Bayesian analysis for extracting properties of the nuclear equation of state from observational data

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Outline

I. Motivation

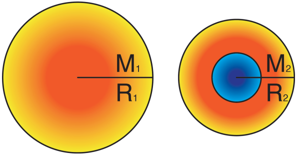
II. Mixed phase construction for cold and dense nuclear matter

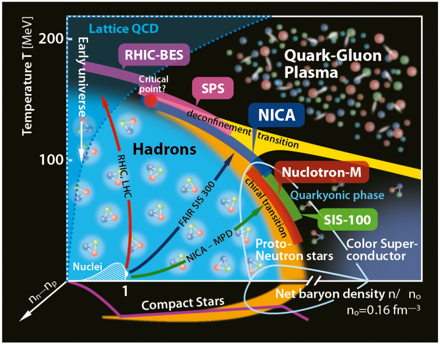
III. Bayesian analysis for extracting properties of the nuclear equation of state from observational data

IV. Conclusions

Motivation

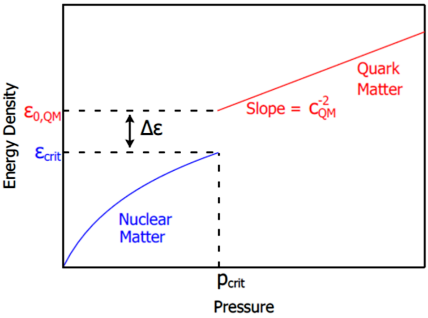
What if we have twins

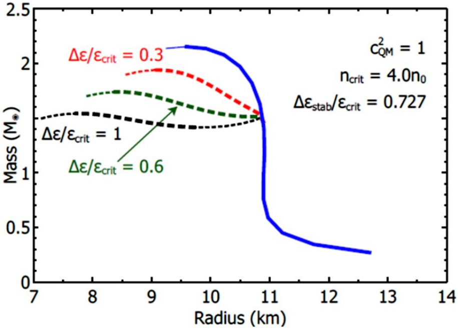




- Does hybrid neutron star exist?
- Does NS twin exist?
- Does CEP exist on QCD phase diagram?
- etc.

Neutron star mass-radius relation





Seidov criterion for instability:

$$\frac{\Delta\varepsilon}{\varepsilon_{crit}} \geq \frac{1}{2} + \frac{3}{2}\frac{P_{crit}}{\varepsilon_{crit}}$$

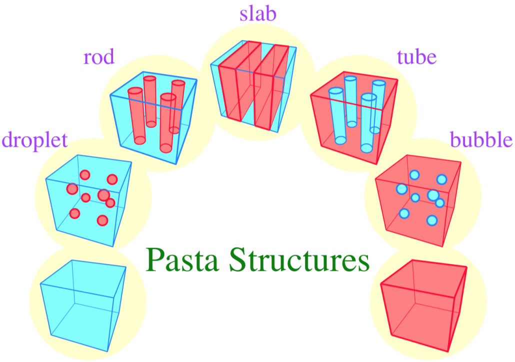
Credit: Mark G. Alford, Sophia Han, and Madappa Prakash. Phys. Rev. D 88, 083013 (2013)

Finite-size effects

Coulomb interaction **vs** **Surface tension**

Tends to break up the like-charged regions into smaller ones

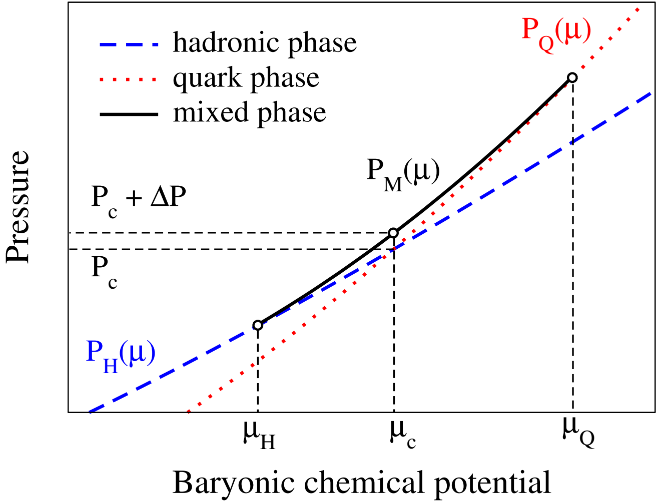
Requires minimization of the surface



The surface tension σ is unknown and used as free parameter.

Yasutake, Maruyama, Tatsumi, Phys. Rev. D80 (2009) 123009

Mimicking the Pasta phase. The idea



Schematic representation of the interpolation function $P_M(\mu)$, it has to go though three points: $P_H(\mu_H)$, $P_c + \Delta P$ and $P_Q(\mu_Q)$.

The Interpolation Method

$$P_M(\mu) = \sum_{q=1}^{N} \alpha_q (\mu - \mu_c)^q + (1 + \Delta_P) P_c$$

where Δ_P is a free parameter representing additional pressure of the mixed phase at μ_c.

