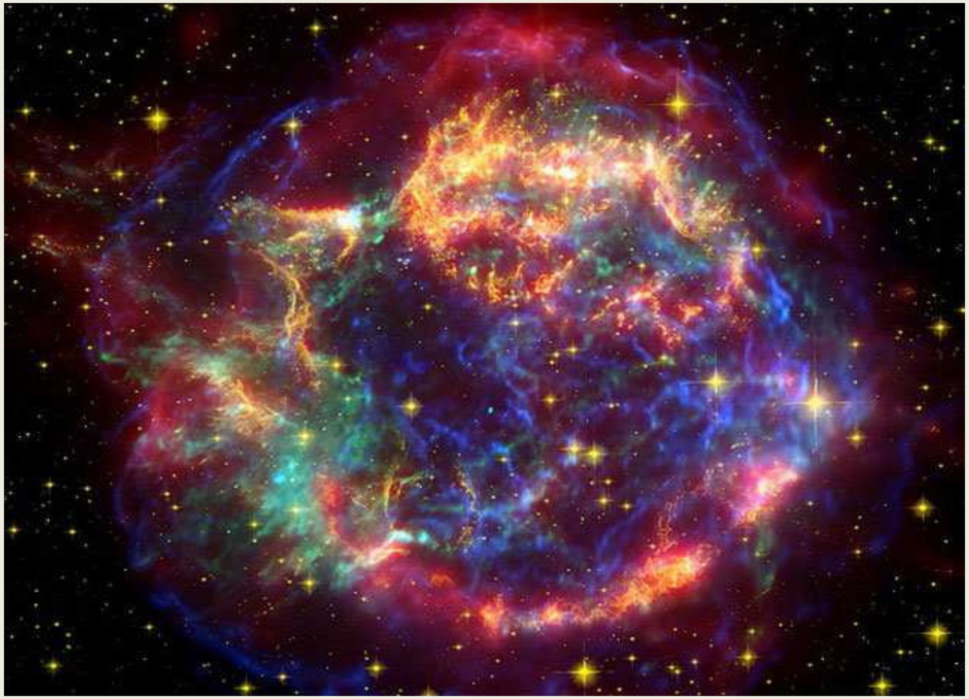
Some aspects of Neutron Stars Cooling Evolution



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Yerevan State University,
AANL CP&IT
(Yeravan, Armenia)

Max Born 40 Symp.
October 9 -12, 2019
Wroclaw, Poland

my co-authors: D.Blaschke, D.Voskresensky,
A. Ayriyan E. Kolomeitsev, K. Maslov

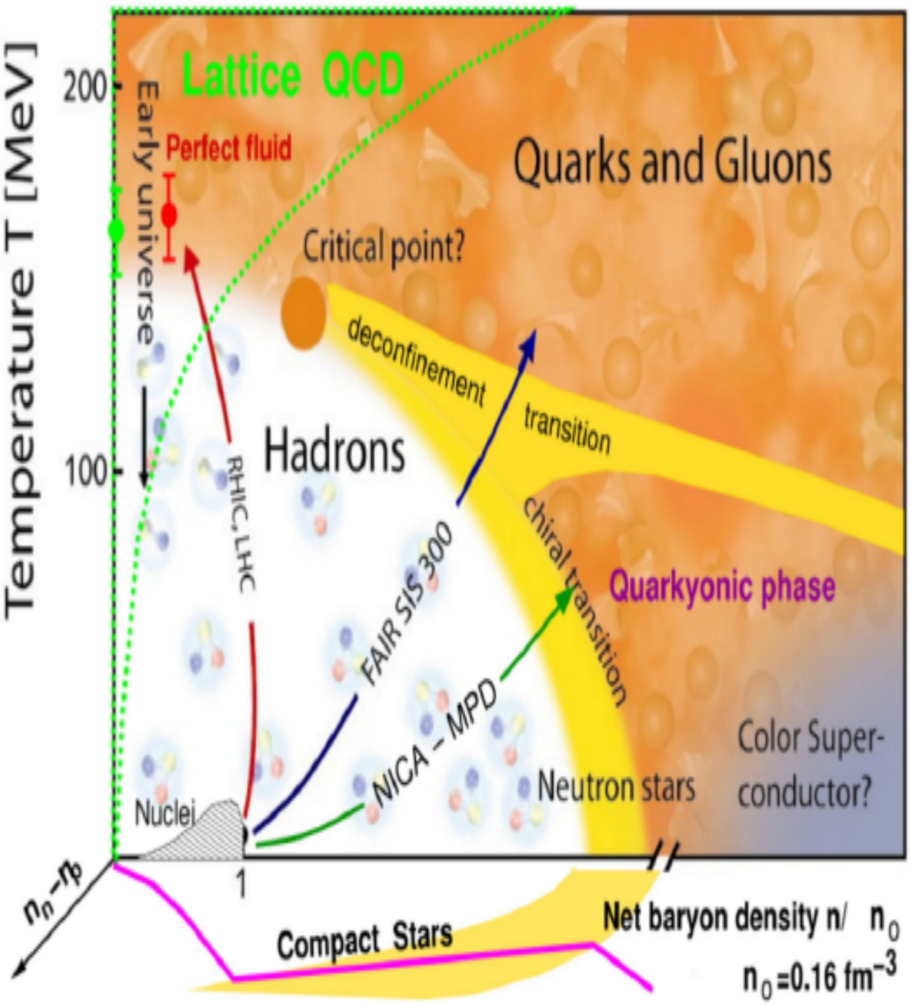
The research was carried out under financial support of the Russian Science Foundation (project #17-12-01427)

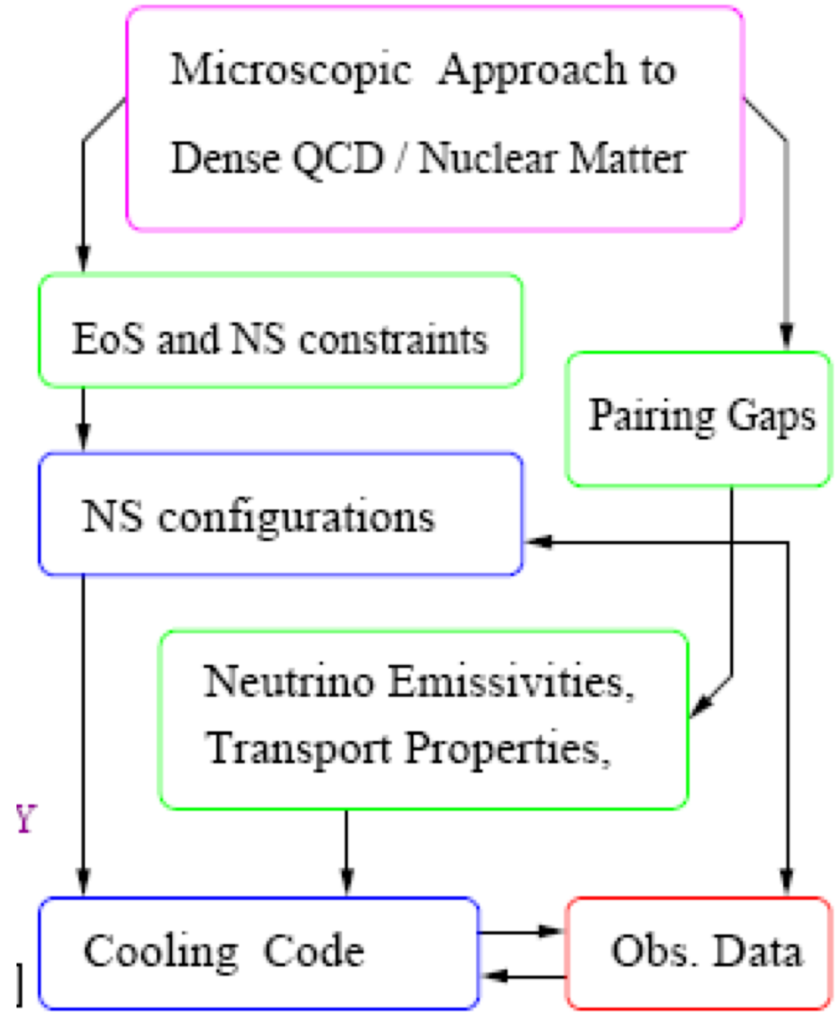
Simulation of Cooling Evolution of Neutron Stars

- Introduction
- Neutron Stars cooling problem
- Simulations algorithm
- *Results for NS cooling*

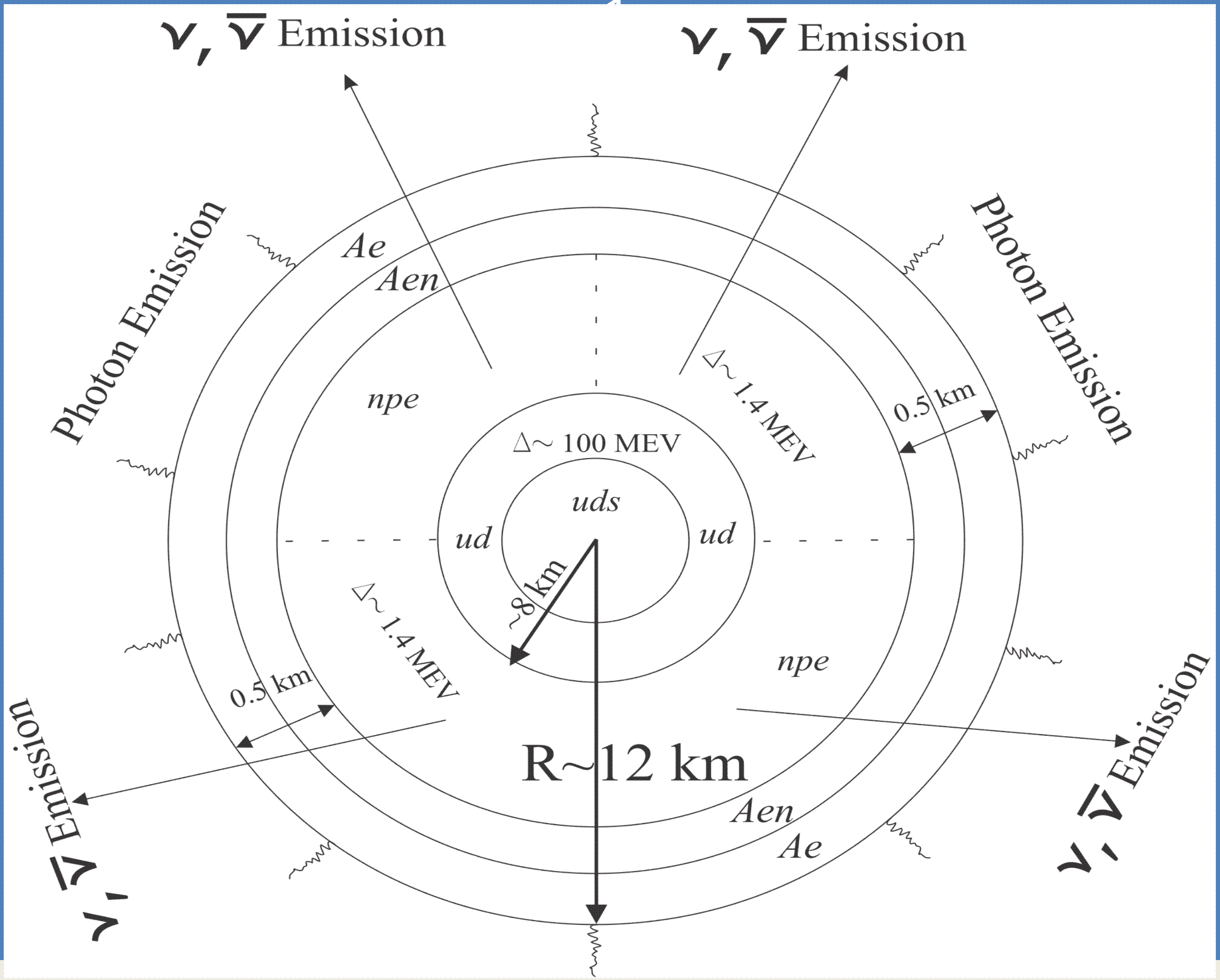
H. Grigorian, D. N. Voskresensky and D. Blaschke
Eur. Phys. J. A **52: 67 (2016).**

Phase Diagramm & Cooling Simulation





Structure Of Hybrid Star



Static neutron star mass and radius

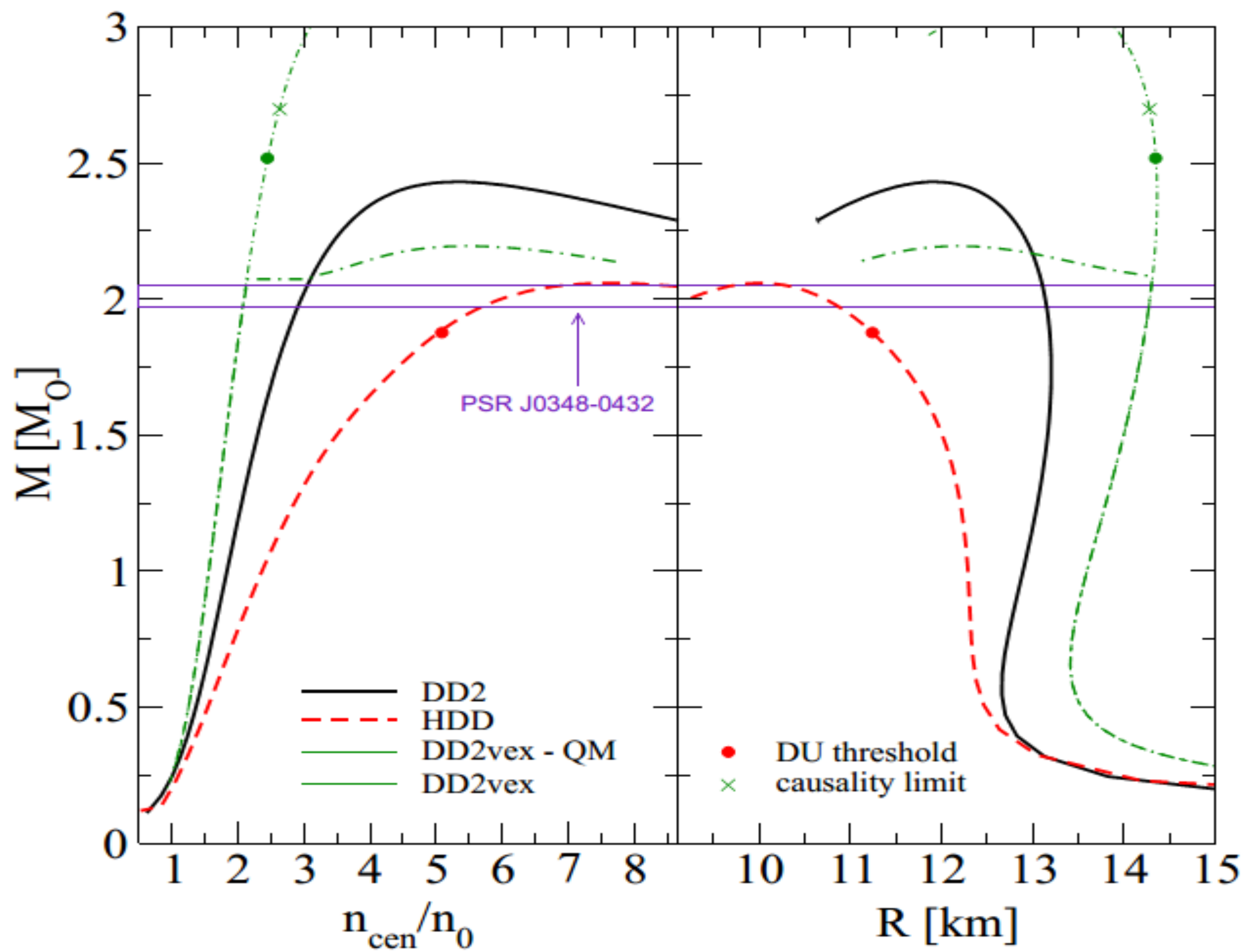
The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations[1,2]:

$$\begin{cases} \dfrac{dP(r)}{dr} = -\dfrac{GM(r)\varepsilon(r)}{r^2} \dfrac{\left(1 + \dfrac{P(r)}{\varepsilon(r)}\right)\left(1 + \dfrac{4\pi r^3 P(r)}{M(r)}\right)}{\left(1 - \dfrac{2GM(r)}{r}\right)}; \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r); \\ \dfrac{dN_B(r)}{dr} = 4\pi r^2 \left(1 - \dfrac{2GM(r)}{r}\right)^{-1/2} n(r). \end{cases}$$

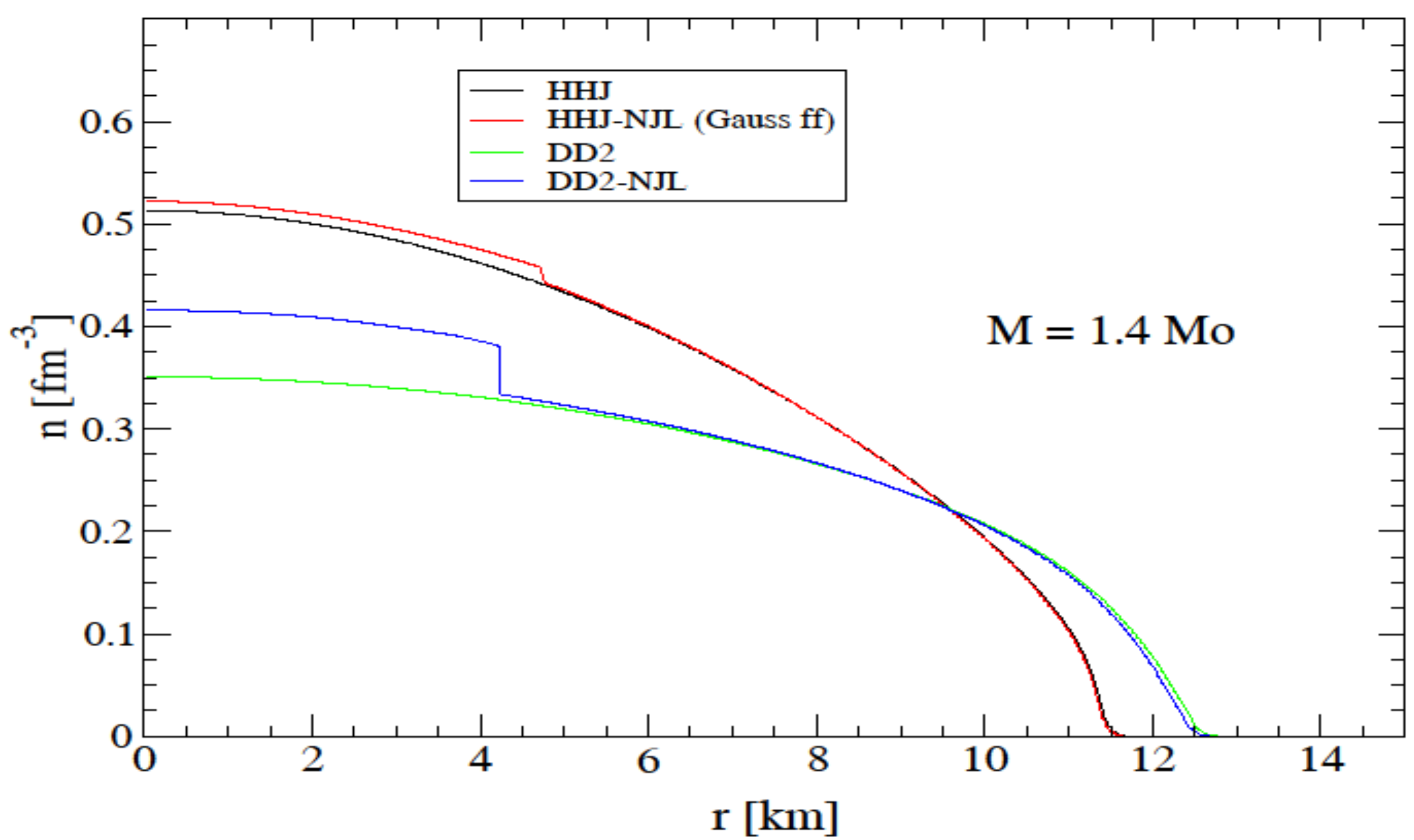
[1] R. C. Tolman, Phys. Rev. **55**, 364 (1939).

