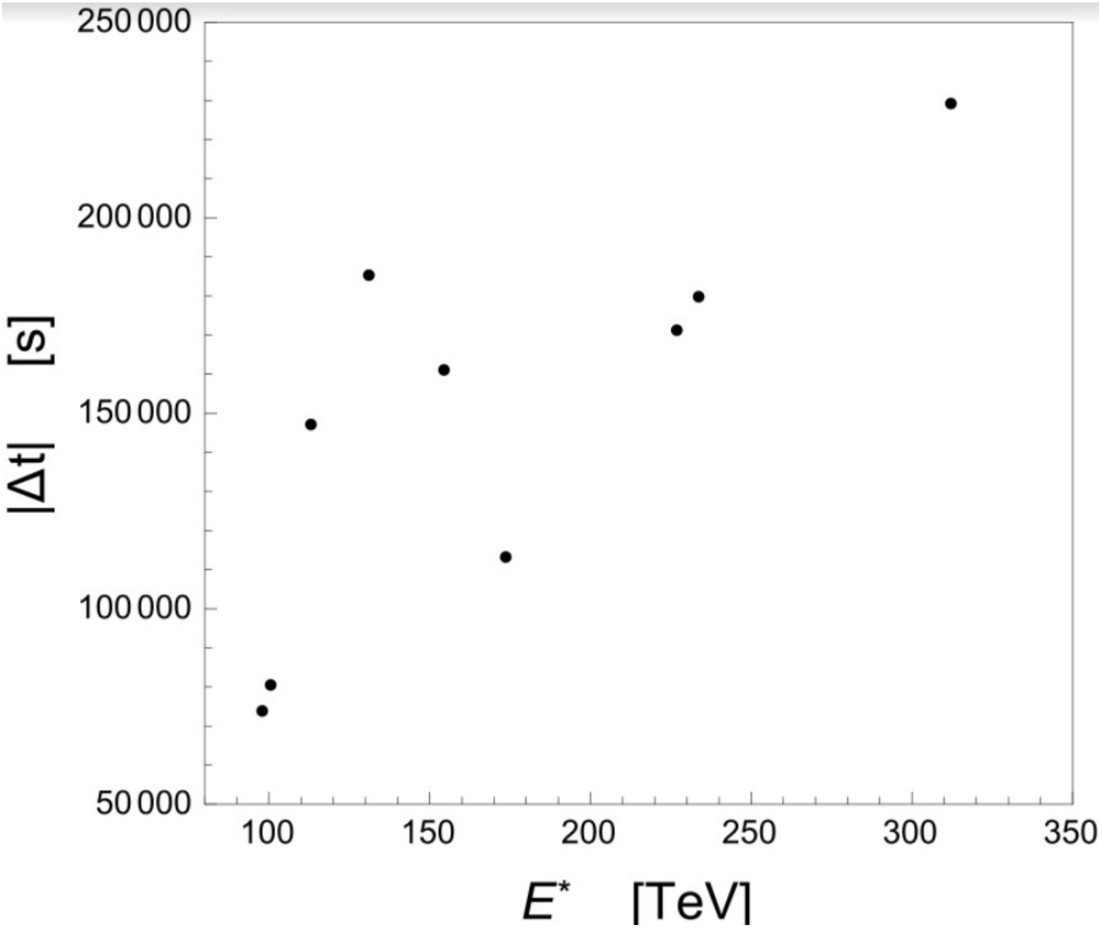