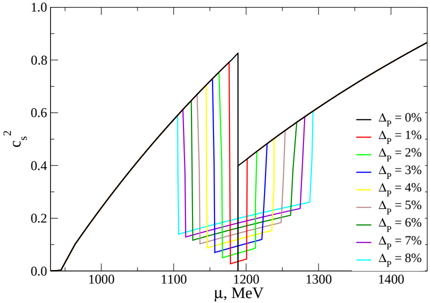
$$P_H(\mu_H) = P_M(\mu_H) \qquad P_Q(\mu_Q) = P_M(\mu_Q)$$

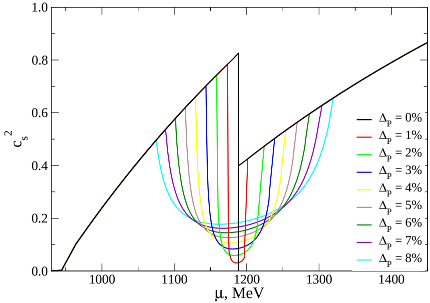
$$\frac{\partial^q}{\partial \mu^q} P_H(\mu_H) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_H) \qquad \frac{\partial^q}{\partial \mu^q} P_Q(\mu_Q) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_Q)$$

where $q = 1, 2, \ldots, k$. All $N + 2$ parameters (μ_H, μ_Q and α_q, for $q = 1, \ldots, N$) can be found by solving the above system of equations, leaving one parameter (ΔP) as a free one.

Ayriyan and Grigorian, *EPJ Web Conf.* **173**, 03003 (2018)
Abgaryan, Alvarez-Castillo, Ayriyan et al. *Universe* **4(9)**, 94 (2018)

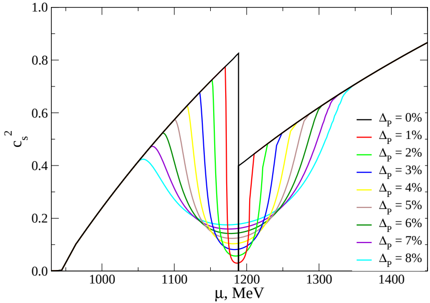
The Interpolation Method



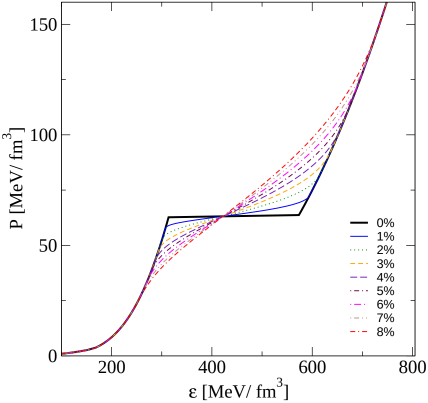


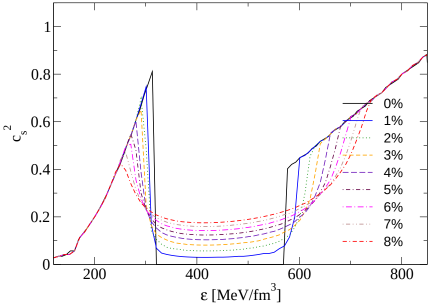
The squared speed vs chemical potential given by the interpolation with $k = 1$ (upper left) $k = 2$ (upper right) and $k = 3$ (right).

Abgaryan, Alvarez-Castillo, Ayriyan, Blaschke and Grigorian. Universe 4(9) (2018), 94

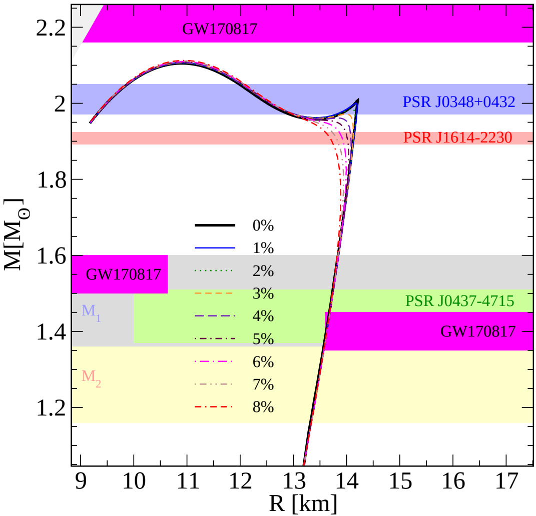


The results of pasta mimicking





The results of pasta effects

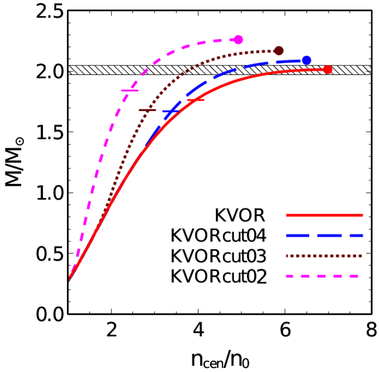


Third family robust against Δ_P up to around 5%!

Abgaryan, Alvarez-Castillo, Ayriyan et al. Universe 4(9), 94 (2018)

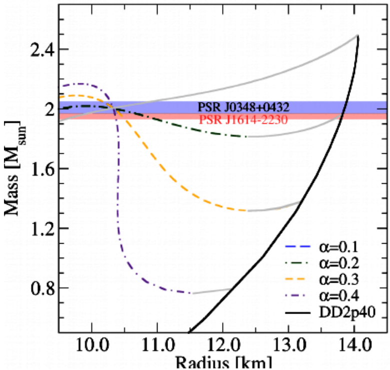
The realistic hadron and quark matter models