[2] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

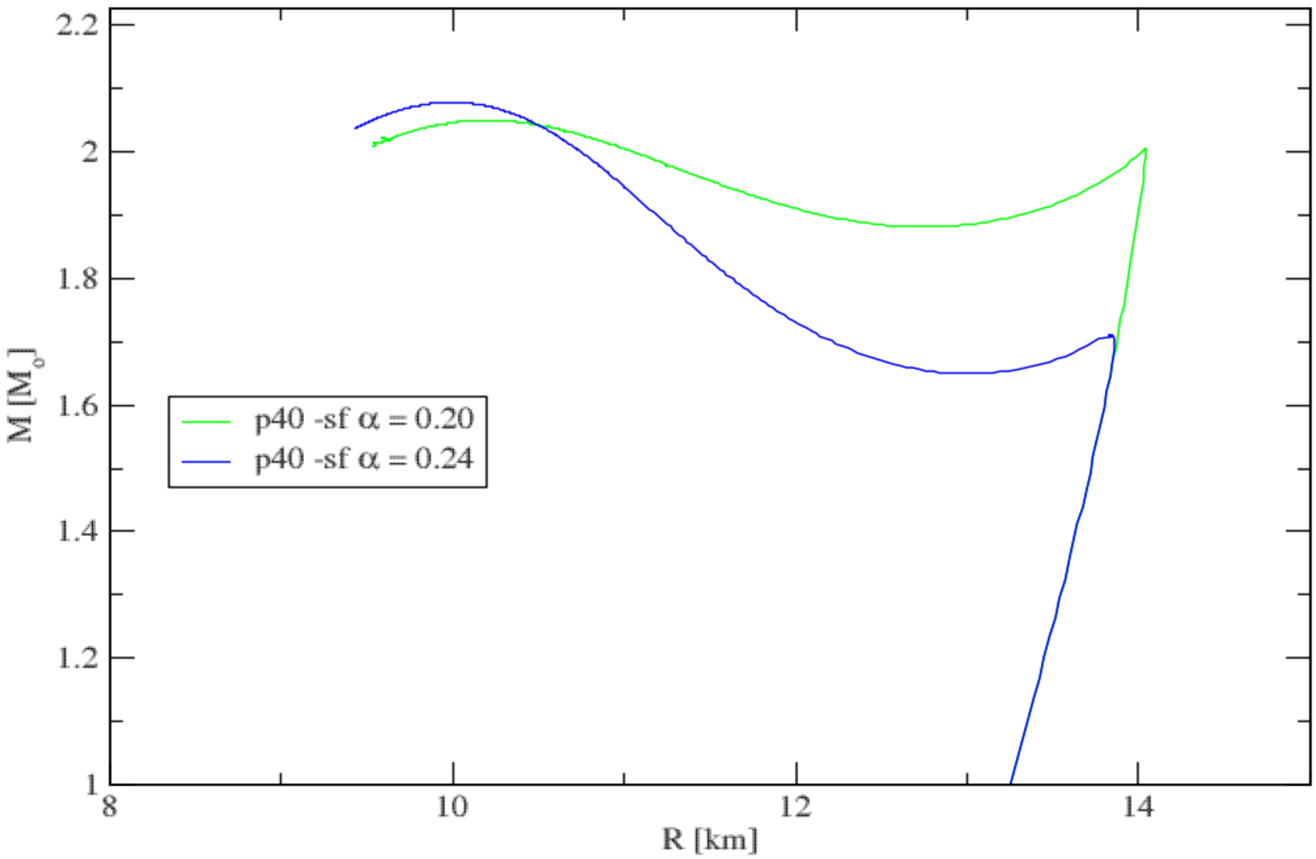
Stability of stars
HDD, DD2 & DDvex-NJL EoS model



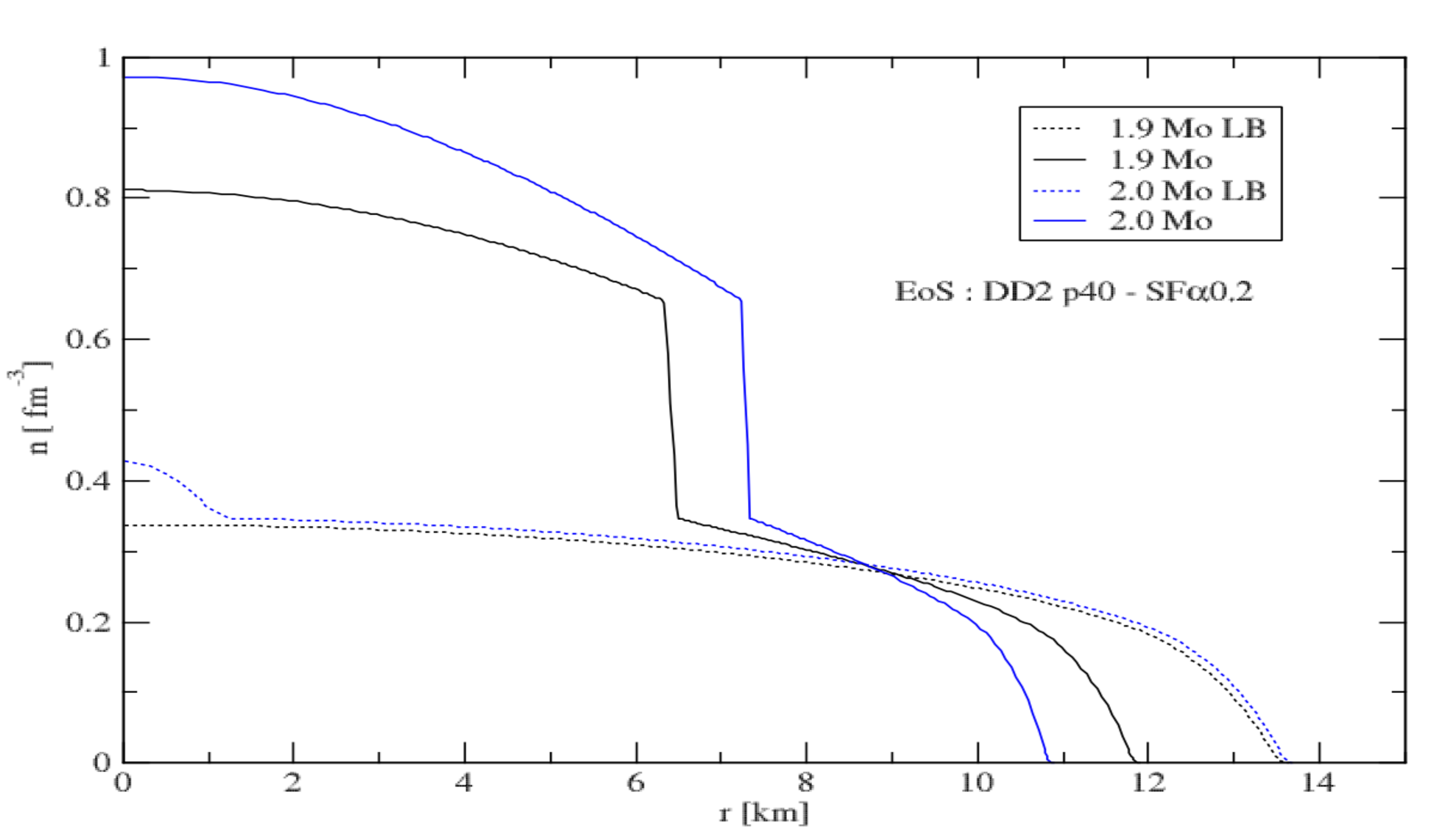
Different Configurations with the same NS mass



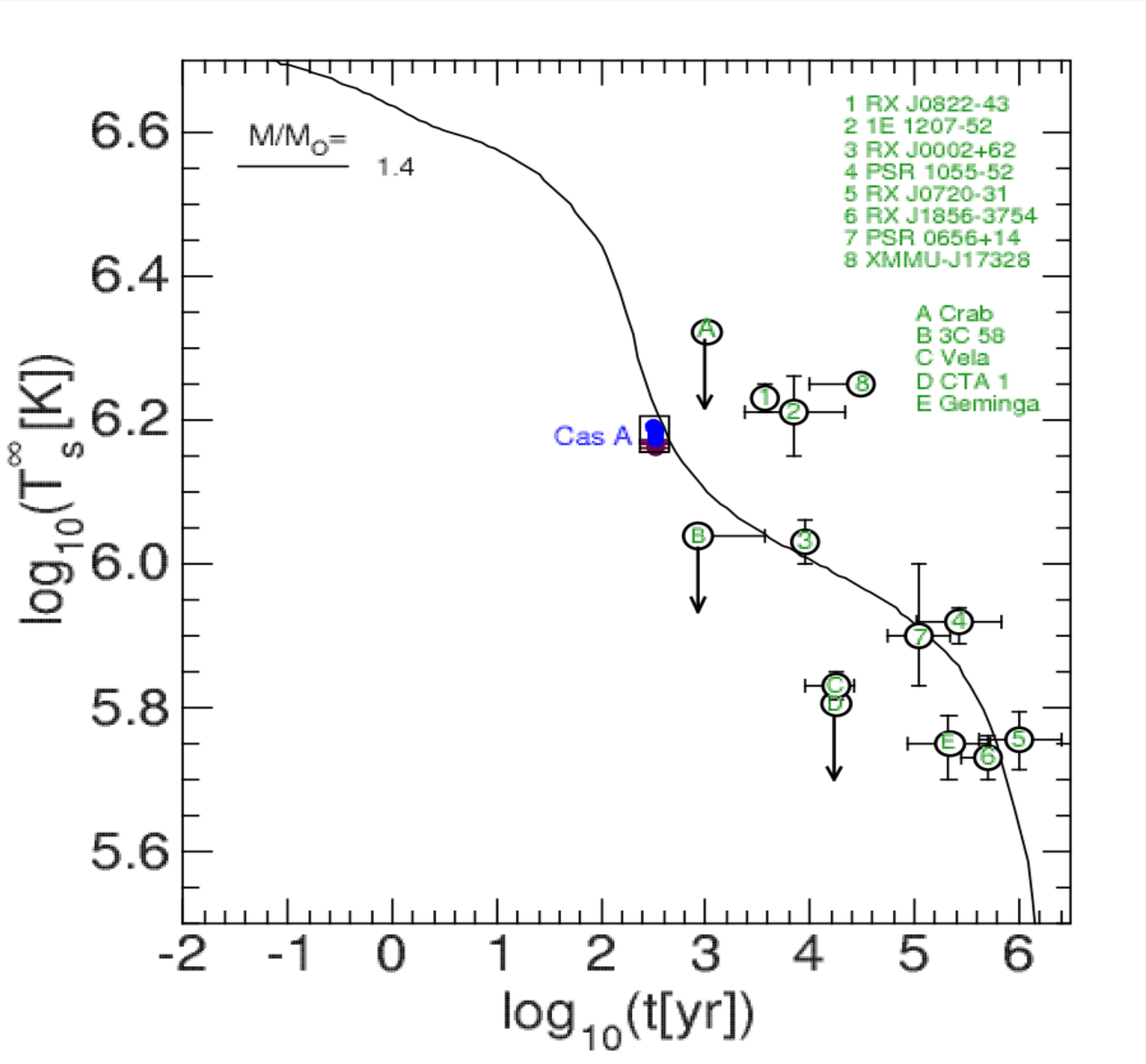
High Mass Twin CS



Different Configurations with the same NS mass



Surface Temperature & Age Data



Cooling Mechanism

$$\frac{dU}{dt} = \sum_i C_i \frac{dT}{dt} = -\varepsilon_\gamma - \sum_j \varepsilon_\nu^j$$

Cooling Processes

- Direct Urca: $n \rightarrow p + e + \bar{\nu}_e$
- Modified Urca: $n + n \rightarrow n + p + e + \bar{\nu}_e$
- Photons: $\rightarrow \gamma$
- Bremsstrahlung: $n + n \rightarrow n + n + \nu + \bar{\nu}$

Cooling Evolution

The energy flux per unit time l(r) through a spherical slice at distance r from the center is:

$$l(r) = -4\pi r^2 k(r) \frac{\partial (T e^{\Phi})}{\partial r} e^{-\Phi} \sqrt{1 - \frac{2M}{r}}.$$

The equations for energy balance and thermal energy transport are:

$$\frac{\partial}{\partial N_B}(l e^{2\Phi}) = -\frac{1}{n}\left(\epsilon_\nu e^{2\Phi} + c_V \frac{\partial}{\partial t}(T e^{\Phi})\right)$$

$$\frac{\partial}{\partial N_B}(T e^{\Phi}) = -\frac{1}{k}\frac{l e^{\Phi}}{16\pi^2 r^4 n}$$

where n = n(r) is the baryon number density, $N_B = N_B(r)$ is the total baryon number in the sphere with radius r

$$\frac{\partial N_B}{\partial r} = 4\pi r^2 n \left(1 - \frac{2M}{r}\right)^{-1/2}$$

F. Weber: Pulsars as Astro. Labs ... (1999);

D. Blaschke Grigorian, Voskresensky, A& A 368 (2001)561.

Equations for Cooling Evolution

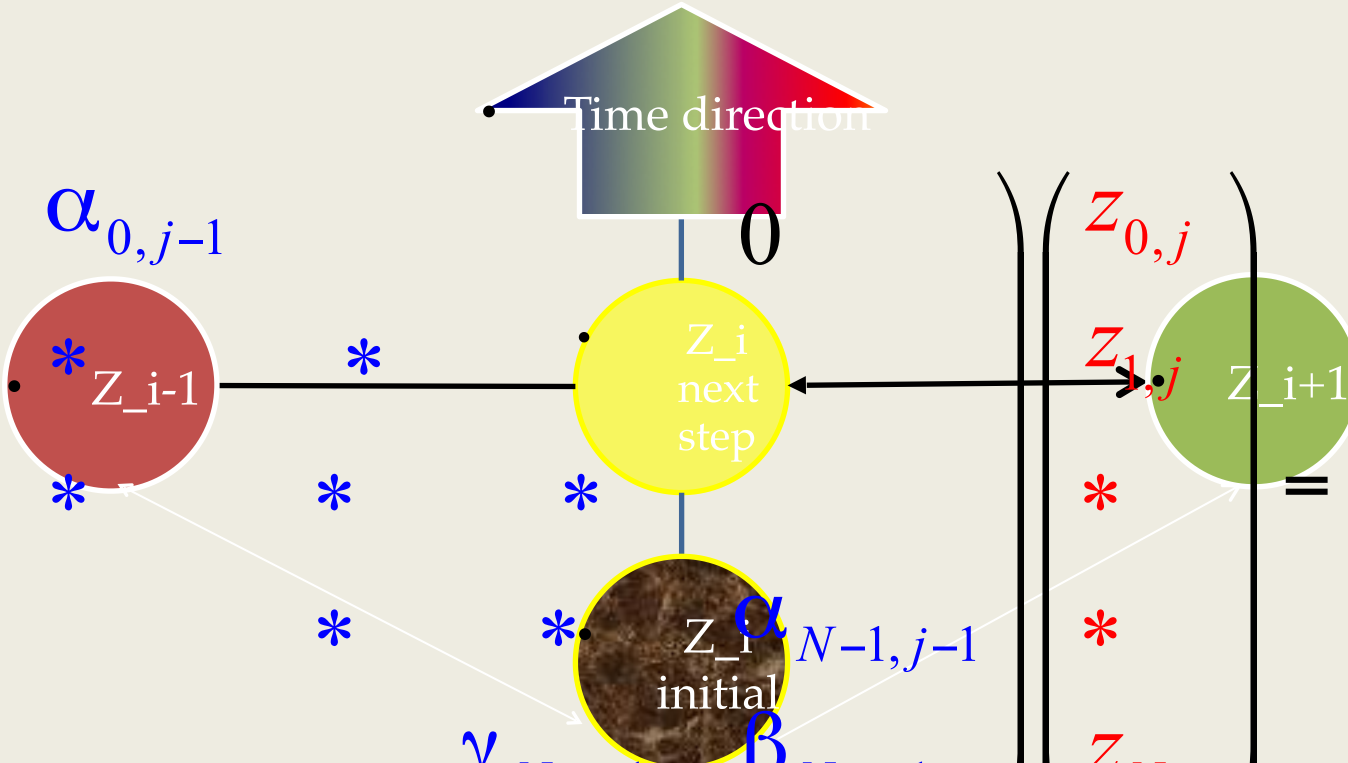
$$\begin{cases} \dfrac{\partial z(\tau, a)}{\partial \tau} = A(z,a)\dfrac{\partial L(\tau, a)}{\partial a} + B(z,a) \\ L(\tau, a) = C(z,a)\dfrac{\partial z(\tau, a)}{\partial a} \end{cases}$$

$$z(\tau, a) = \log T(\tau, a)$$

$$L_{i\pm 1/2} = \pm \frac{C_i + C_{i\pm 1}}{2} \frac{z_{i\pm 1} - z_i}{\Delta a_{i-1/2(1\boxed{?}1)}}$$

$$\frac{\partial L_i}{\partial a} = 2 \frac{L_{i+1/2} - L_{i-1/2}}{\Delta a_i + \Delta a_{i-1}}$$