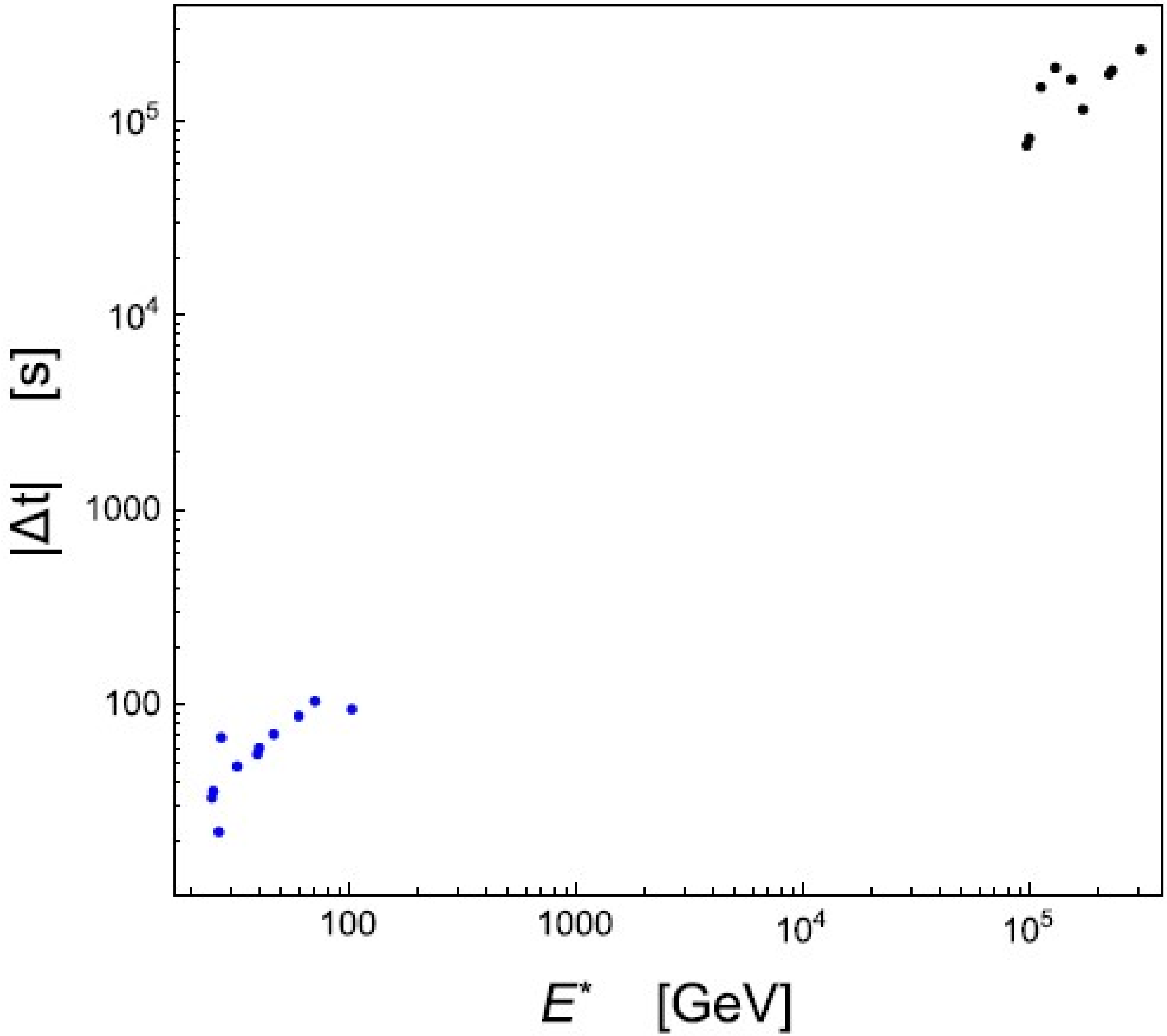