The hadron EoS model KVOR with modification of stiffness



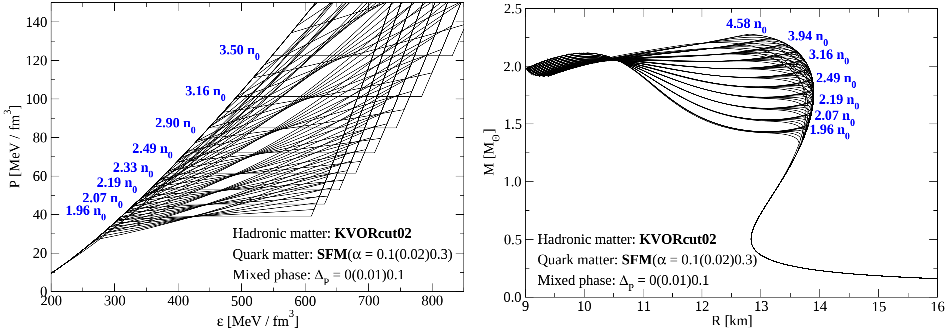
Maslov, Kolomeitsev, Voskresensky, Nucl.Phys. A950 (2016)
Kolomeitsev & Voskresensky, Nuc. Phys. A 759 (2005)

The quark EoS model SFM with available volume fraction parameter



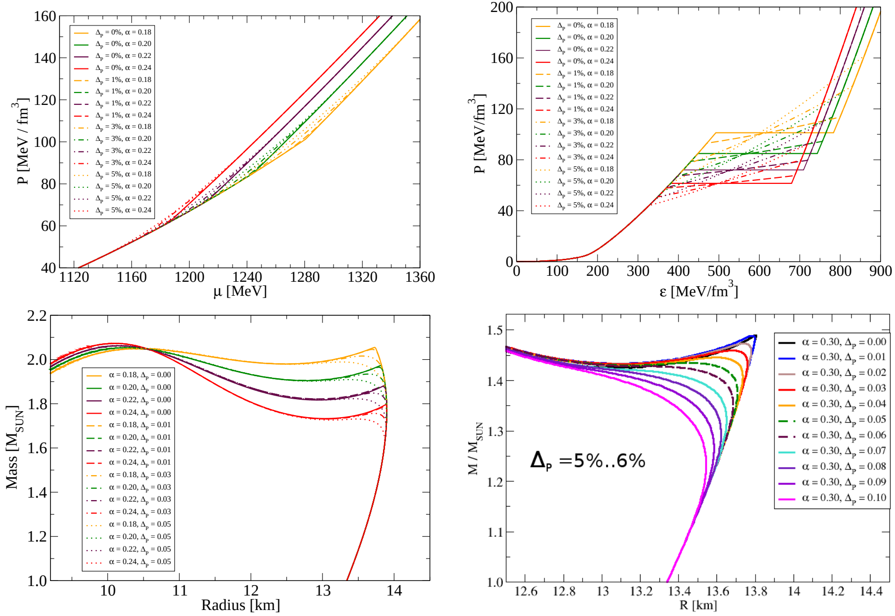
Kaltenborn, Bastian, Blaschke, Phys. Rev. D 96, 056024 (2017)

Robustness of third family solutions

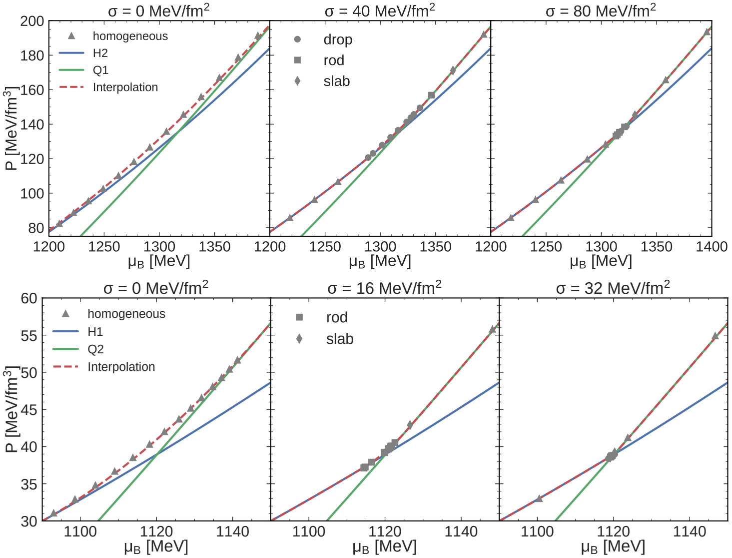


Ayriyan, Bastian, Blaschke, Grigorian, Maslov, Voskresensky. PRC 97, 045802 (2018)