Finite difference scheme



$$\begin{pmatrix} \beta_{0,j-1} & \alpha_{0,j-1} & & 0 & \\ \gamma_{1,j-1} & * & * & & \\ & * & * & * & \\ & & * & * & \alpha_{N-1,j-1} \\ 0 & & & \gamma_{N,j-1} & \beta_{N,j-1} \end{pmatrix} \begin{pmatrix} z_{0,j} \\ z_{1,j} \\ * \\ * \\ z_{N,j} \end{pmatrix} = \begin{pmatrix} \delta_{0,j-1} \\ \delta_{1,j-1} \\ * \\ * \\ \delta_{N,j-1} \end{pmatrix}$$

$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

Crust Model

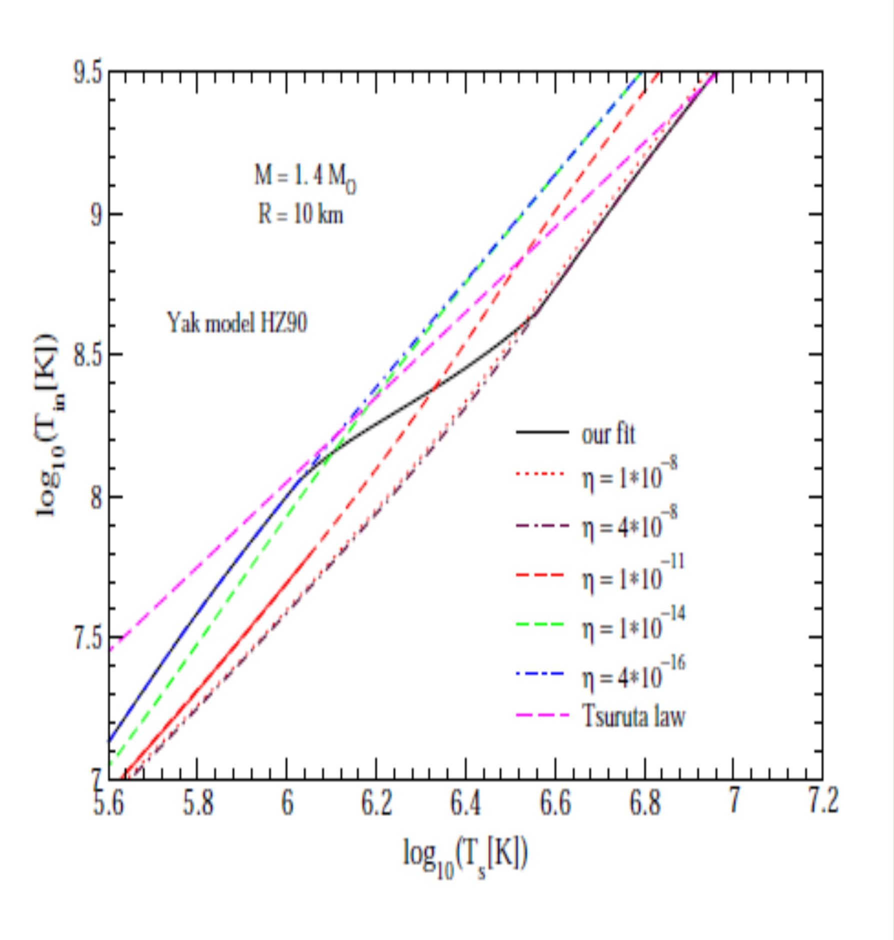
Time dependence of the light element contents in the crust

$$\Delta M_{\rm L}(t) = e^{-t/\tau}\,\Delta M_{\rm L}(0)$$

Blaschke, Grigorian, Voskresensky, A&A 368 (2001)561.

Page, Lattimer, Prakash & Steiner, Astrophys.J. 155,623 (2004)

Yakovlev, Levenfish, Potekhin, Gnedin & Chabrier, Astron. Astrophys, 417, 169 (2004)



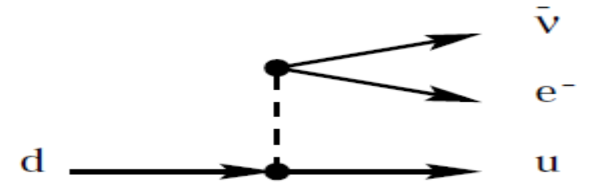
Neutrino emissivities in quark matter:

- Quark direct Urca (QDU) the most efficient processes

$d \to u + e + \bar{\nu}$ and $u + e \to d + \nu$

$\epsilon_\nu^{\rm QDU} \simeq 9.4 \times 10^{26} \alpha_s u Y_e^{1/3} \zeta_{\rm QDU}\, T_9^6$ erg cm^{-3} s^{-1},

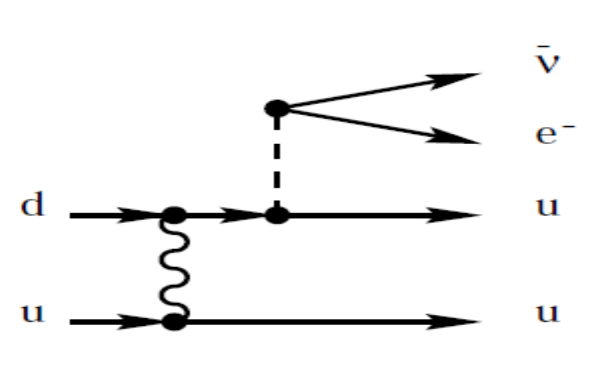
Compression n/n0 ≃ 2 , strong coupling αs ≈ 1



- Quark Modified Urca (QMU) and Quark Bremsstrahlung

$d + q \to u + q + e + \bar{\nu}$ and $q_1 + q_2 \to q_1 + q_2 + \nu + \bar{\nu}$

$\epsilon_\nu^{\rm QMU} \sim \epsilon_\nu^{\rm QB} \simeq 9.0 \times 10^{19} \zeta_{\rm QMU}\, T_9^8$ erg cm^{-3} s^{-1}.



- Suppression due to the pairing

QDU : $\zeta_{\rm QDU} \sim$ exp$(-\Delta_q/T)$

QMU and **QB** : $\zeta_{\rm QMU} \sim$ exp$(-2\Delta_q/T)$ for $T < T_{{\rm crit},q} \simeq 0.57\, \Delta_q$

- Enhanced cooling due to the pairing

- $e + e \to e + e + \nu + \bar{\nu}$ (becomes important for $\Delta_q/T >> 1$)

$\epsilon_\nu^{ee} = 2.8 \times 10^{12}\, Y_e^{1/3} u^{1/3} T_9^8$ erg cm^{-3} s^{-1},

Quark PBF

Neutrino emissivities in hadronic matter:

- Direct Urca (DU) the most efficient processes

$$\epsilon_{DU} = M_{DU} * (m_p^*)(m_n^*) * \Gamma^2_{wN} * (n_e)^{1/3}(T_9)^6 * R_D;$$
$$M_{DU} = 4 \times 10^{27} \quad erg/s/cm^3$$

- Modified Urca (MU) and Bremsstrahlung

$$\epsilon_{MUp} = F_M * M_p * (m_p)^3(m_n^*)(T_9)^8(n_e)^{1/3} * R_{MUp}(v_n, v_p);$$
$$\epsilon_{nnBS} = P_{nnBS} * R^{nn}_{BS}(v_n) * \Gamma^2_w \Gamma^4_s (n_b)^{4/3}(T_9)^8(m_n^*)^4/(\omega)^3;$$

- Suppression due to the pairing

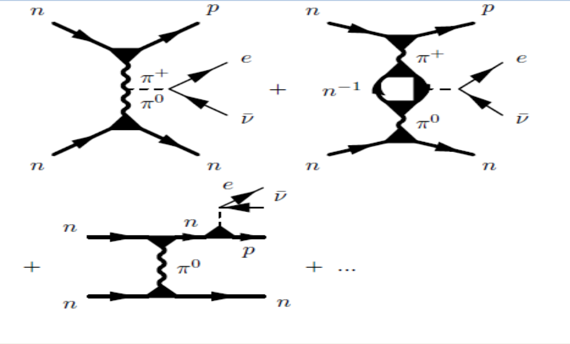
$$v_N = \Delta_N(T)/T = \sqrt{1-\tau_N}\left(1.456 - \frac{0.157}{\sqrt{\tau_N}} + \frac{1.766}{\tau_N}\right)$$

- Enhanced cooling due to the pairing

$$\begin{aligned}\epsilon_\nu^{\mathrm{NPBF}} &= 6.6 \times 10^{28}(m_n^*/m_n)(\Delta_n(T)/\mathrm{MeV})^7 \; u^{1/3} \\ &\times \xi \; I(\Delta_n(T)/T) \;\mathrm{erg\;cm^{-3}s^{-1}}, \\ \epsilon_\nu^{\mathrm{PPBF}} &= 0.8 \times 10^{28}(m_p^*/m_p)(\Delta_p(T)/\mathrm{MeV})^7 \; u^{2/3} \\ &\times \; I(\Delta_p(T)/T) \;\mathrm{erg\;cm^{-3}s^{-1}},\end{aligned}$$

Medium Effects In Cooling Of Neutron Stars

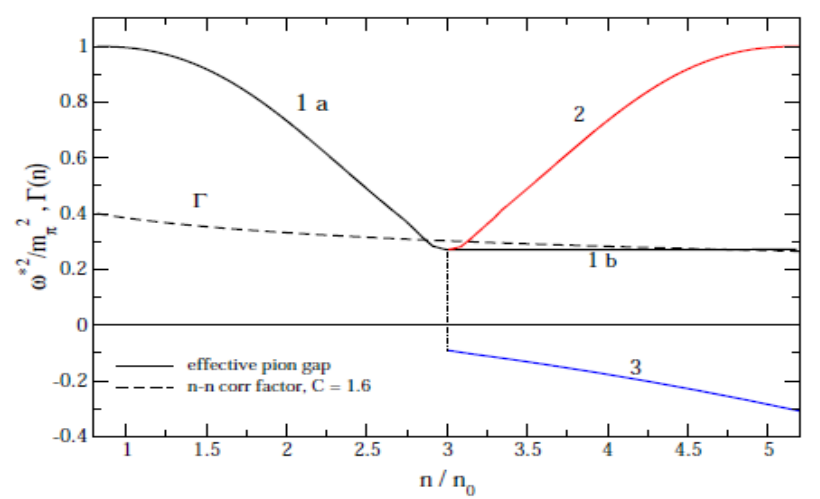
- Based on Fermi liquid theory (Landau (1956), Migdal (1967), Migdal et al. (1990))
- MMU – insted of MU



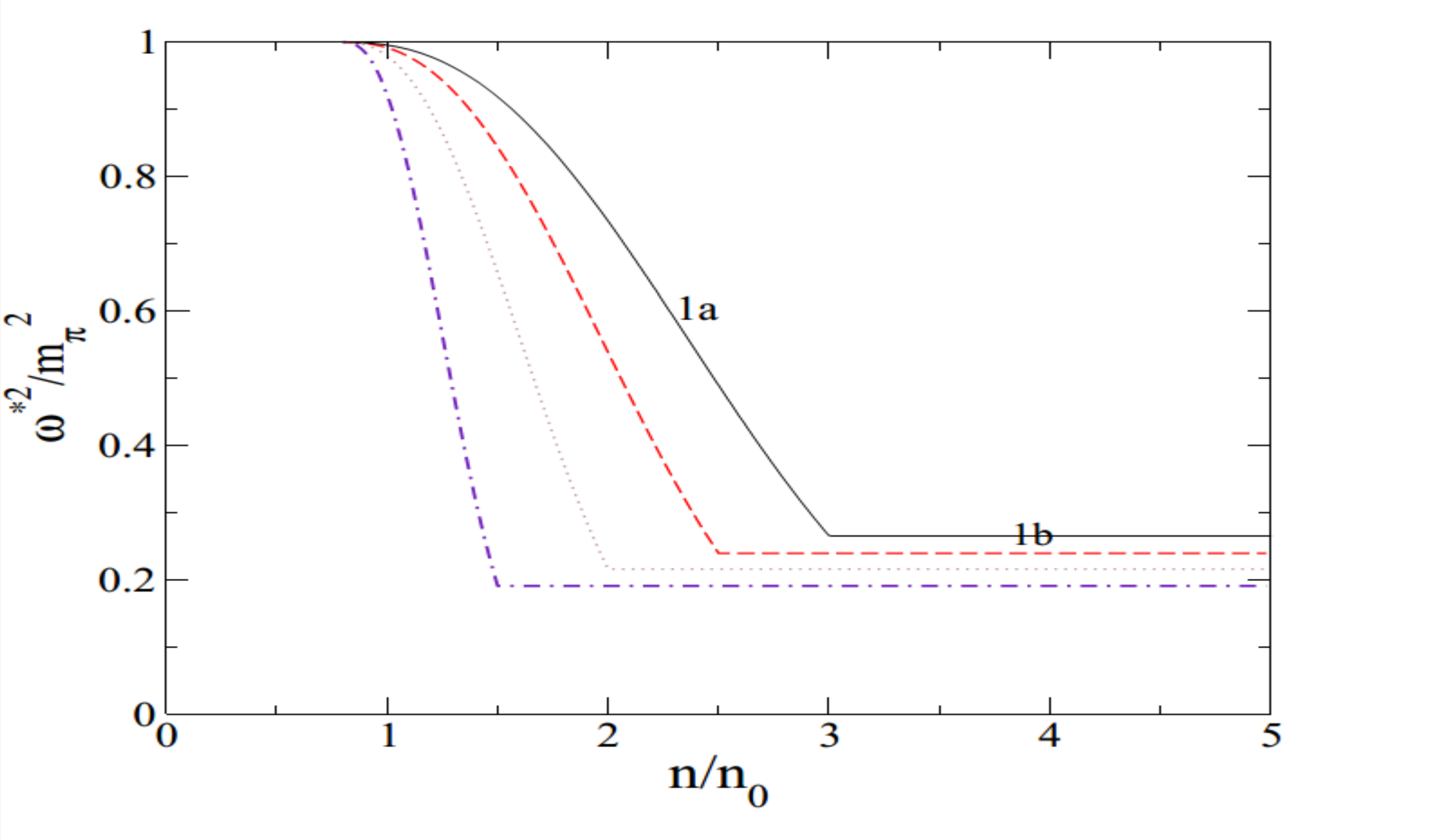
$$\frac{\varepsilon_\nu[\mathrm{MMU}]}{\varepsilon_\nu[\mathrm{MU}]} \sim 10^3\ (n/n_0)^{10/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^8},$$

- Main regulator in Minimal Cooling

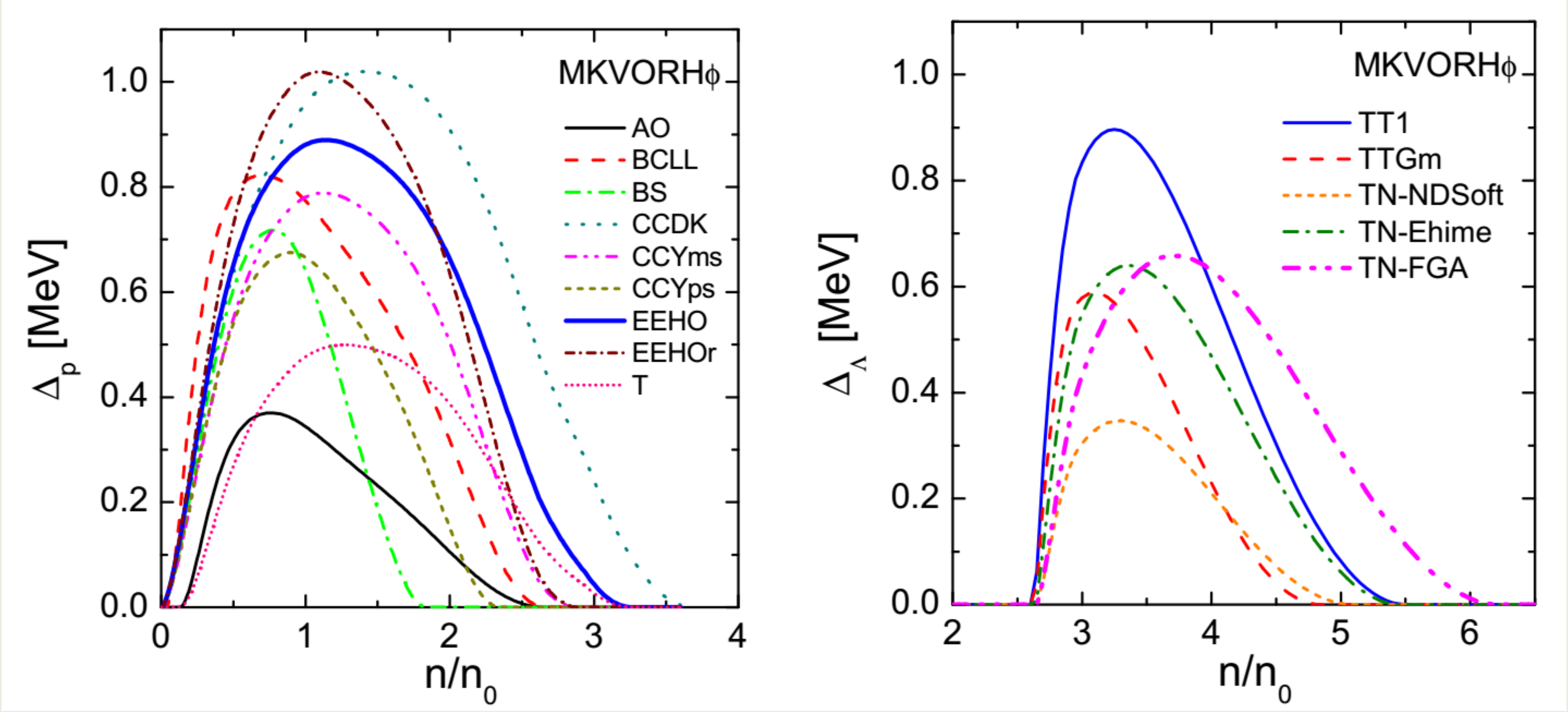
$$\varepsilon_\nu[\mathrm{MpPBF}] \sim 10^{29}\ \frac{m_N^*}{m_N}\left[\frac{p_{Fp}}{p_{Fn}(n_0)}\right]\ \left[\frac{\Delta_{pp}}{\mathrm{MeV}}\right]^7 \times \left[\frac{T}{\Delta_{pp}}\right]^{1/2}\ \xi_{pp}^2\ \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{sec}},\quad T < T_{cp}.$$