<u>the correlation found in data is 0.95</u>

particularly amazing considering that we can independently estimate (even if there was in-vacuo dispersion, and therefore some of these are GRB neutrinos) that most likely 3 or 4 of our 9 neutrinos must be background neutrinos, unrelated to GRBs

the false alarm probability is 0.5% (probability of finding such a high correlation if all neutrinos are background neutrinos that happened to fit by accident our GRB-neutrino selection criteria)

GAC+D'Amico+Rosati +Loret, arXiv:1612.02765, NatureAstronomy1,0139

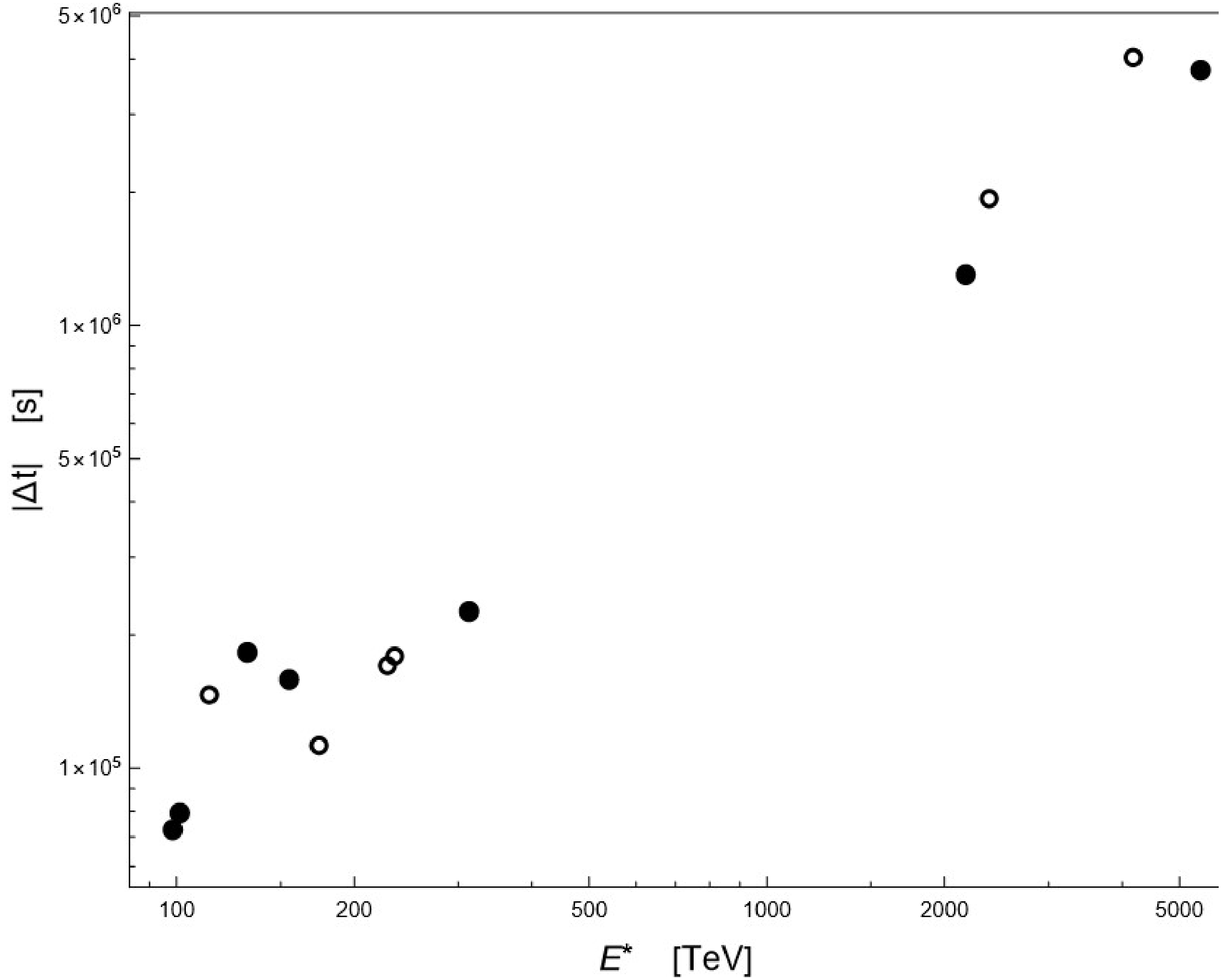


PRELIMINARY

"PRIVATE" COMMUNICATION

NOT FOR CIRCULATION

recent work by the group of B.-Q. Ma and collaborators
[Nature Communications, in press]