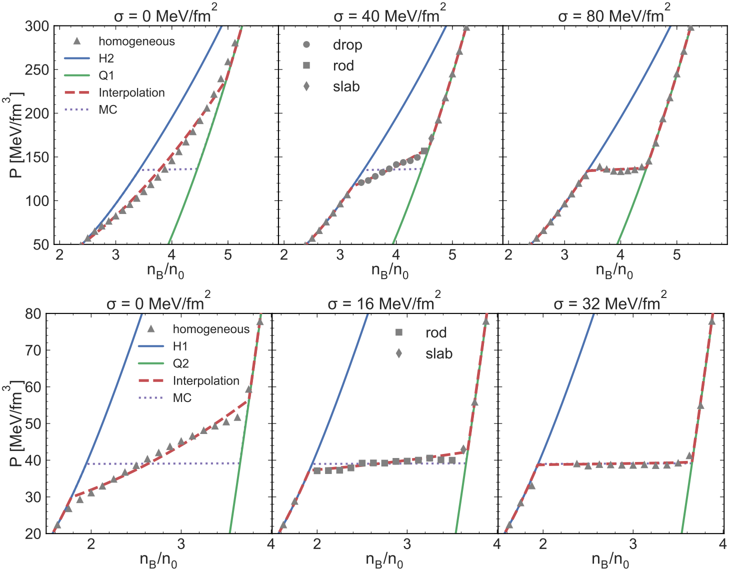
Robustness of third family solutions



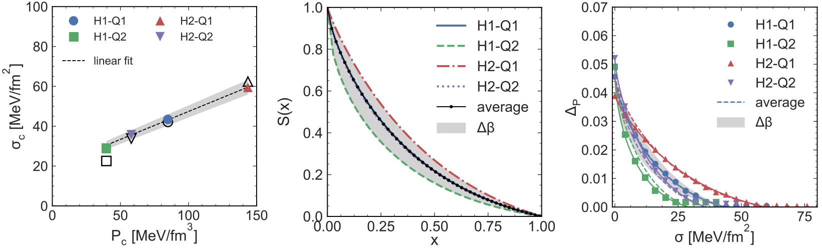
Relation with surface tension parameter



Relation with surface tension parameter



Relation with surface tension parameter



$\sigma_c = d\,(P_c - P_0) + \sigma_0$: $d = 0.45 \pm 0.02$ fm, $P_0 = 40$ MeV/fm^3, and $\sigma_0 = 31.6 \pm 1.19$ MeV/fm^2

$\Delta_P(\sigma) = \Delta_P(0) S(\sigma/\sigma_c; \beta)$: $\overline{\beta} = 0.64$

$S(x;\beta) = e^{-x}(1 - x^{\beta})\theta(1 - x)$

Maslov, Yasutake, Blaschke, Ayriyan, Grigorian, Maruyama, Tatsumi, Voskresensky. PRC100, 025802 (2019)

How to mimic pasta phase for given σ

1. Maxwell constructio to obtain P_c and μ_c
2. Glendaning construction to find $\Delta_P(0)$
3. Use the fit formulas of **Maslov et al PRC100 (2019)** to obtain $\Delta_P(0)$.
4. Use the interpolation formulas of **Ayriyan et al PRC98 (2018)** to obtain the pasta-equivalent hybrid EoS with a mixed phase for a given choice of the surface tension σ

D. Blaschk. **Cooking book for mimicking the pasta phase** presented at MPCS2019 (20.09.2019, Yerevan)

Bayesian Analysis

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.
One of the most frequent case is analysis of probable values of model parameters.

Bayes' theorem:

$$\underbrace{p(H_1 \mid D, I)}_{\text{Posterior}} = \frac{\overbrace{p(D \mid H_1, I)}^{\text{Likelihood}}\, \overbrace{p(H_1 \mid I)}^{\text{Prior}}}{\underbrace{p(D \mid I)}_{\text{Evidence}}}$$

Prior: knowledge before experiment (logically)
Likelihood: Probability for data if the hypothesis was true
Posterior: Probability that the hypothesis is true given the data
Evidence: normalization; important for model comparison

Generally, maximum likelihood (parameters which maximize the probability for data) **does not** give the most likely parameters!!!

Bayesian Analysis

Formulation of set of models (set of hypothesis):
π_i here $i = 0..N-1$

$\downarrow$

Finding the *a priori* probabilities of the models:
$P(\pi_i) = 1/N \quad \text{for} \quad \forall\, i = 0..N-1$

$\downarrow$

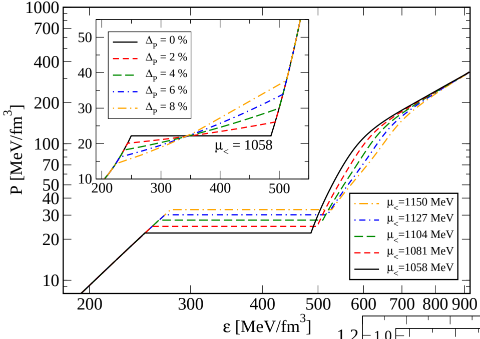
Calculating the coditional probabilities of the events:
$P(E\,|\,\vec{\pi}_i) = \prod_\alpha P(E_\alpha\,|\,\vec{\pi}_i)$,
where α is the index of the observational constraints.

$\downarrow$

Calculating the *a posteriori* probabilities of the models:

$$P(\vec{\pi}_i\,|E) = \frac{P(E|\vec{\pi}_i)P(\vec{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\vec{\pi}_j)P(\vec{\pi}_j)}$$

Equation of State

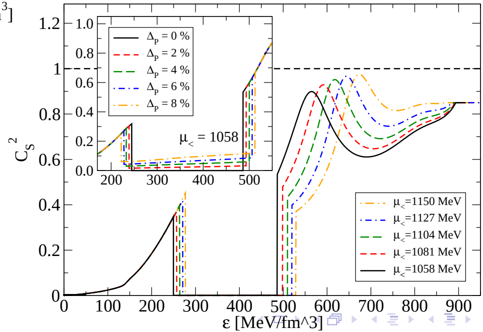


DD2 with excluded volume plus color superconducting two-flavor quark matter, described within a nonlocal covariant chiral quark model.