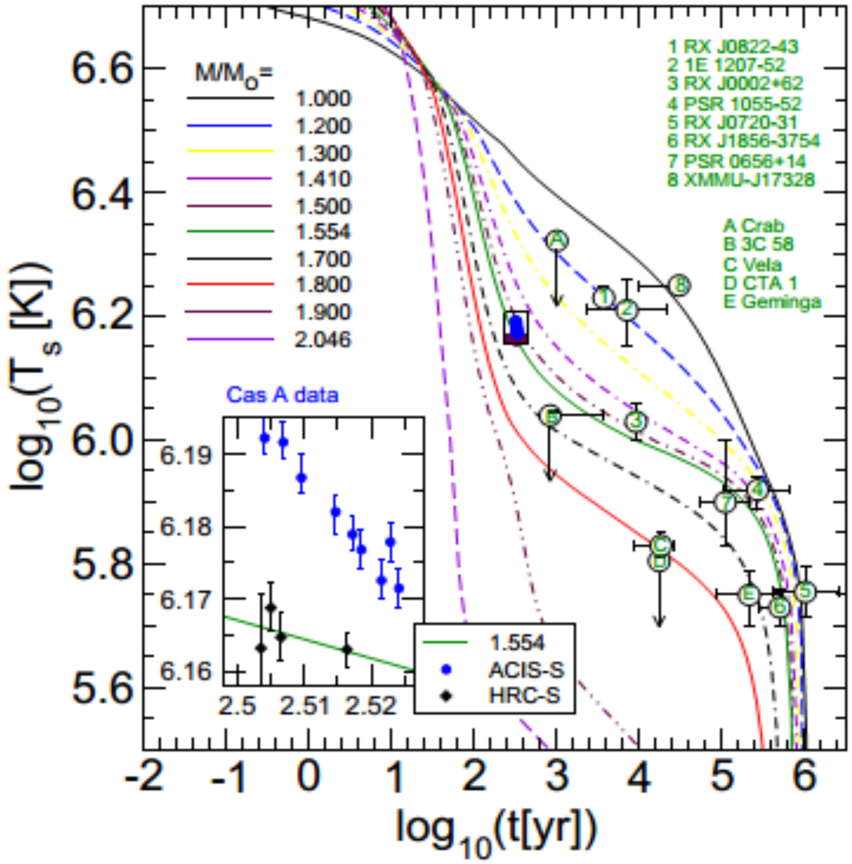
Medium Effects In Cooling Of Neutron Stars