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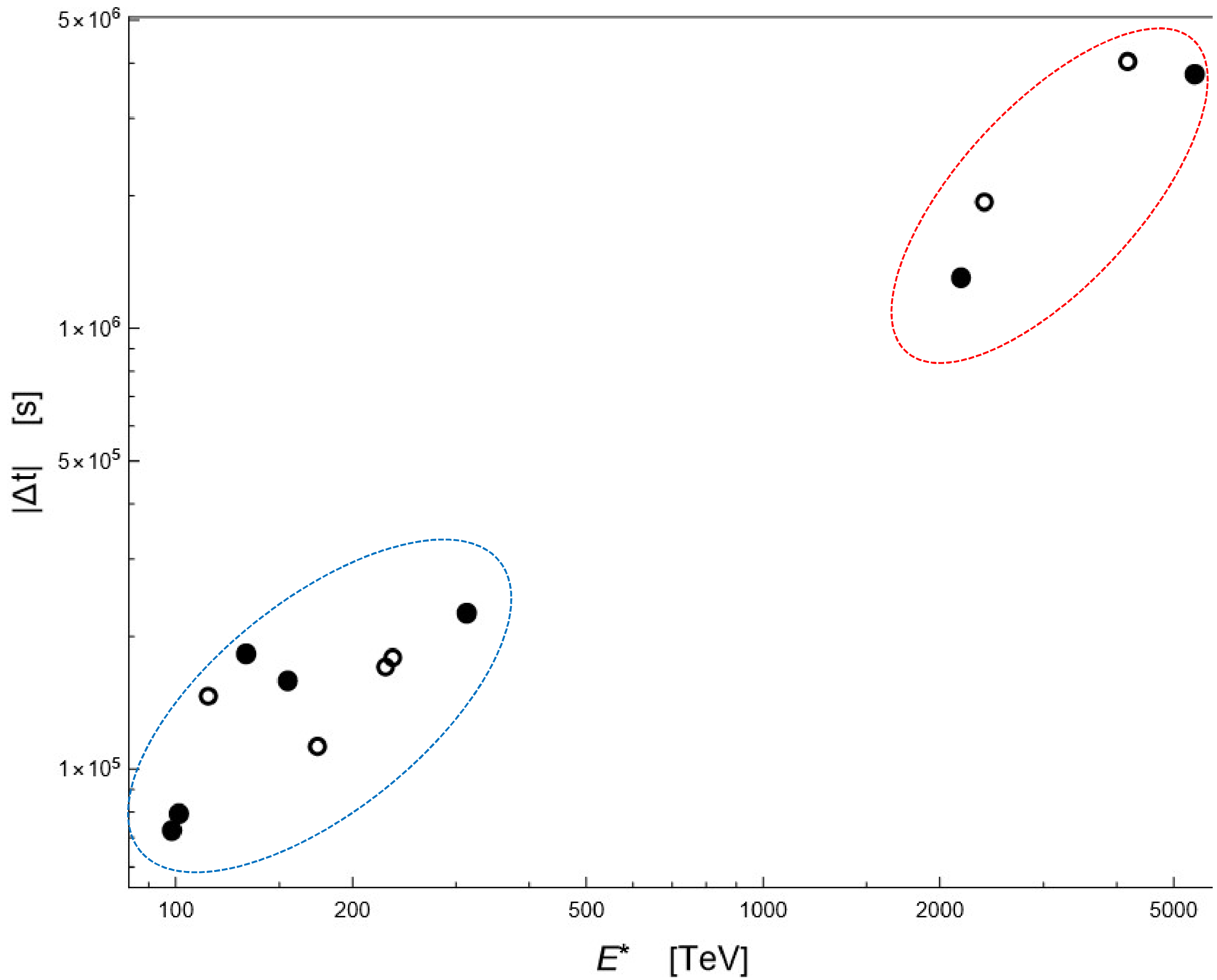

large variety of phenomenological models

* quantum-gravity scale could be bigger or smaller than E_{planck}

* can be brokenSR or deformedSR
    - notice that no quantum-spacetime picture has been shown rigorously to lead to brokenSR
    - notice that threshold anomalies (e.g. anomalous transparency… $\gamma\gamma \rightarrow e^+e^-$) are only possible with brokenSR (protected by a theorem in any deformedSR scenario, GAC,PhysRevD85,084034)
    - for time-of-flight analyses techniques borrowed from propagation of light in media might not apply to deformedSR

*the redshift dependence may be different from the Jacob-Piran ansatz

*the effects can be spin/helicity/polarization dependent

*the effects can be particle-type dependent (different for photons and neutrinos)

*the effects should be fuzzy but theory work at present only provides essentially the deformation of the lightcone, without being able to establish the fuzziness of the deformed lightcone

CLOSING REMARKS

the "preliminary statistical evidence" is strong enough to encourage us to think about alternative phenomenological models, giving a better description of the data situation…would have to be a case such that my simple-minded in-vacuo-dispersion formula is like the Bohr-Somerfeld description of atoms:

- what about the 31GeV event from GRB090510? Should we ascribe it to a remarkable conspiracy?
is the effect intrinsically statistical/non-systematic? Does the effect depend on polarization?
Does the effect depend on direction? Do we need to look beyond the Jacob-Piran formula?
(most of the data that give more strength to the statistical evidence are from very distant GRBs)

- 4 out of our 9 neutrinos are "early neutrinos"…are they background? Or does the effect for neutrino have both signs? If so why does the effect have only one sign for photons?