$$P(\mu) = P(\mu;\eta(\mu), B(\mu)) = -\Omega^{MFA}(\eta(\mu)) - B(\mu)$$

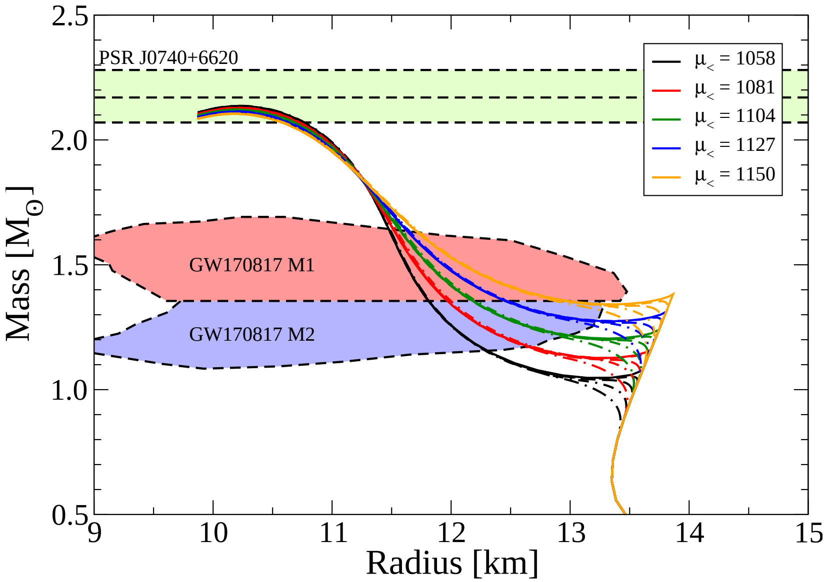
$$\Omega^{MFA} = \frac{\bar{\sigma}^2}{2G_S} + \frac{\bar{\Delta}^2}{2H} - \frac{\bar{\omega}^2}{2G_V} - \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \ln\det\left[\, S^{-1}(\bar{\sigma}, \bar{\Delta}, \bar{\omega}, \mu_{fc})\right]$$

$$\frac{d\Omega^{MFA}}{d\bar{\Delta}} = 0\,, \quad \frac{d\Omega^{MFA}}{d\bar{\sigma}} = 0\,, \quad \frac{d\Omega^{MFA}}{d\bar{\omega}} = 0\,.$$

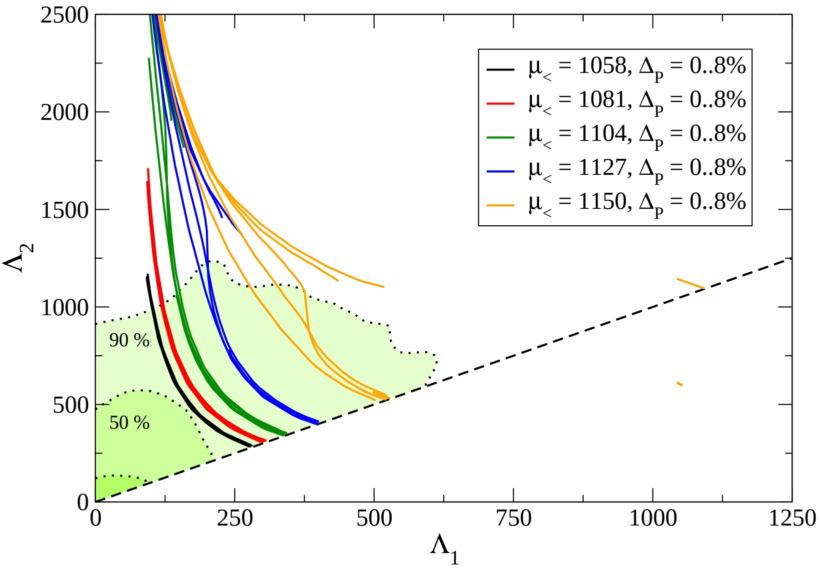


Alvarez-Castillo, Blaschke, Grunfeld, and Pagura, PRD99, 063010 (2019)