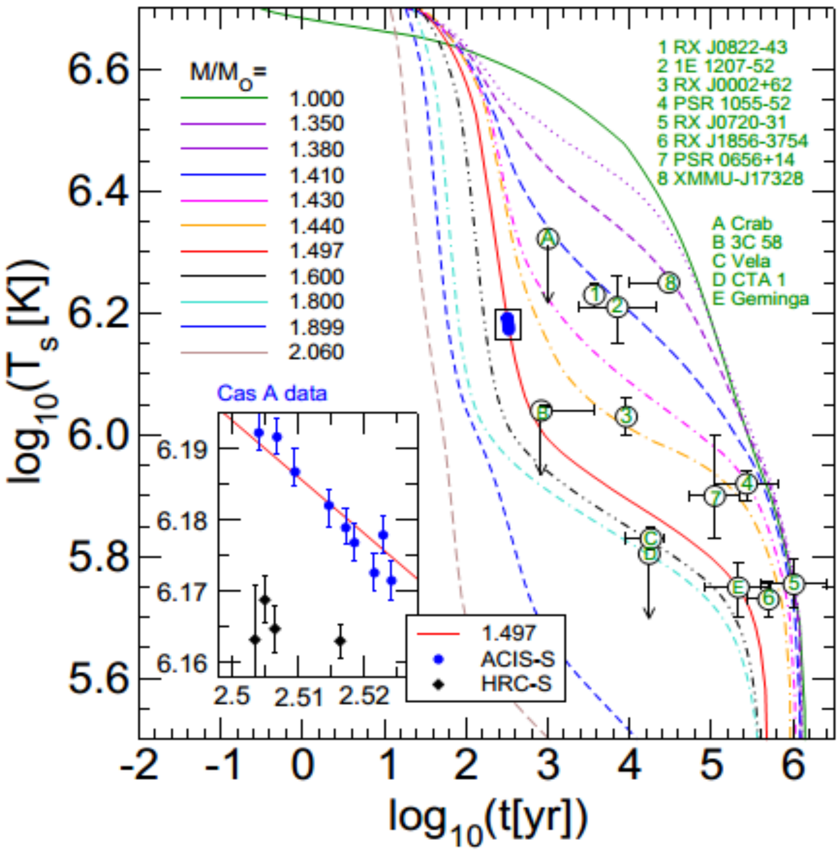


MKVORHp — Gap models

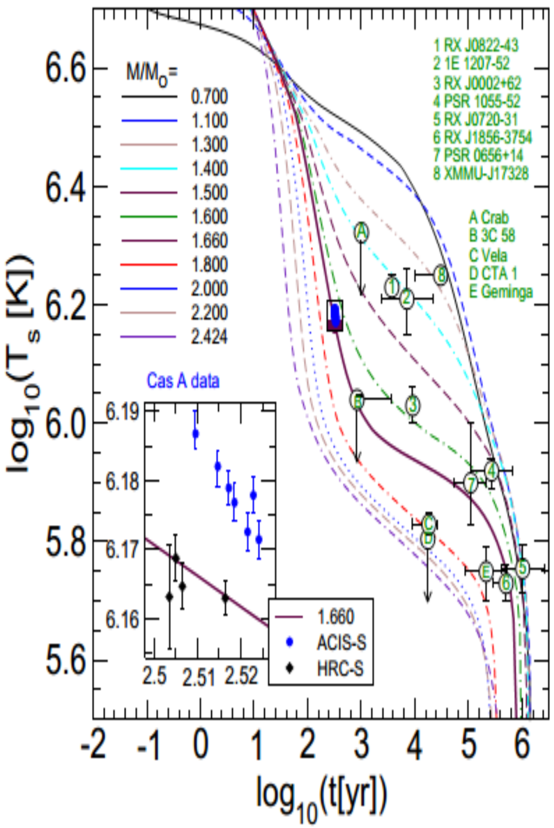


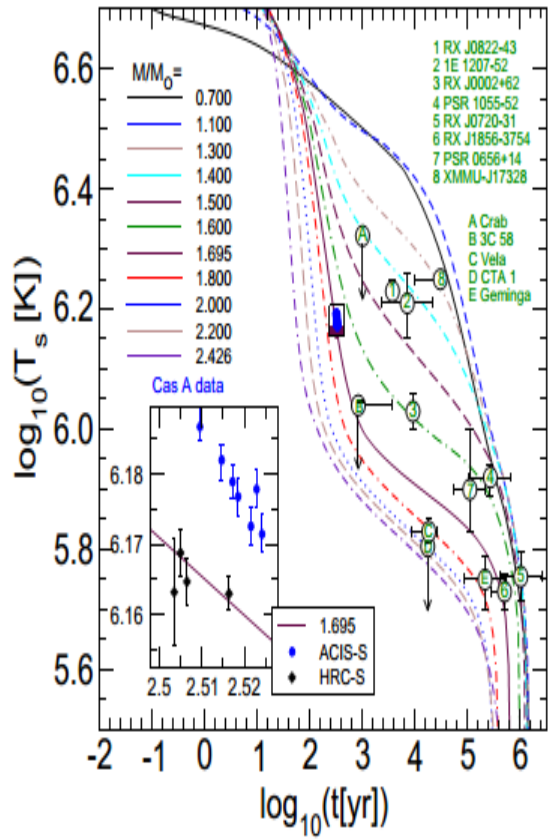
HDD – AV18 , Yak.
ME nc = 3 n0

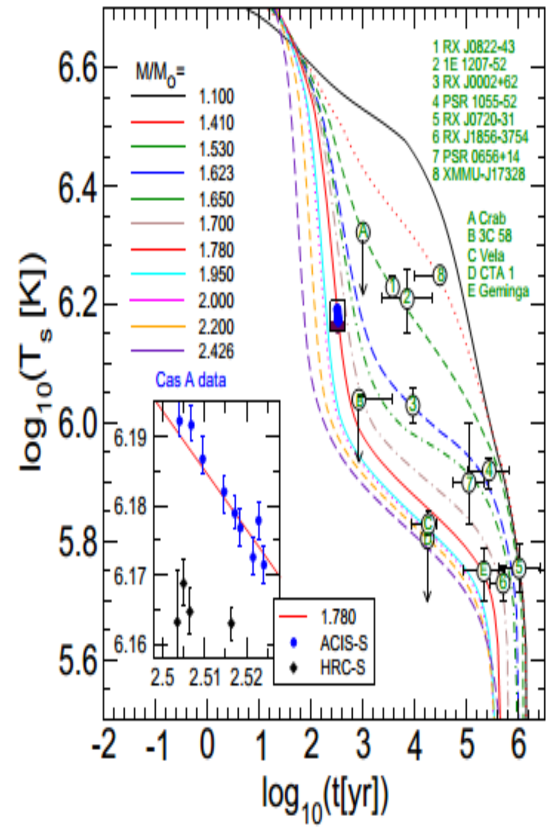




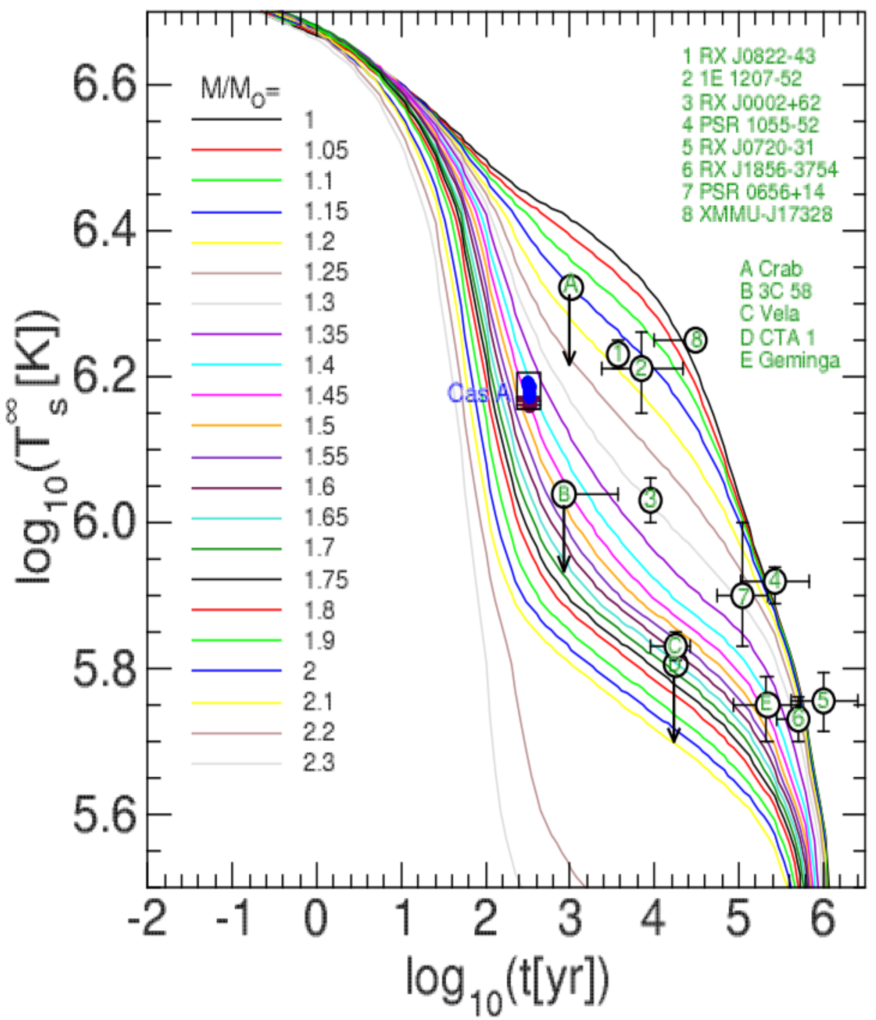
DD2 – EEHOr
ME-nc=1.5,2.0,2.5n0

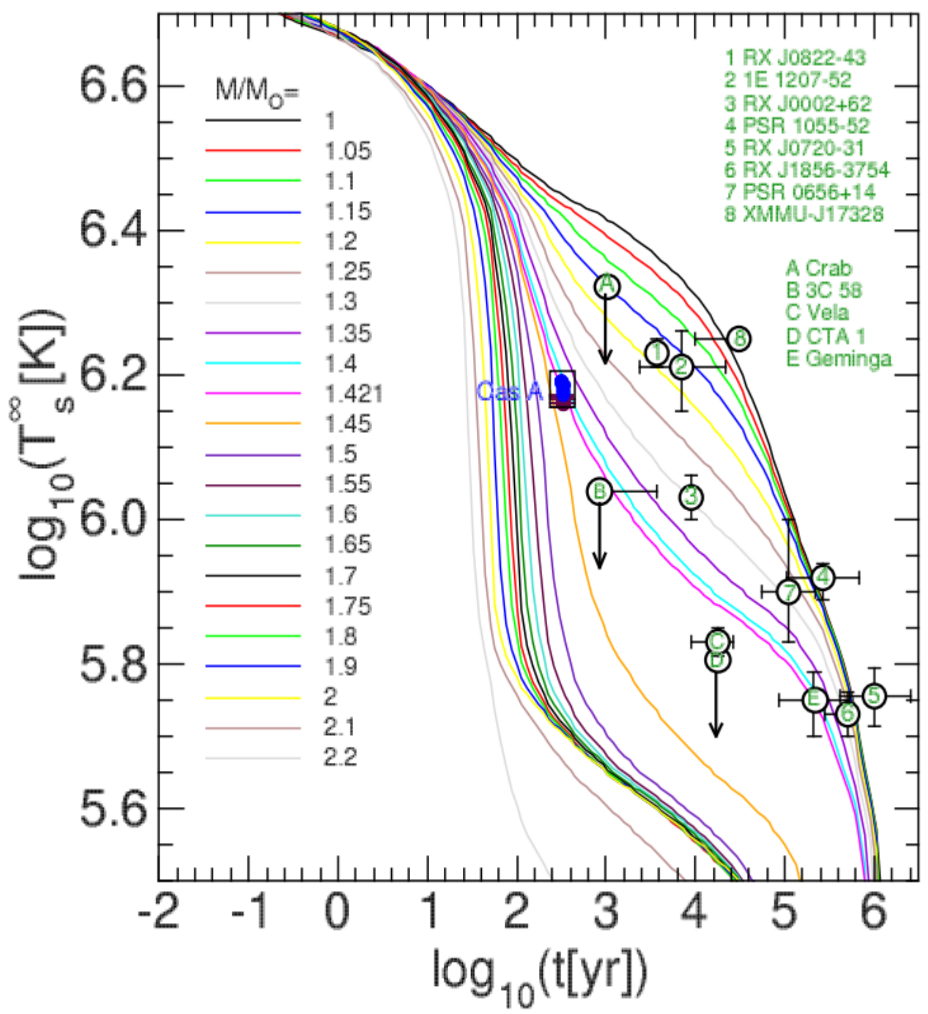




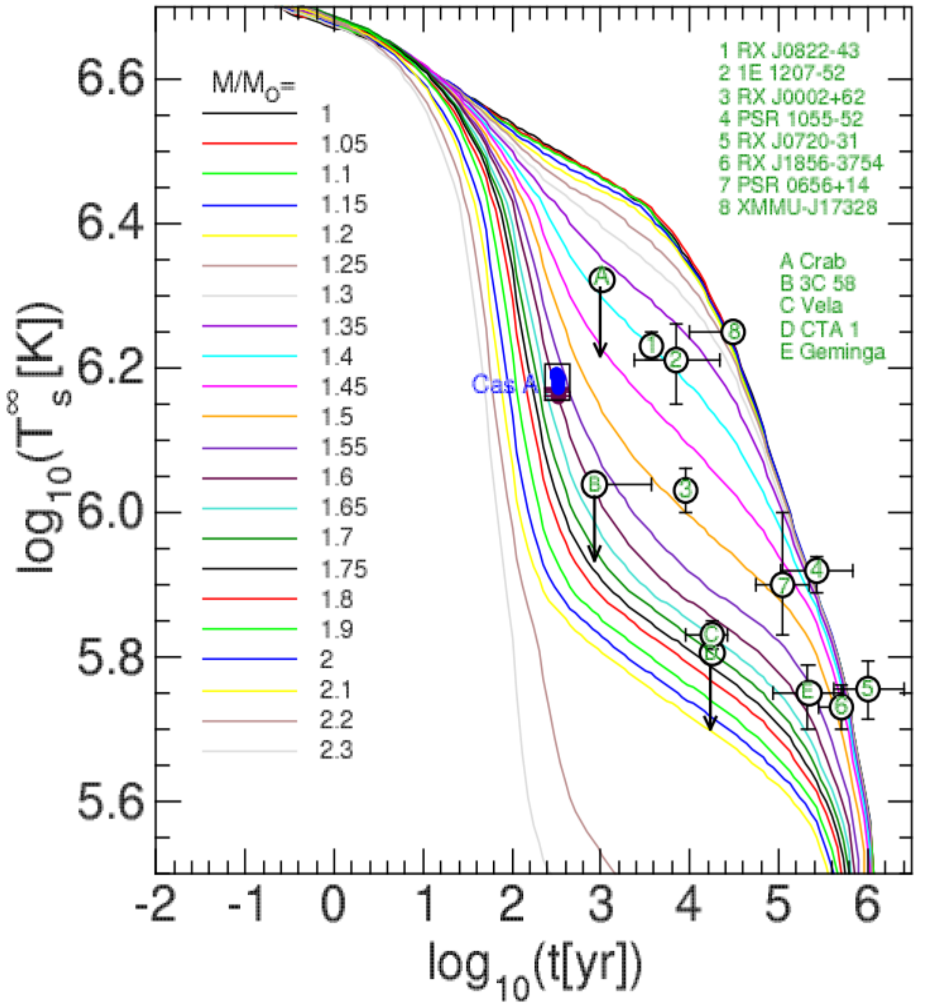


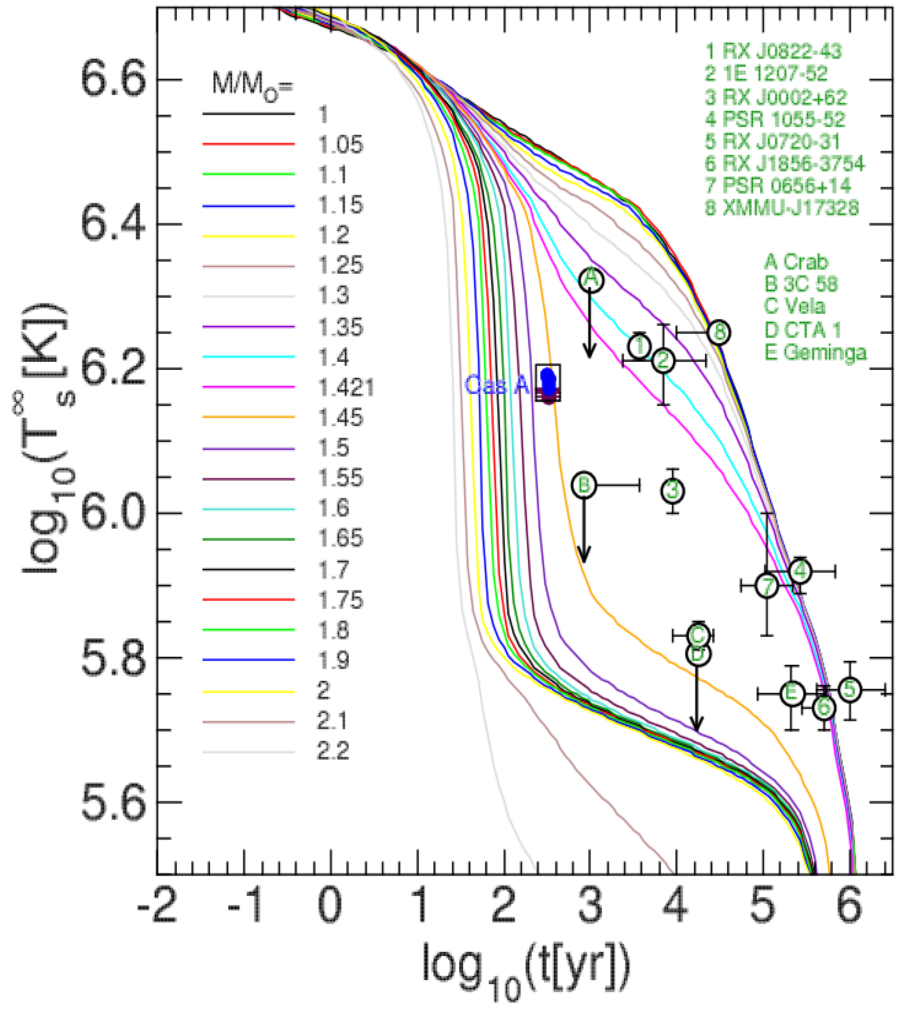
MKVOR – BCLL, TN-FGA ME-nc=3.0n0



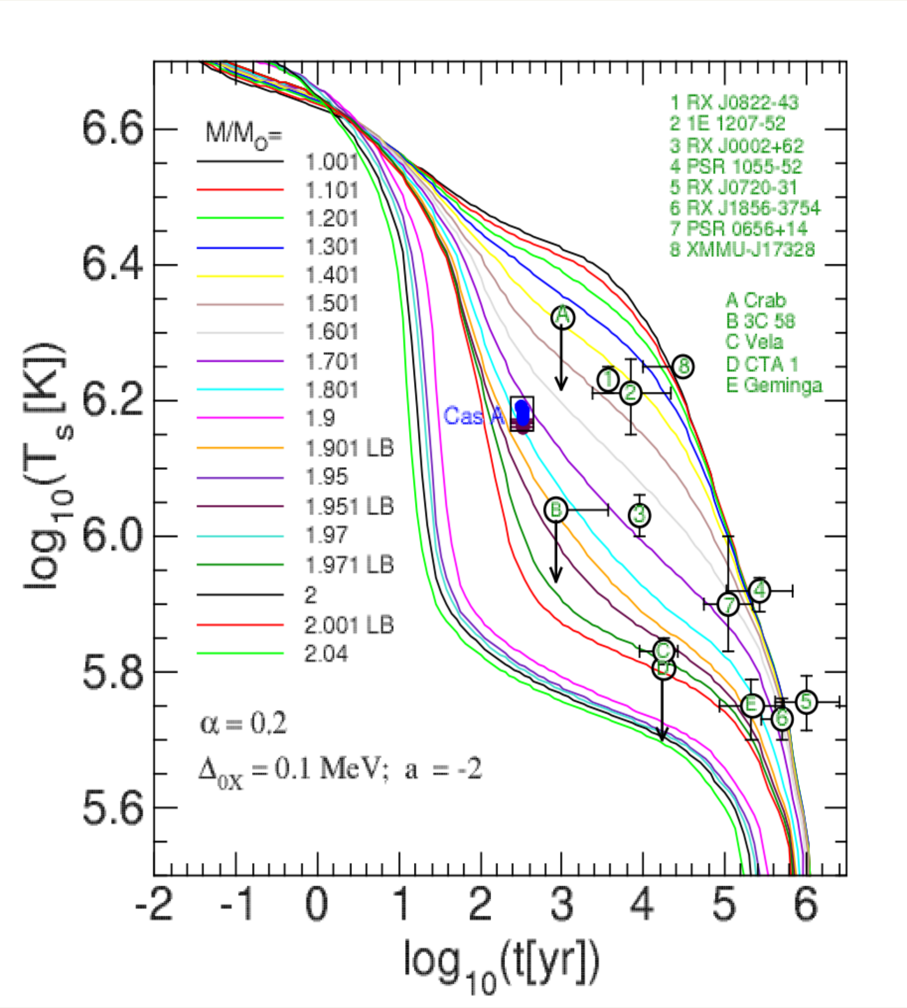


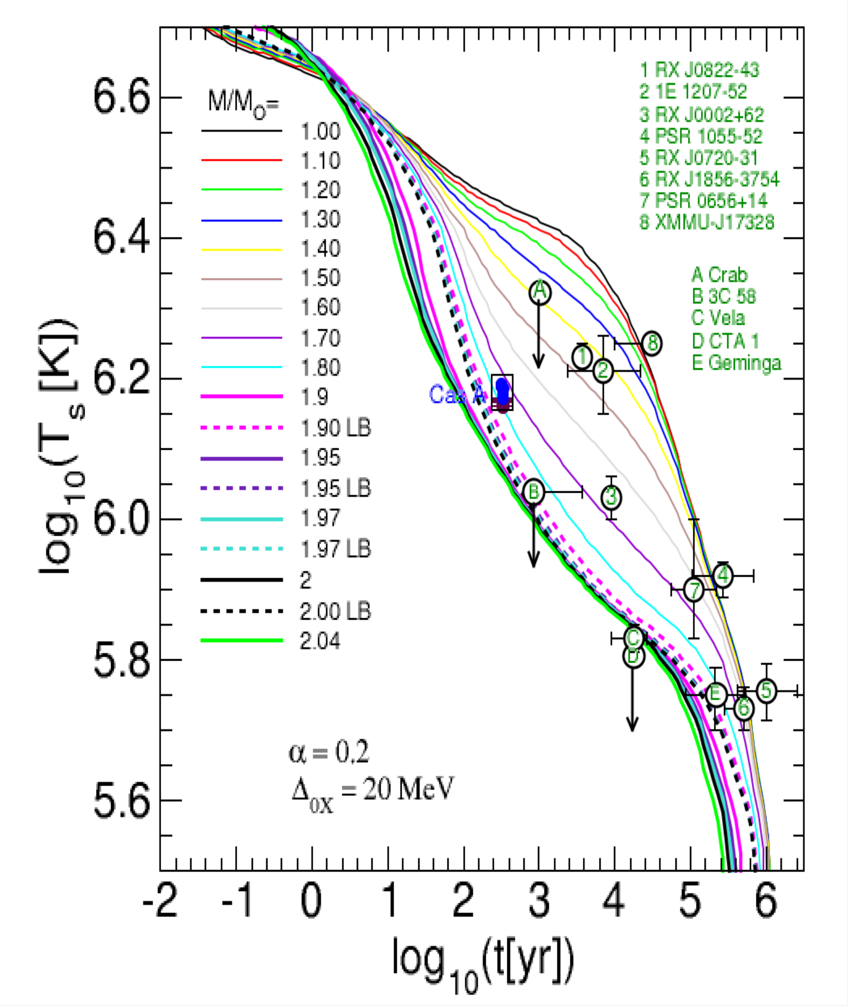
MKVOR Hyp – EEHOr, TN-FGA ME-nc=3.0n0



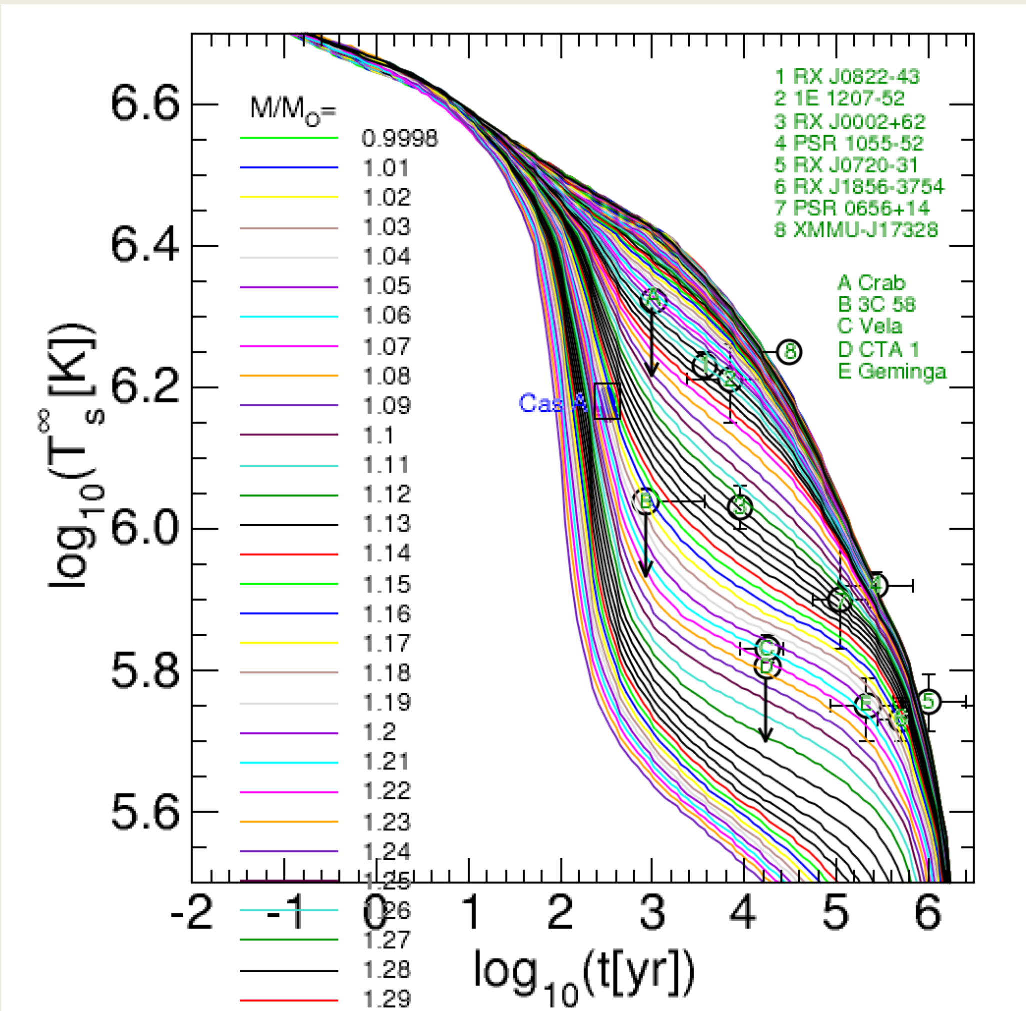


Cooling of Twin CS

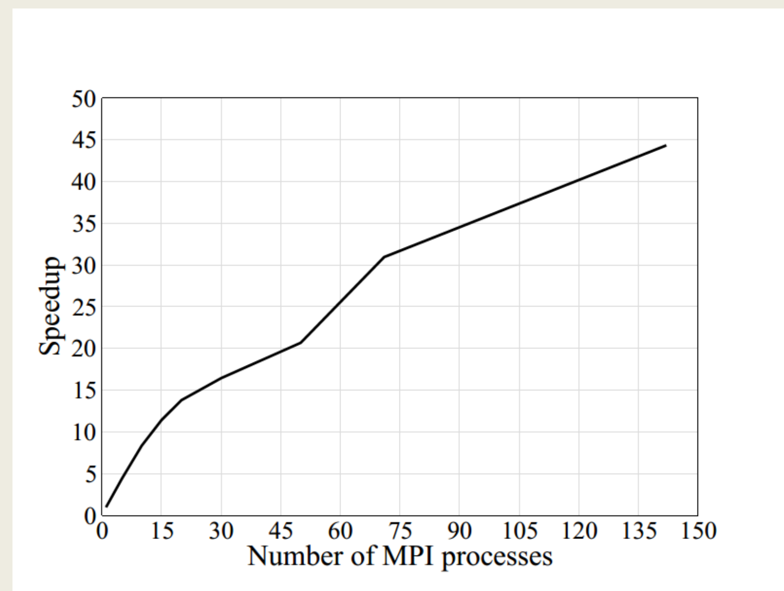




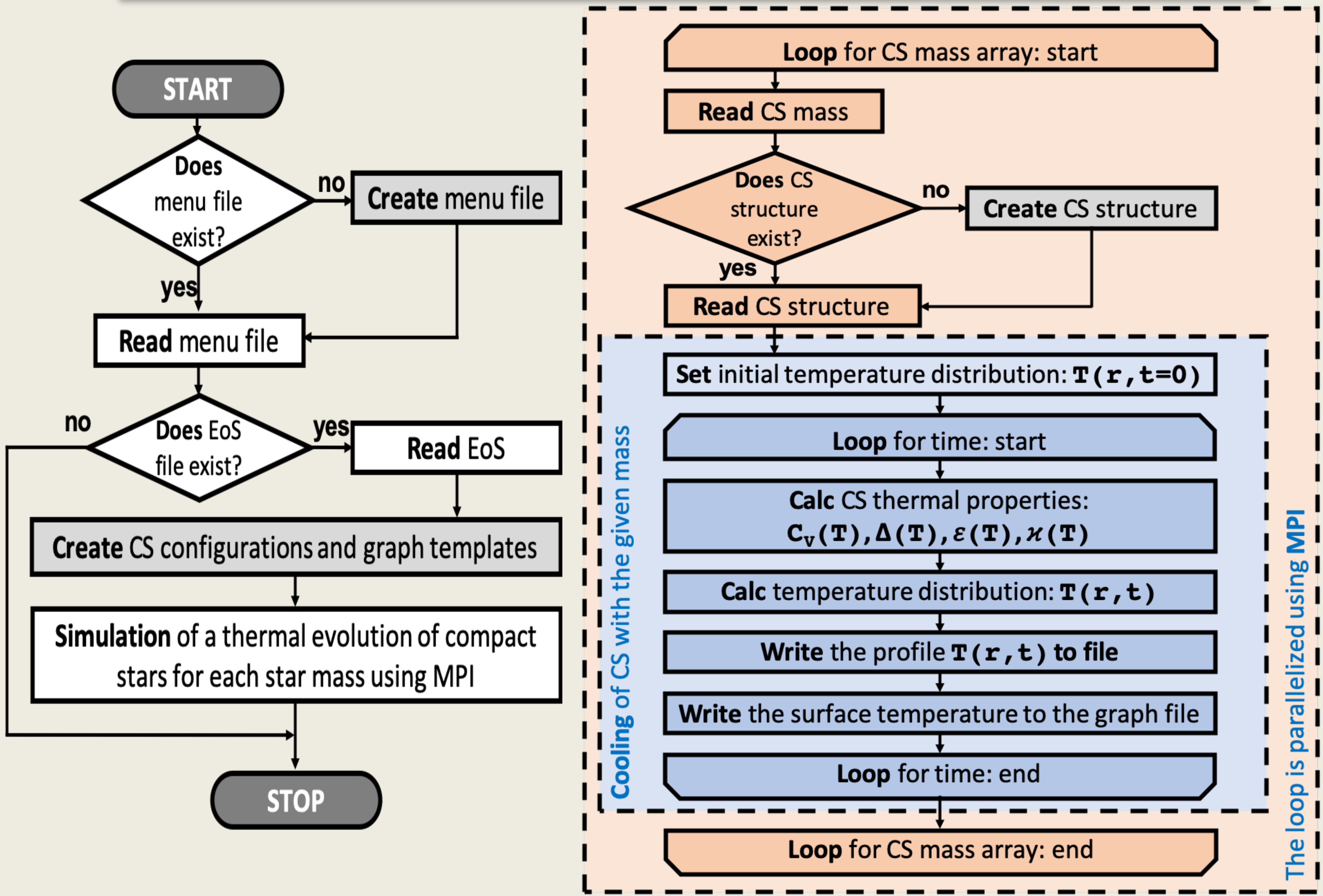
Results produced with use of MPI Technology



142 configurations has been calculated in 0m49s on the 142 processes.
On 1 process it takes 36m14s
– acc is ~ 44 times