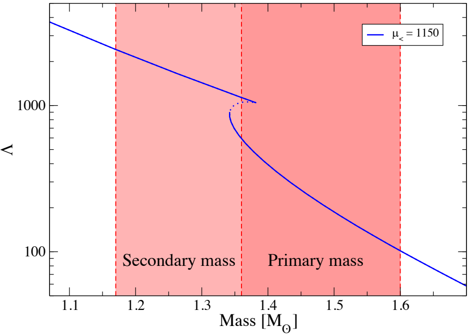
Equation of State at $M - R$ diagram

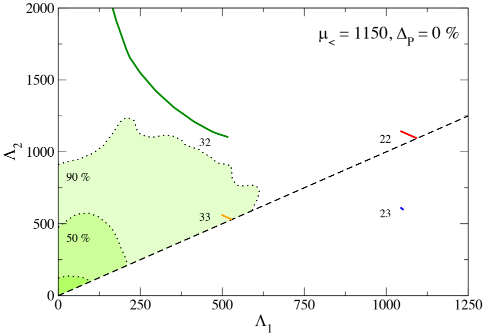


Equation of State at Λ_1–Λ_2 diagram

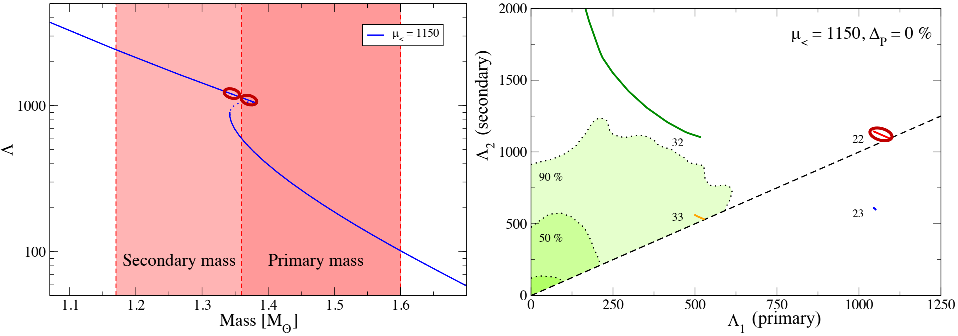


Equation of State at Λ_1–Λ_2 diagram

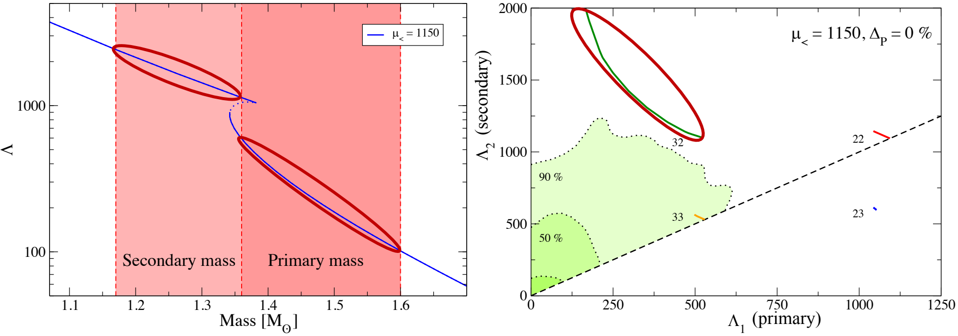




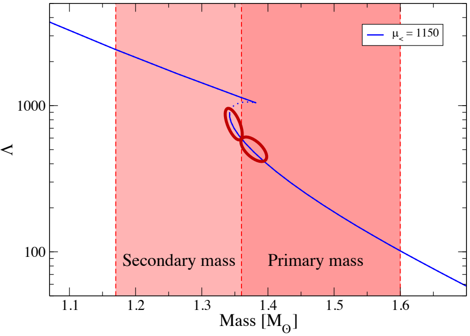
Equation of State at Λ_1–Λ_2 diagram

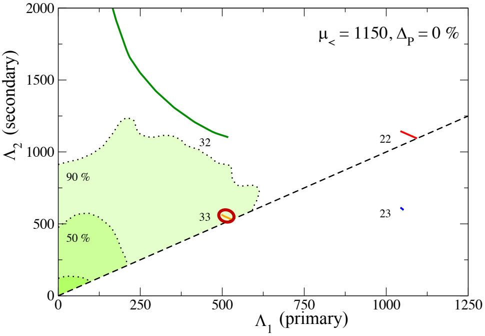


Equation of State at Λ_1–Λ_2 diagram

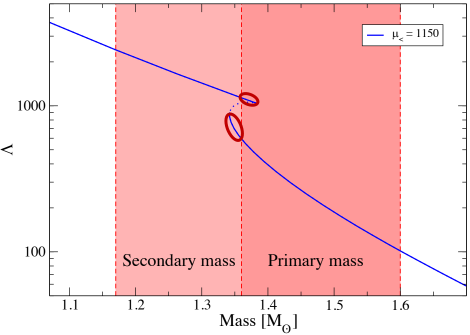


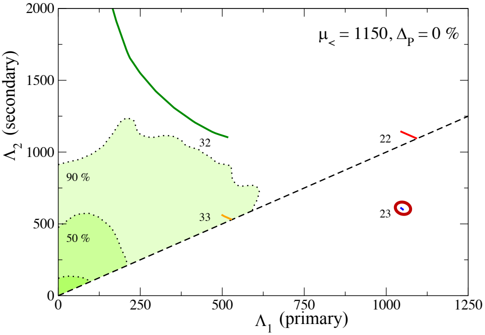
Equation of State at Λ_1–Λ_2 diagram





Equation of State at Λ_1–Λ_2 diagram





The same phenomena were found in Montana, Tolos, Hanauske, Rezzolla. PRD99, 103009 (2019) for polytropic models

More interesting results have been achieved by Prof. Armen Sedrakian for triplet of compact stars produced by the fourth family.

The region $\Lambda_2 < \Lambda_1$ was called unphysical at Abbott *et al.* PRL121 (2018).

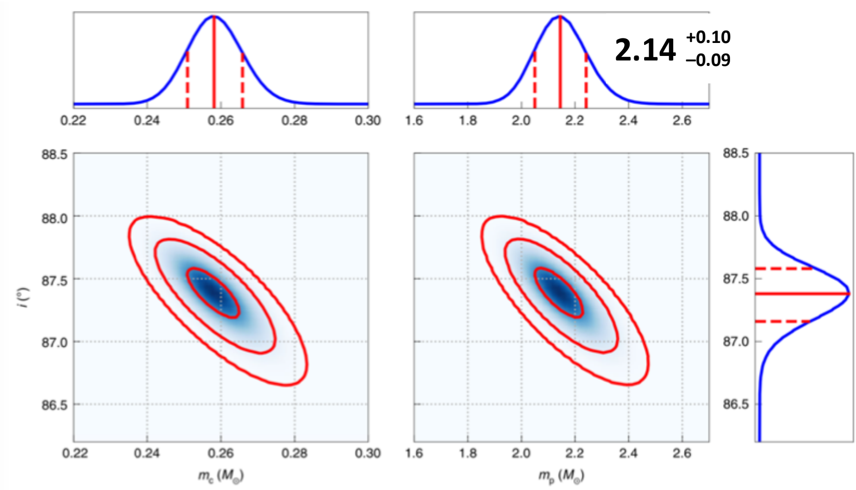
Vector of Parameters

The set of parameters of models could be represented in the parameter space with introduction of the vector of parameters, each vector is one fixed model from considered types of EoS model and transition construction:

$$\overrightarrow{\pi}_i = \left\{ \mu_{<(j)}, \Delta_{P(k)} \right\},$$

where $i = 0..N-1$ and $i = N_2 \times j + k$ and $j = 0..N_1 - 1$, $k = 0..N_2 - 1$ and N_1 and N_2 are number of values of model parameters $\mu_<$ and Δ_P correspondingly.

Likelihood of a EoS model for the mass constraint

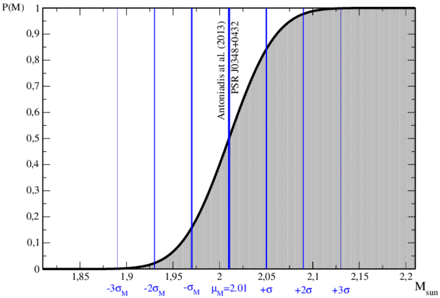


Cromartie et. al. Nature Astronomy (2019), doi: 10.1038/s41550-019-0880-2

Likelihood of a EoS model for the mass constraint

$$P\left(E_M \,|\pi_i\right) = \Phi(M_i, \mu_A, \sigma_A)$$

here M_i is maximum mass of the given by π_i, and $\mu_A = 2.17\ \mathrm{M}_\odot$ and $\sigma_A = 0.105\ \mathrm{M}_\odot$ is the mass measurement of PSR J0740+6620 $2.17^{+0.11}_{-0.10}\ \mathrm{M}_\odot$ [Cromartie *et al.*, arXiv:1904.06759 (2019)].

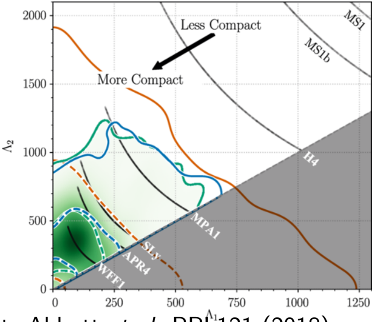


Note, that here we replace previously used mass measurement for two solar mass pulsar J0348+0432 $2.01^{+0.04}_{-0.04}\ \mathrm{M}_\odot$ [Antoniadis *et al.*, Science **340**, 6131 (2013)].

Likelihood of a EoS model for the Λ_1–Λ_2 constraint

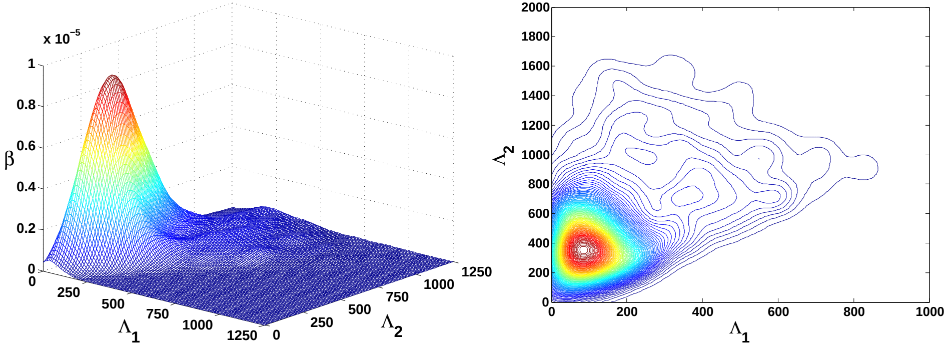
$$P\left(E_{GW}\,|\pi_i\right) = \int_{l_{22}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{23}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau$$
$$+ \int_{l_{32}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{33}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau,$$

where l_{ps} are the length of the line at Λ_1–Λ_2, the indecies p and s determine to which family of compact stars the GW170817 components belong. The parameter τ is, for instance, central density of a star.



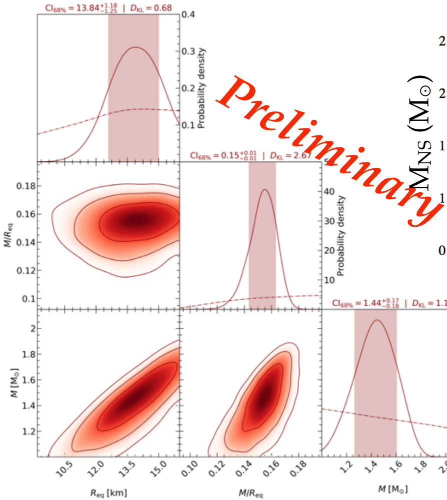
Credt: Abbott *et al.* PRL121 (2018)

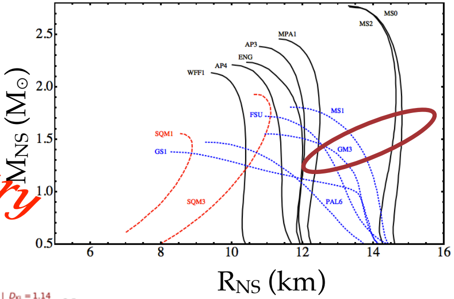
$\beta(\Lambda_1, \Lambda_2)$



The PDF $\beta(\Lambda_1, \Lambda_2)$ has been reconstructed by the method Gaussian kernel density estimation with Λ_1–Λ_2 data given at LIGO web-page https://dcc.ligo.org/LIGO-P1800115/public.