Program Algorithm



Model parameters – DD2

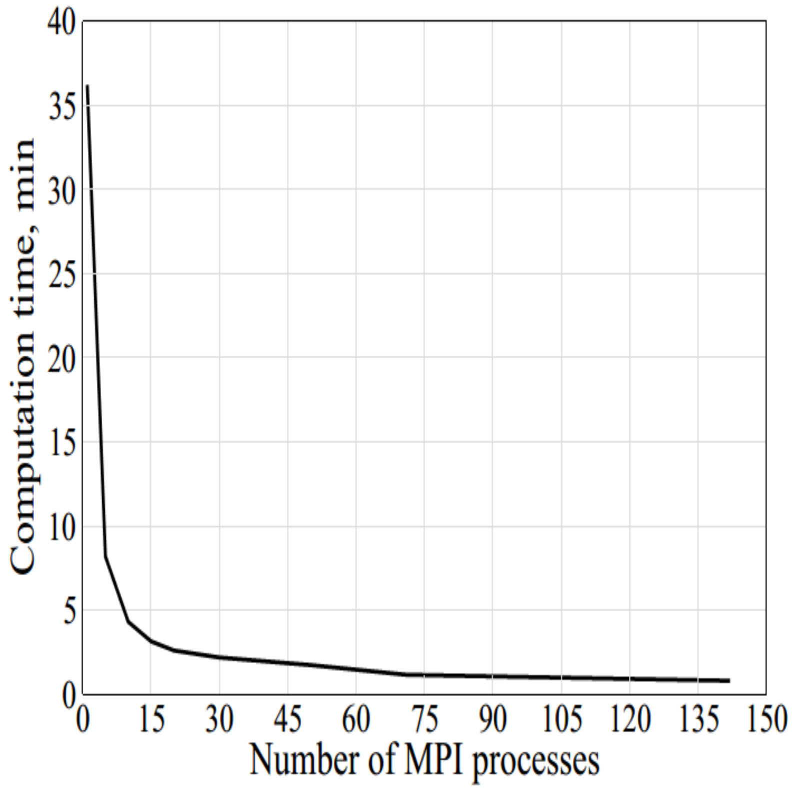
Menu_dd2_2017n.dat

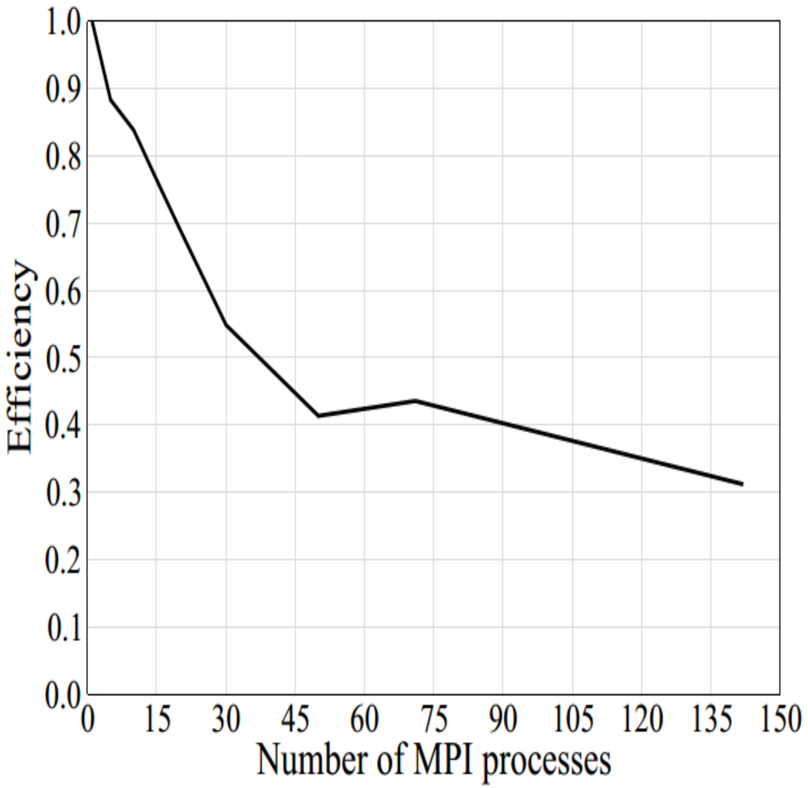
```
Model Parametrs

The HOME directory is : .\Data\DD2\Configs-2
The EV UOTPUT directory : .\Data\DD2\17-12-2019\EV-DD2-pi-F4-o3-D
Make EoS file : 0
Make new config. file : 0
Read full EoS from a file : 1
Read from : .\EoS\DD2_HG
Hadronic EoS
LWalecka (0)  NLW (1) HDD (3) BSk20 (4): 3
Normal Shell : 0
Quark EoS SM model (1) Bag model (0) : 0
In case of SM  GF (0)  GL(1)  NJL (2) : 0
with Quark core : 1
without Mixed phase : 1
Superconducting Quark core : 1
Quark Star : 0
Medium effects : 1
Pion condensate : 1
Crust Model (Yakovlev - Y Tsuruta - T our - G)  : G
Gaps in Hadrons Model (Yakovlev - Y AV18 - A Schwenk - U Armen-fit - F)  : F
for F-fit p-Gap
1-AO
2-BCLL
3-BS
4-CCDK
5-CCYms
6-CCYps
7-EEHO
8-EEHOr
9-T
: 4
for F-fit n-Gap
2-AWP2
3 - AWP3
4 - CCDK
5 - CLS
6 - GIPSF
7 - MSH
8 - SCLBL
9 - SFB
0 - WAP : o
XGaps in 2SC QModel constant 0 - 0
 constant 0.1 MeV - 1
constant 0.05 MeV - 5
constant 0.03 MeV - 3
 rising 0.03 +  MeV - A
 incrising 0.03 - MeV - B
       constant 0.03 ++ MeV - C
 constant 0.03 -- MeV - D
 : C
```

Menu_dd2_2017n.dat

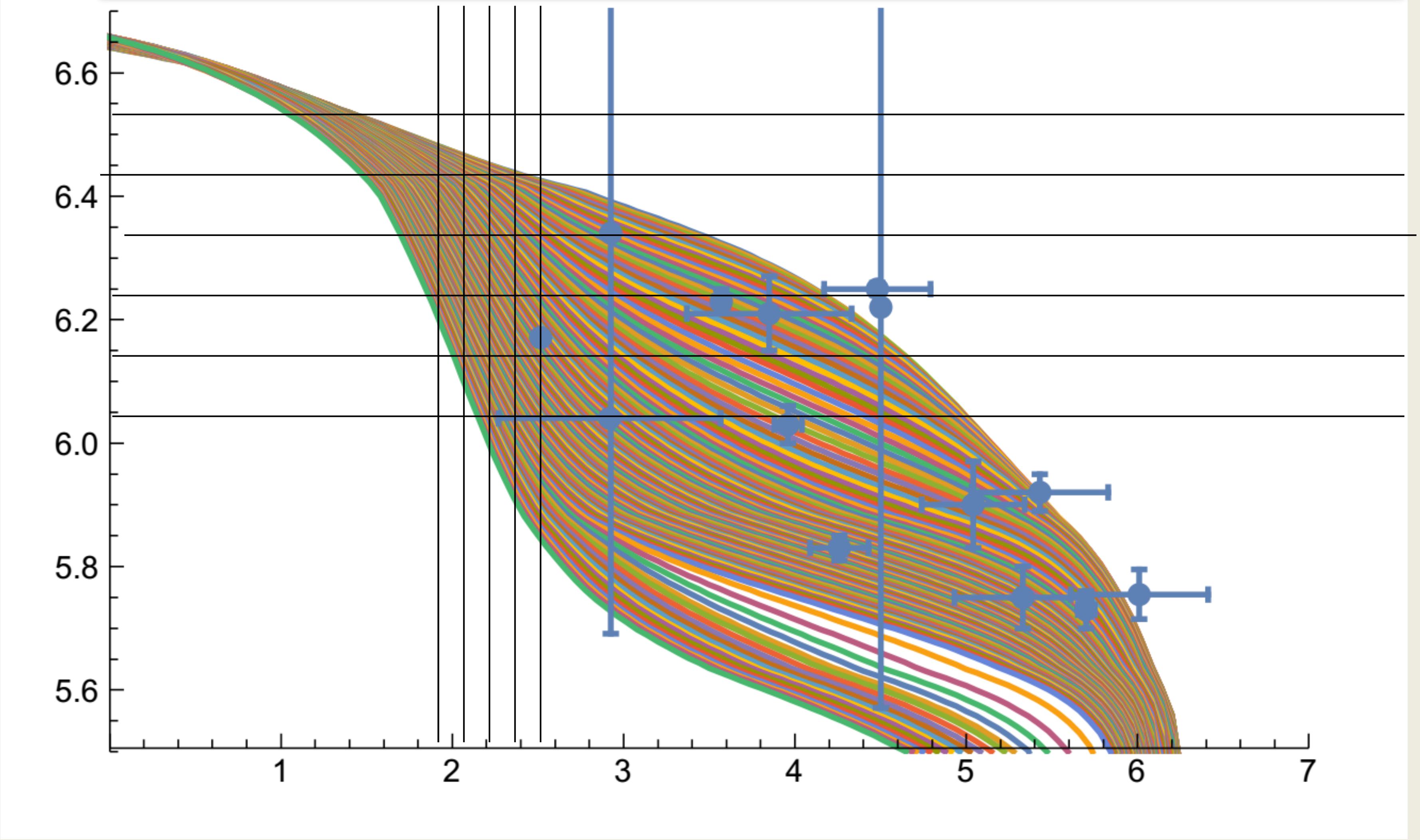
```
Gap factors in HM
Protons  1S0p : 1
Neutrons 1S0n : 1
Neutrons 3P2n : 0.1

End time point log10(t/yr) : 8

initial temperatur in MeV : 0.5

minimal value of log Temperature : 5.5

Print output files for LogN-LogS : 0

Print profiles for the time points : 0

Number of points : 7
0 0 0 0 0 0 0

 The Masses [Mo] of Configurations to be Cooled

Number of points : 51
1.450
0.5 0.51 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59
0.6 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69
0.7 0.71 0.72 0.73 0.74 0.75 0.76 0.77 0.78 0.79
0.8 0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89
0.9 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99



1.0 1.01 1.02 1.03 1.04 1.05 1.06 1.07 1.08 1.09
1.10 1.11 1.12 1.13 1.14 1.15 1.16 1.17 1.18 1.19
1.20 1.21 1.22 1.23 1.24 1.25 1.26 1.27 1.28 1.29
1.30 1.31 1.32 1.33 1.34 1.35 1.36 1.37 1.38 1.39
1.40 1.41 1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49
1.50 1.51 1.52 1.53 1.54 1.55 1.56 1.57 1.58 1.59
1.60 1.61 1.62 1.63 1.64 1.65 1.66 1.67 1.68 1.69
1.70 1.71 1.72 1.73 1.74 1.75 1.76 1.77 1.78 1.79
1.80 1.81 1.82 1.83 1.84 1.85 1.86 1.87 1.88 1.89
1.90 1.91 1.92 1.93 1.94 1.95 1.96 1.97 1.98 1.99
2.00 2.01 2.02 2.03 2.04
2.05 2.06 2.07 2.08 2.09
2.10 2.11 2.12 2.13 2.14 2.15 2.16 2.17 2.18 2.19
2.20
2.21 2.22 2.23 2.24 2.25 2.26 2.27 2.28 2.29
2.30 2.31 2.32 2.33 2.34 2.35 2.36 2.37 2.38 2.39
2.40 2.41

1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2
```

Calculation Time and efficiency

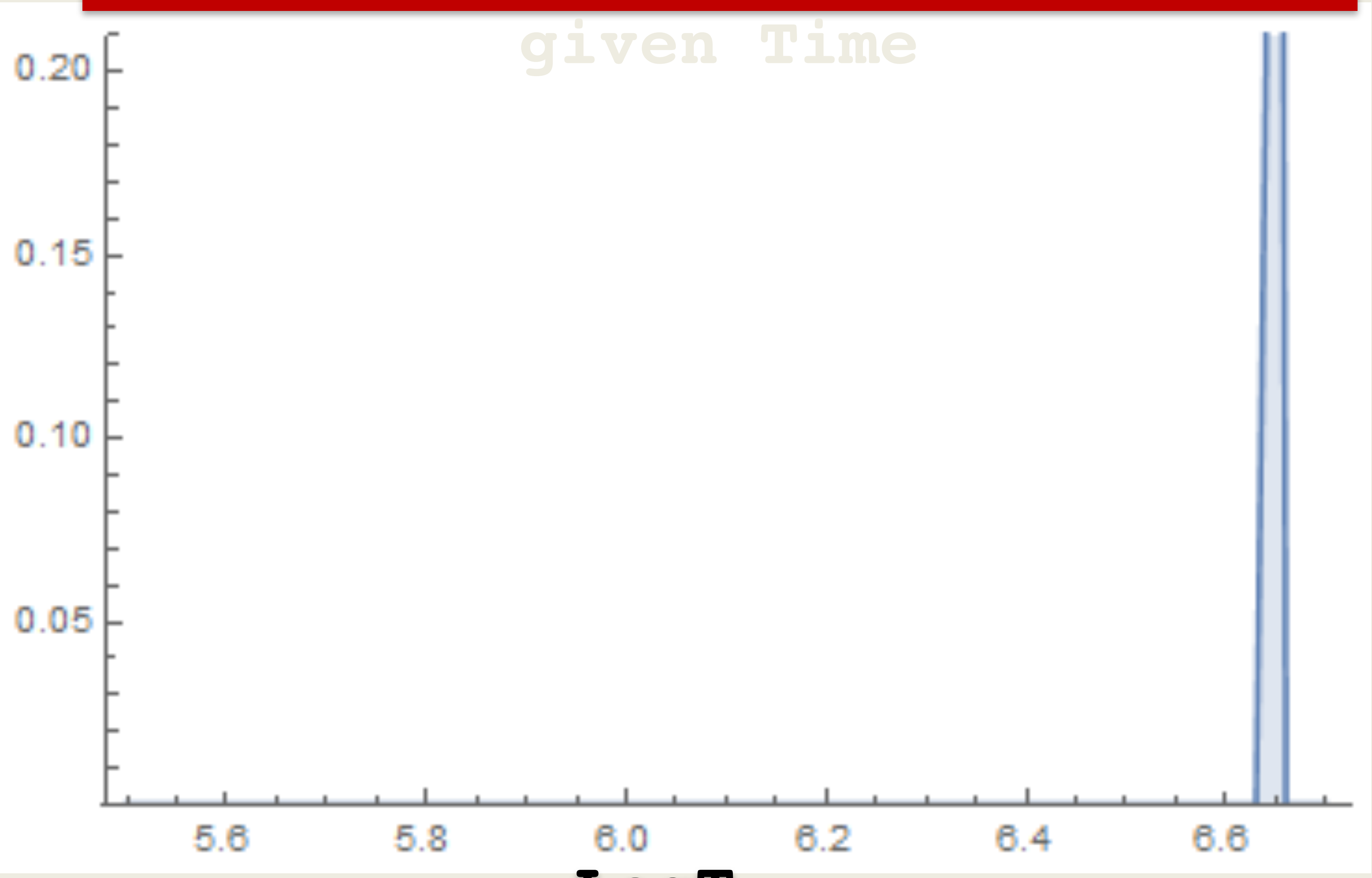




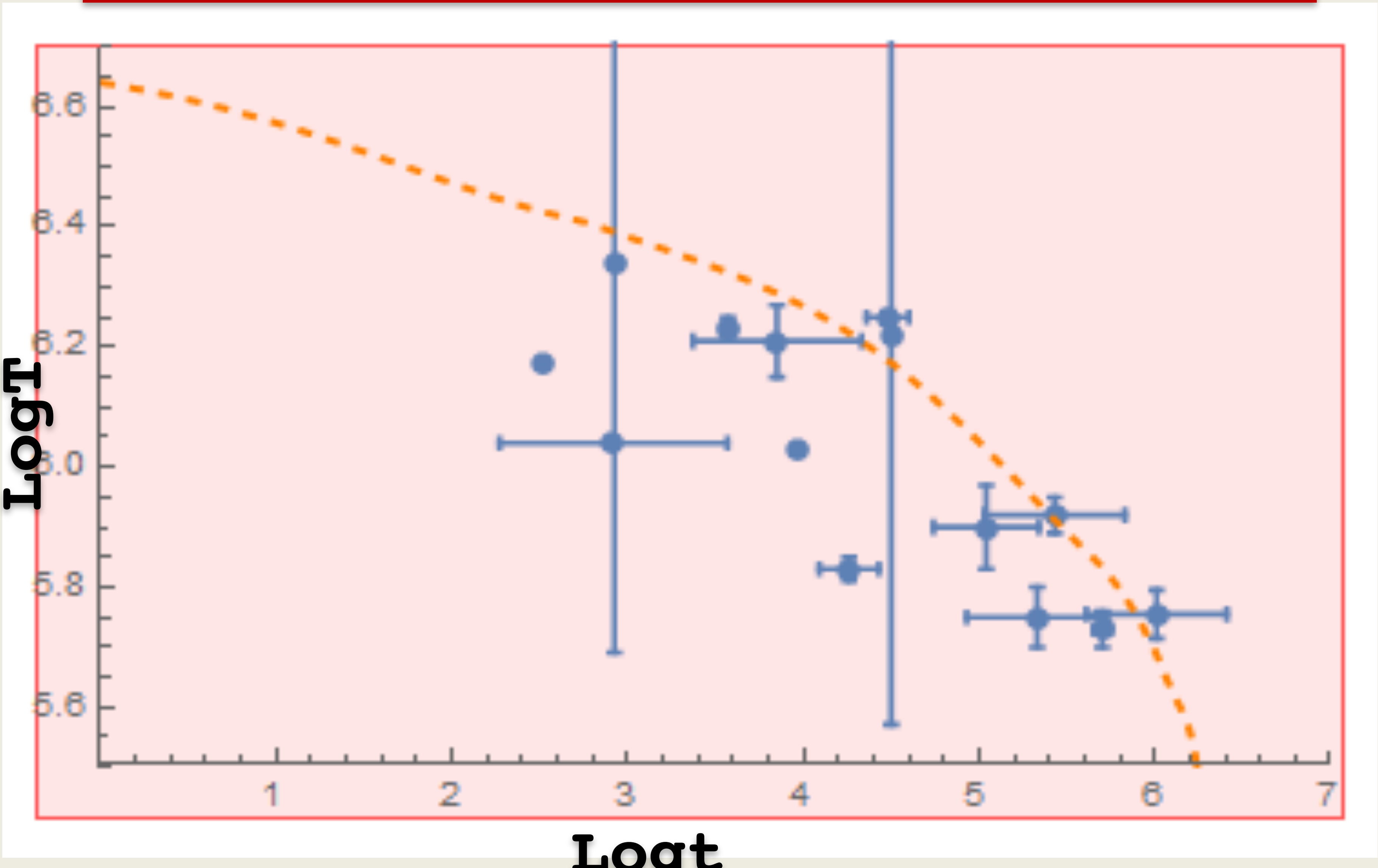
Distribution of Evolution tracks via Temperature at given Time



Distribution of Evolution tracks via Temperature at given Time

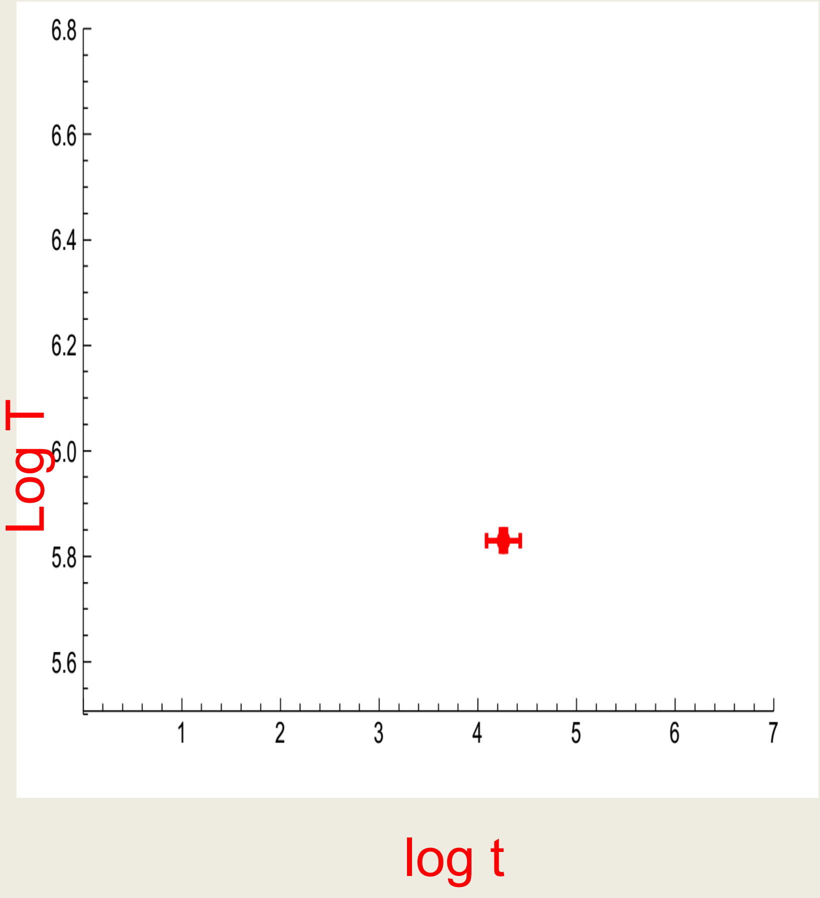


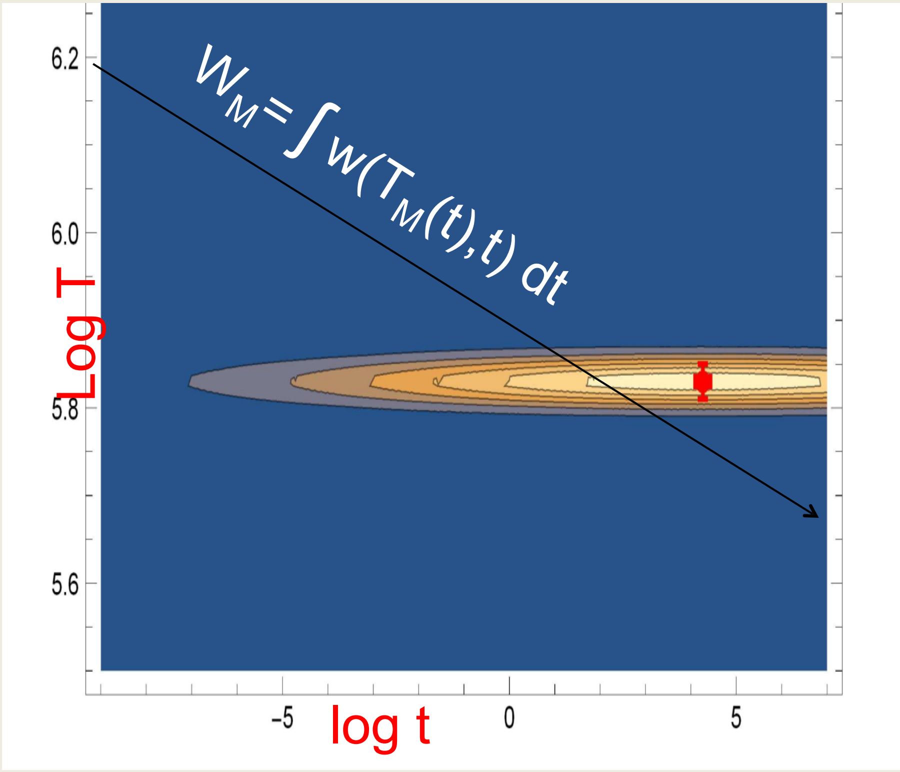
Evolution tracks for different NS Masses



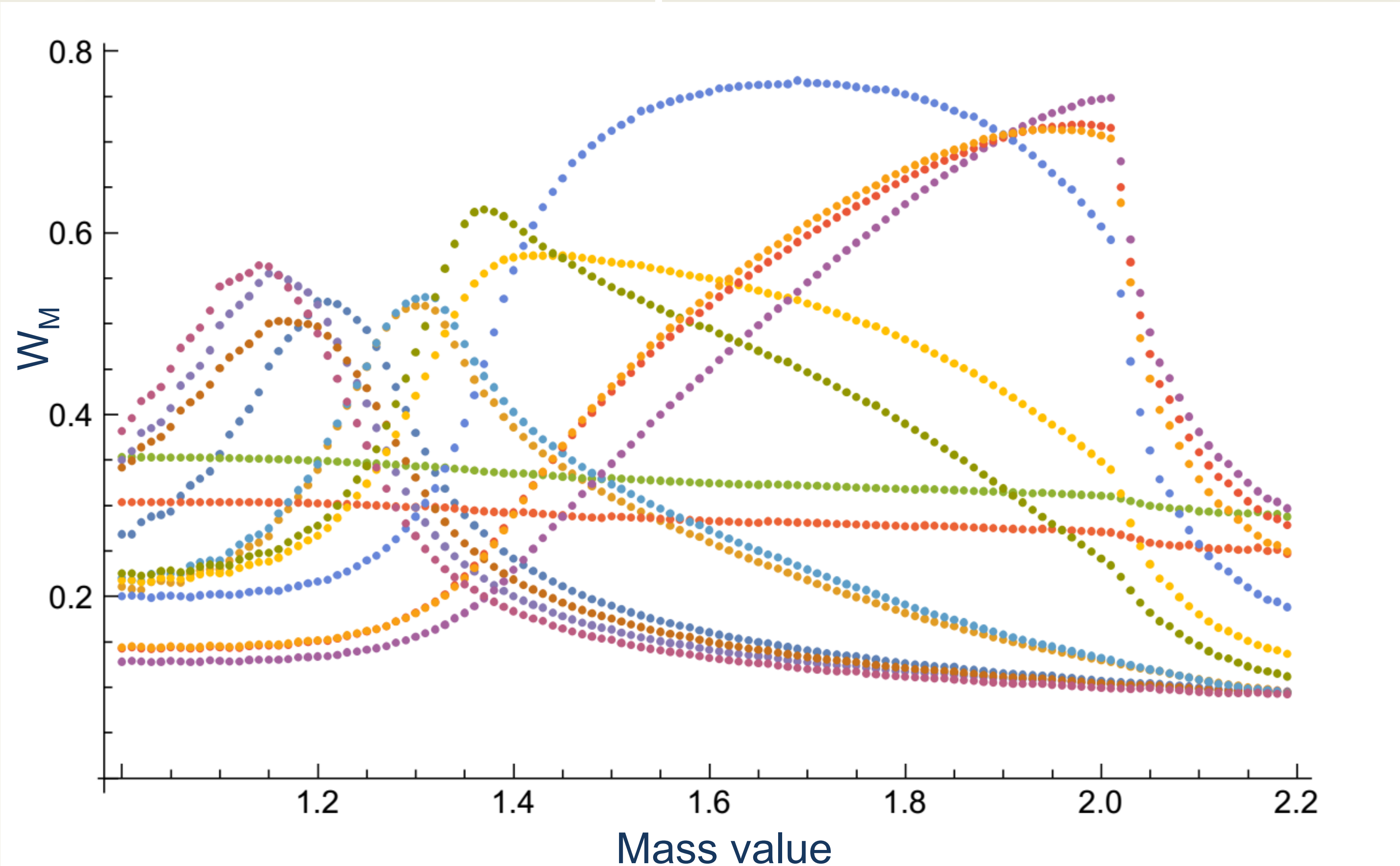
Weighting of Data point on the Temperature – Age Diagram

$$w(T,t) = \mathrm{Exp}\{(\log T-\log T_D)^2/\sigma_T + (\log t -\log t_D)^2/\sigma_t\}$$

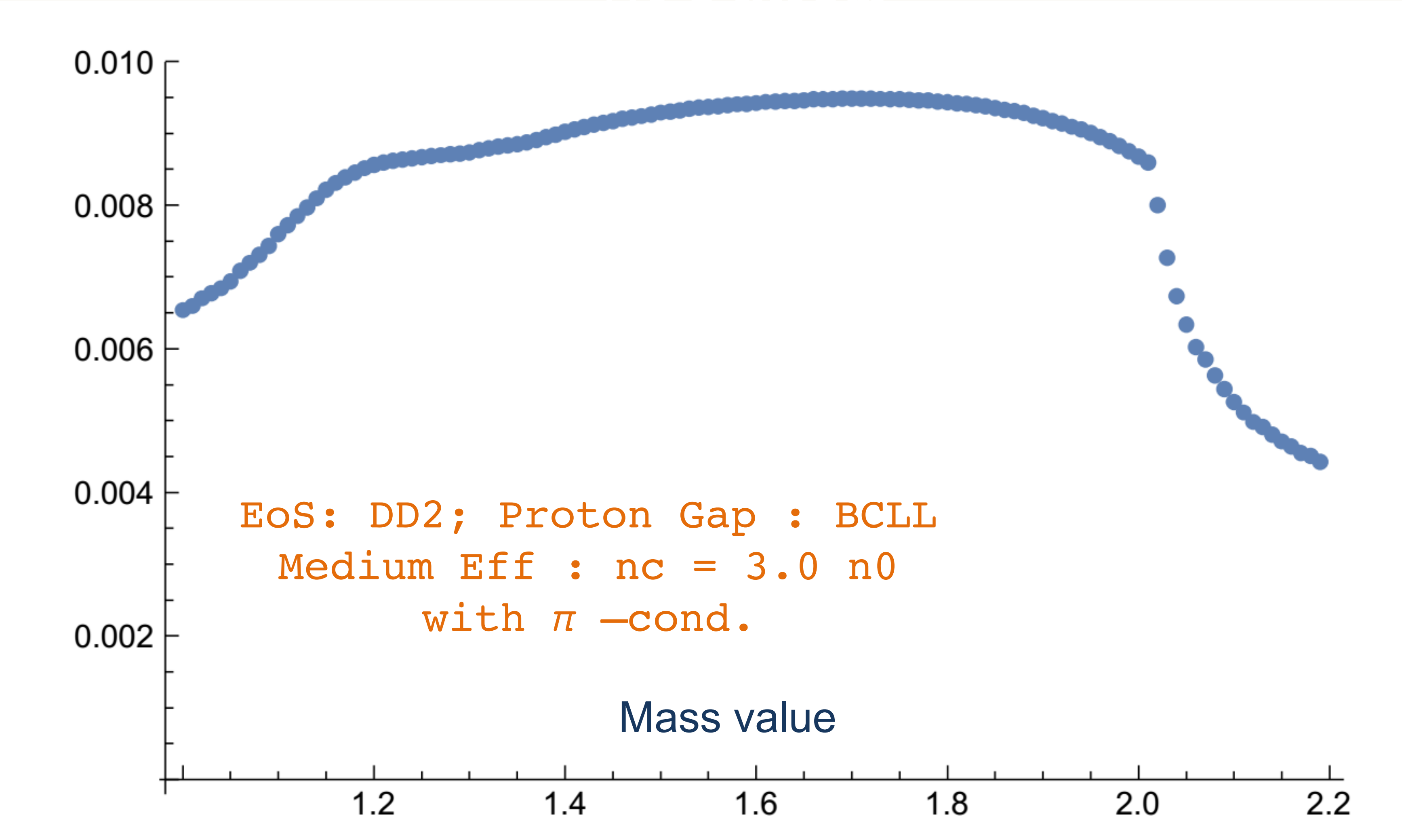




Expected Mass value for the Data points on the T – t



Expected Mass value for the Data points on the T – t Diagram

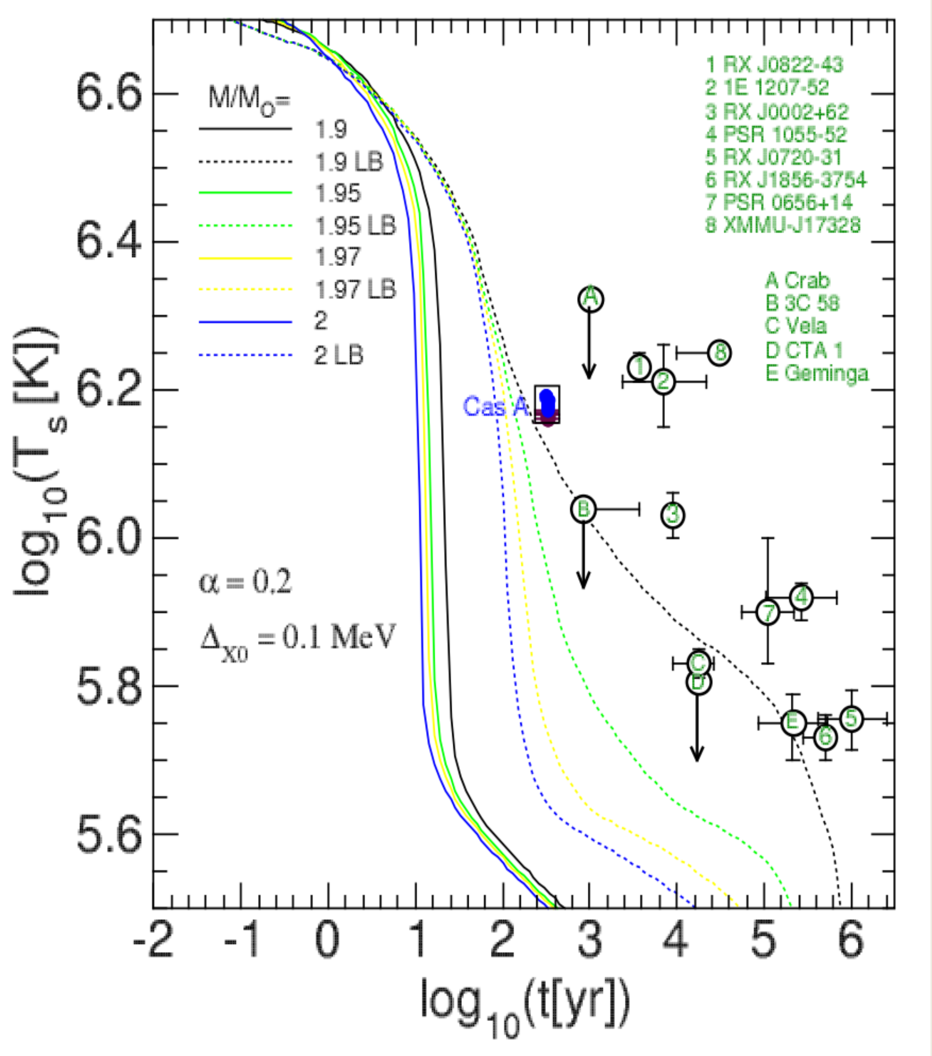


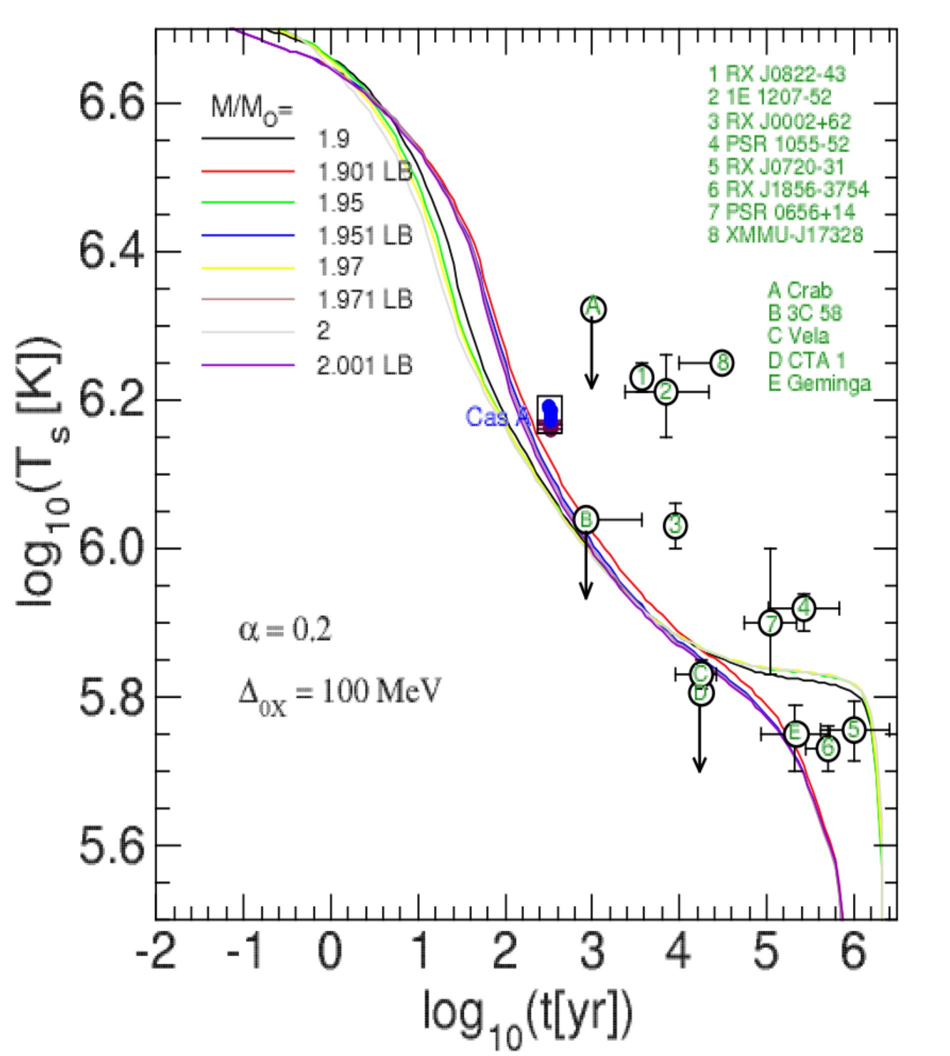
Conclusions

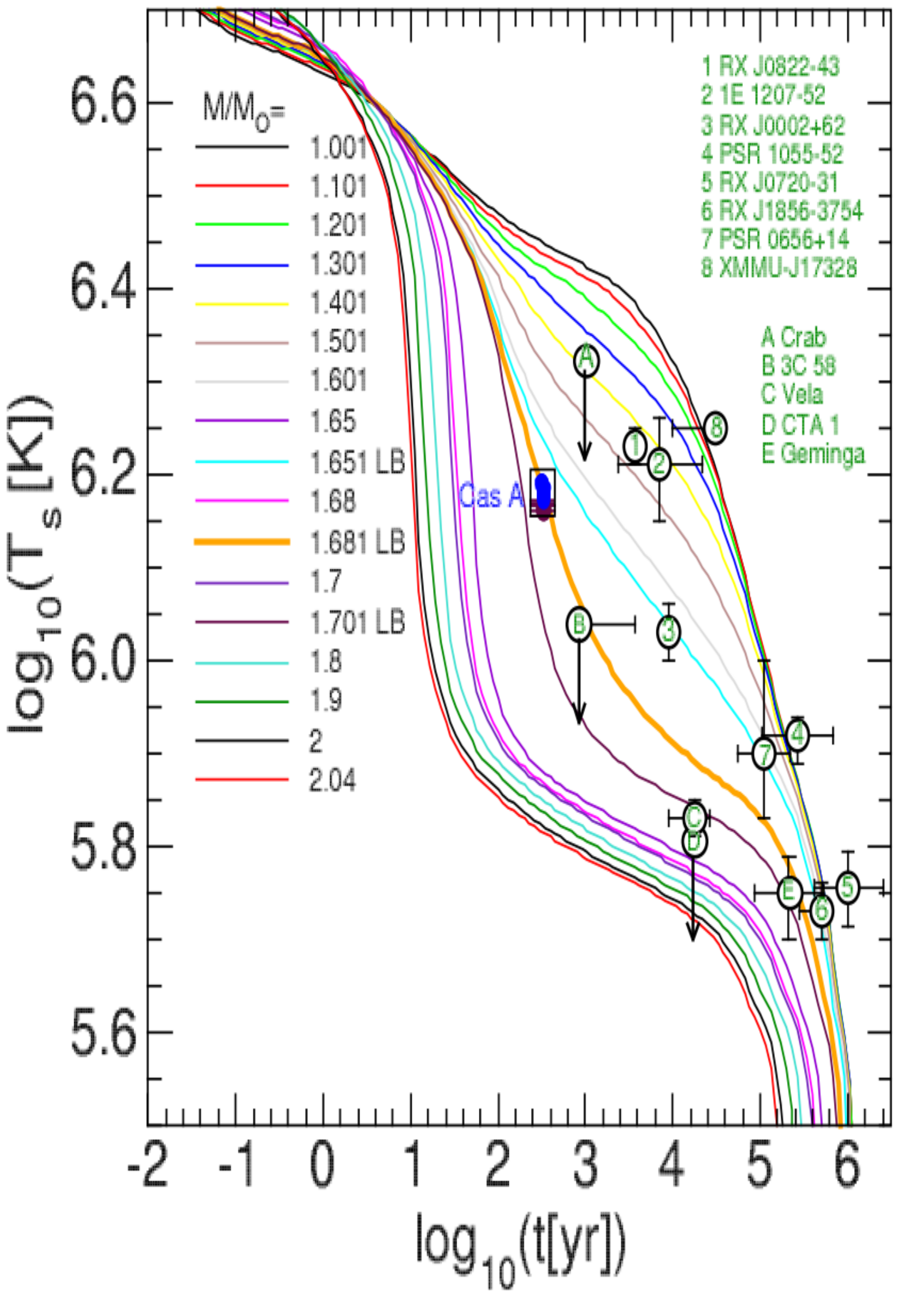
- All known cooling data including the Cas A rapid cooling consistently described by the "nuclear medium cooling" scenario
- Influence of stiffness on EoS and cooling can be balanced by the choice of corresponding gap model.
- In case of existence of III CSF high-mass twin stars could show different cooling behavior depending on core superconductivity
- Parallelization allowed to make the calculations for statistical analyses of models in reasonable time

Thank YOU!!!!!

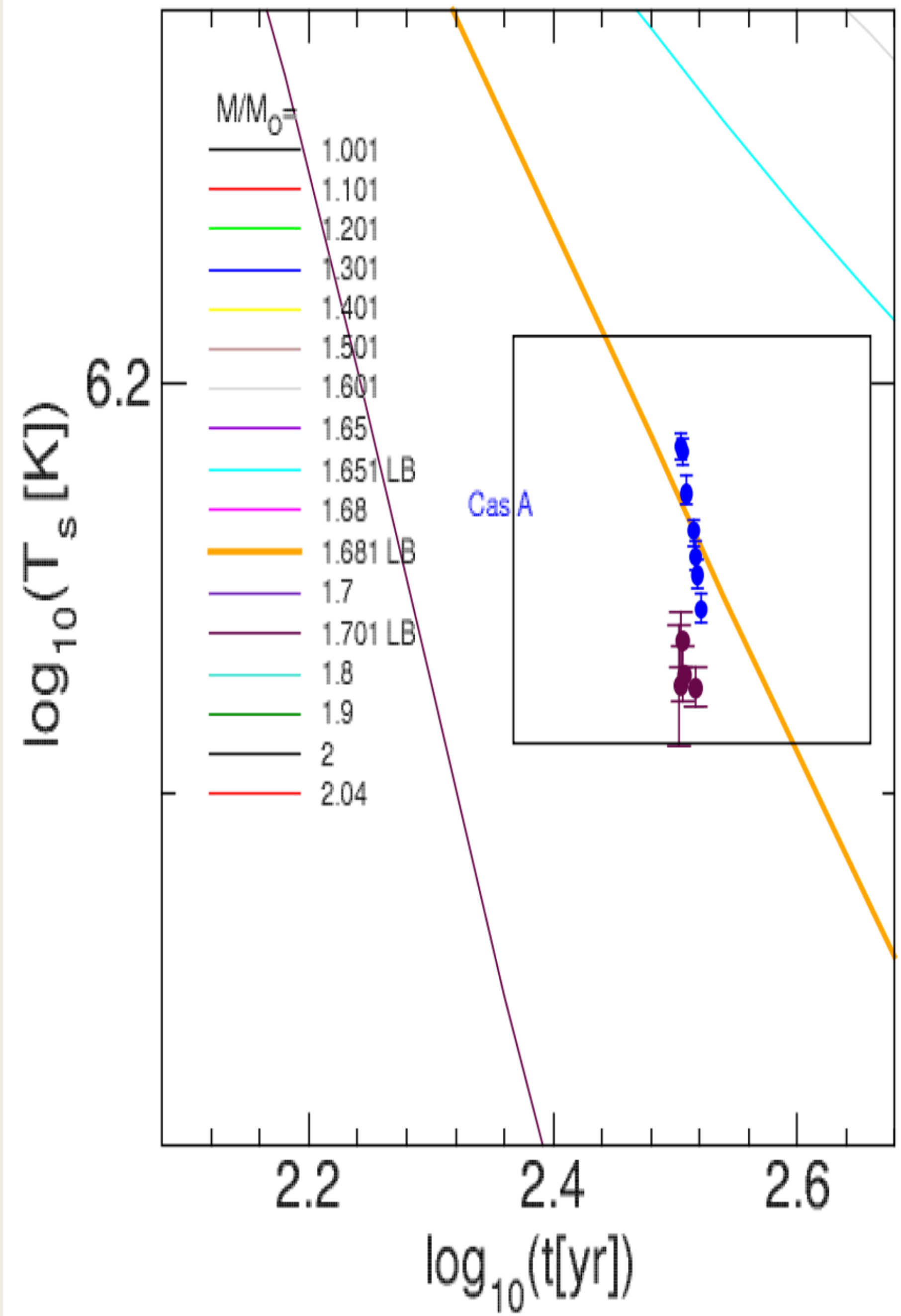
Highmass Twins: QM SC Effect





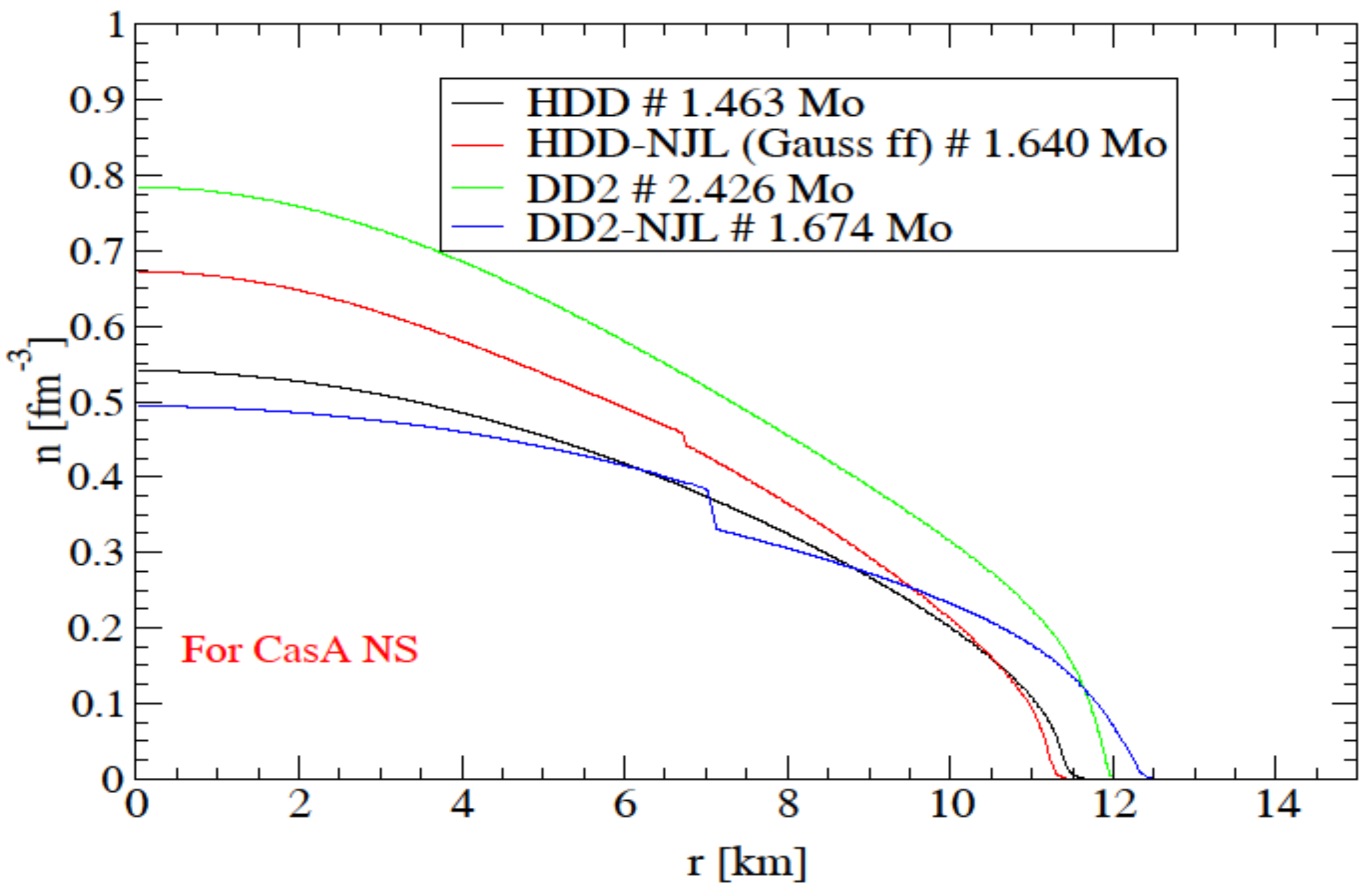


6.6
6.4
6.2
6.0
5.8
5.6
log10(Ts [K])
-2 -1 0 1 2 3 4 5 6
log10(t[yr])
M/MO=
1.001
1.101
1.201
1.301
1.401
1.501
1.601
1.65
1.651 LB
1.68
1.681 LB
1.7
1.701 LB
1.8
1.9
2
2.04
1 RX J0822-43
2 1E 1207-52
3 RX J0002+62
4 PSR 1055-52
5 RX J0720-31
6 RX J1856-3754
7 PSR 0656+14
8 XMMU-J17328
A Crab
B 3C 58
C Vela
D CTA 1
E Geminga
Cas A

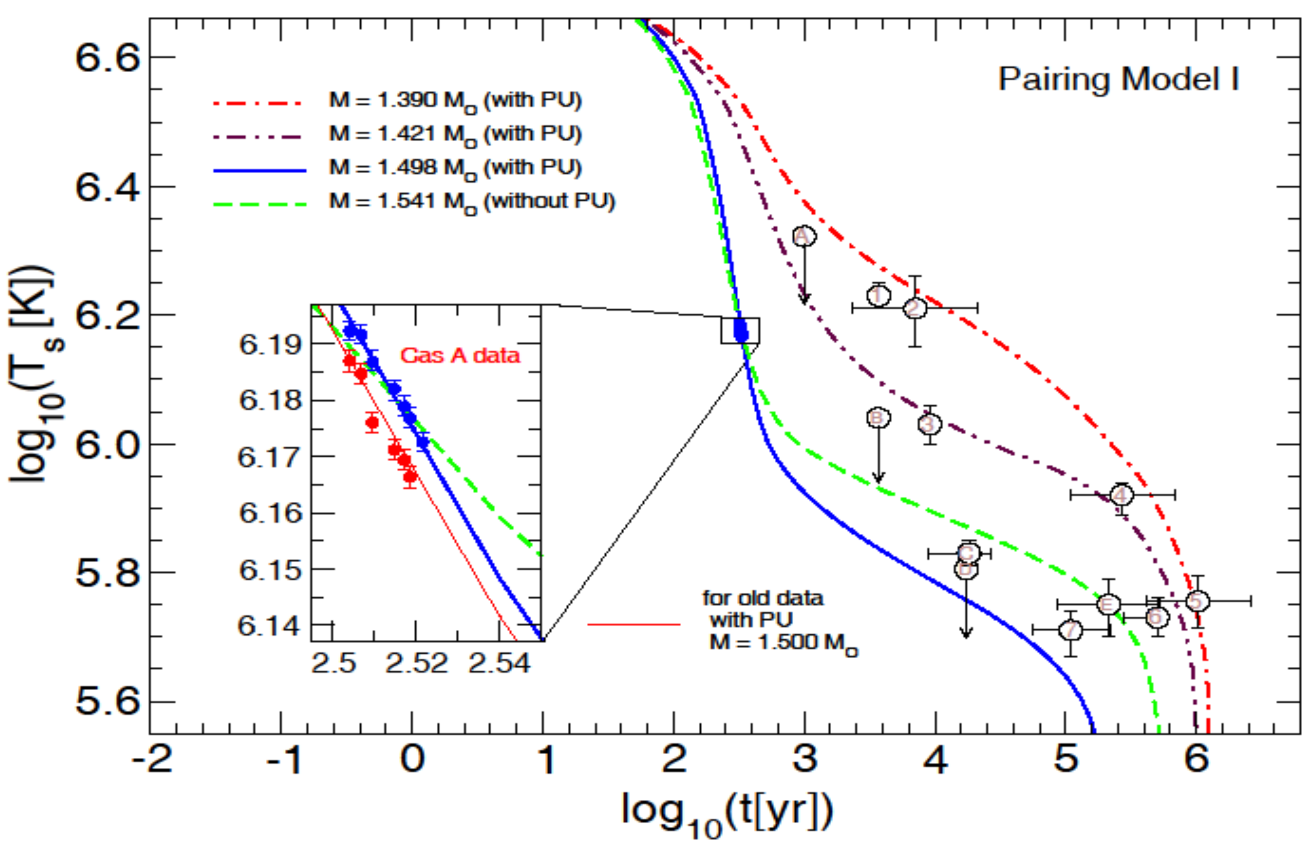


6.2
log10(Ts [K])
2.2 2.4 2.6
log10(t[yr])
M/MO=
1.001
1.101
1.201
1.301
1.401
1.501
1.601
1.65
1.651 LB
1.68
1.681 LB
1.7
1.701 LB
1.8
1.9
2
2.04
Cas A

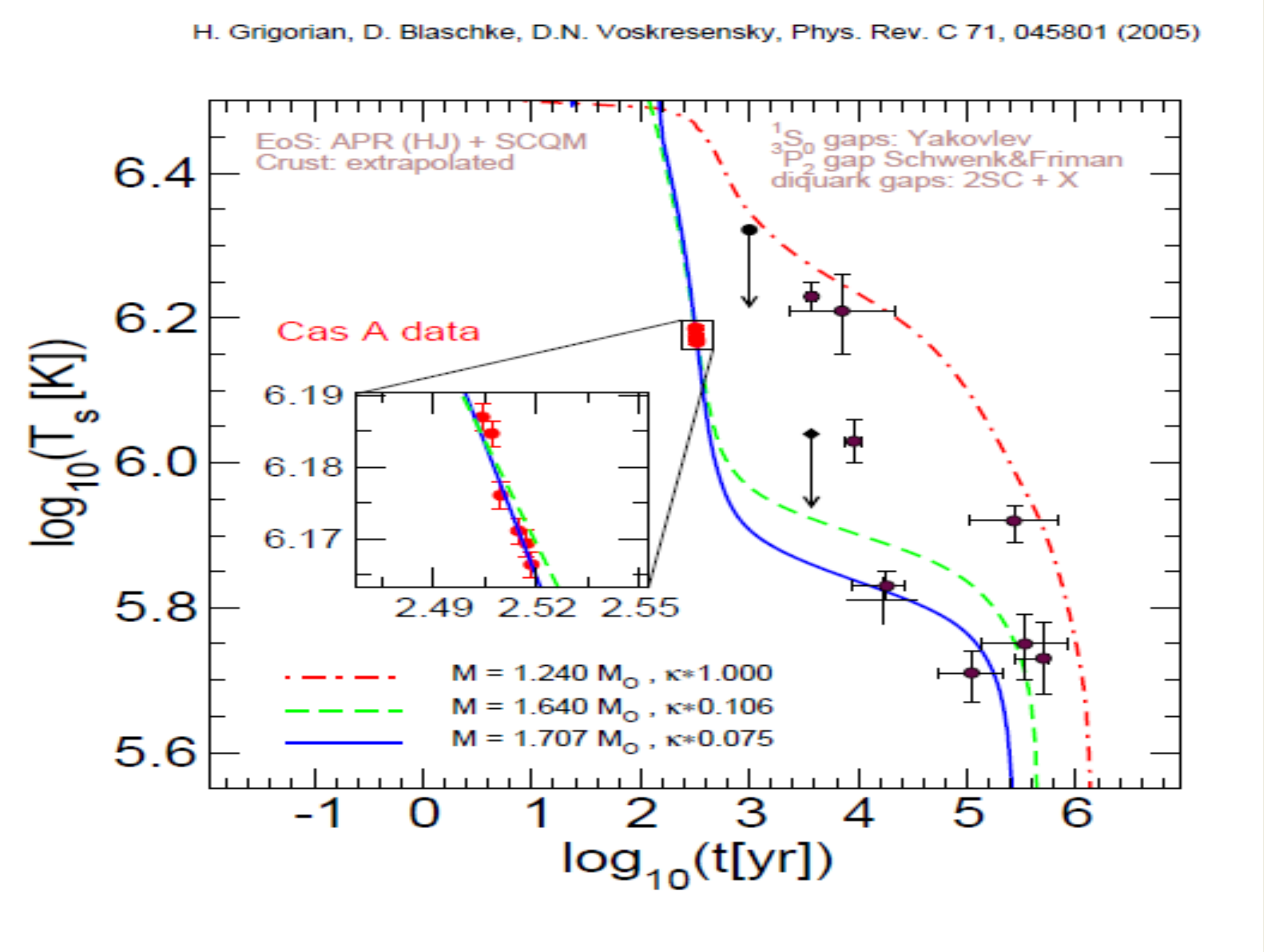
Possible internal structure of CasA



Cas A as an Hadronic Star



Cas A As An Hybrid Star



MKVOR – EoS model