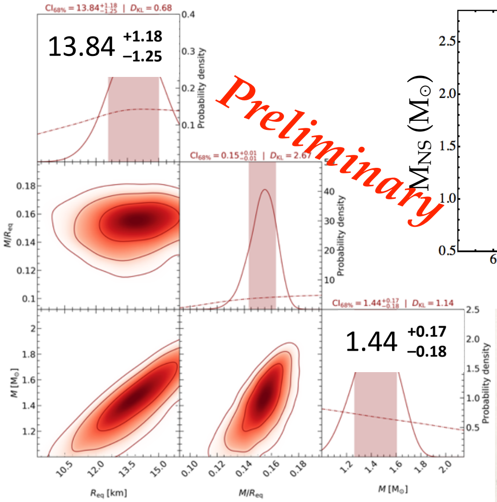
Likelihood of a model for the fictitious M-R constraint

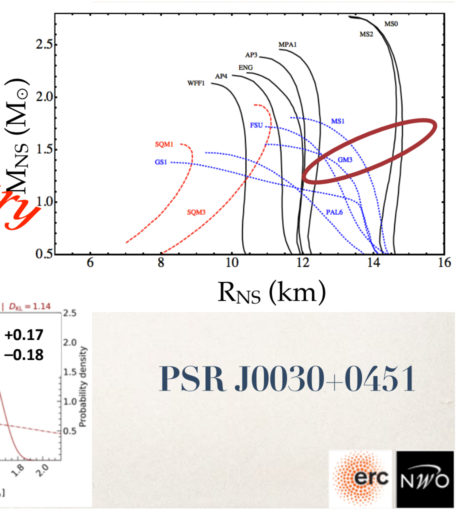




[Guillot. Talk at the Workshop "NSs and their environments", (April 8, 2019)]

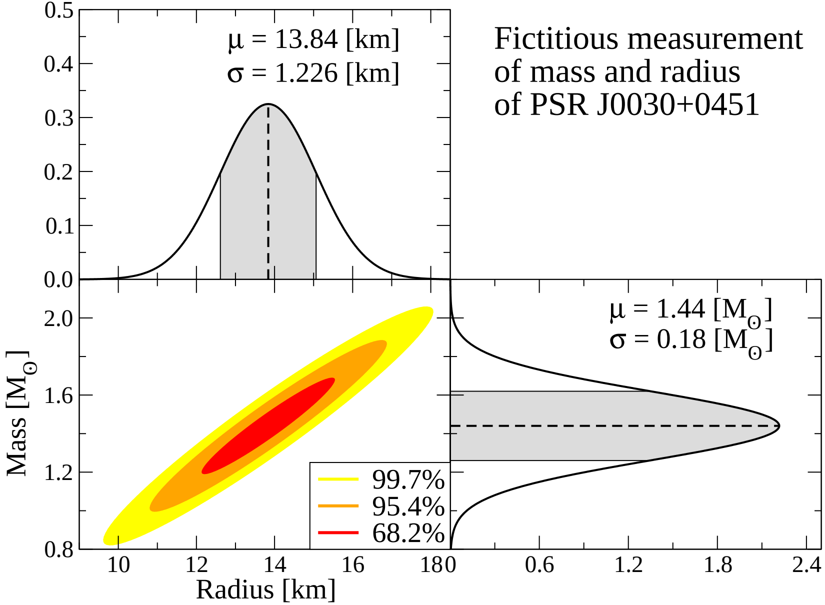
Likelihood of a model for the fictitious M-R constraint



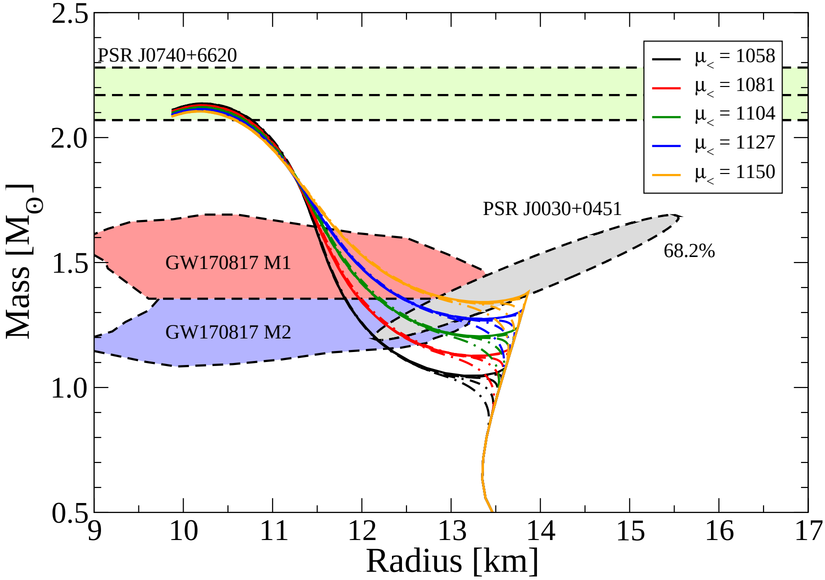


[Guillot. Talk at the Workshop "NSs and their environments", (April 8, 2019)]

Likelihood of a model for the fictitious *M*-*R* constraint



Likelihood of a model for the fictitious M-R constraint



Likelihood of a model for the fictitious M-R constraint

The fictitious M-R measurement has been implemented, inspired by the preliminary results of NICER observation of M-R of the PSR J0030+0451.

$$P\left(E_{MR}\left|\pi_i\right.\right) = \int_{l_2} \mathcal{N}(\mu_R,\, \sigma_R,\, \mu_M,\, \sigma_M, \rho) d\tau + \int_{l_3} \mathcal{N}(\mu_R,\, \sigma_R,\, \mu_M,\, \sigma_M, \rho) d\tau,$$

where $\mu_R = 13.84$, $\sigma_R = 1.2276$, $\mu_M = 1.44$, $\sigma_M = 0.18$, and the correlation parameter $\rho = 0.9566$, winch corresponds to $8^{\rm o}$ of th ellipse rotation. l_2 and l_3 are length of the lines at M-R diagram of the second and third families correspondingly.

Posterior distribution

The full likelihood for the given π_i can be calculated as a product of all likelihoods, since the considered constraints are independent of each other

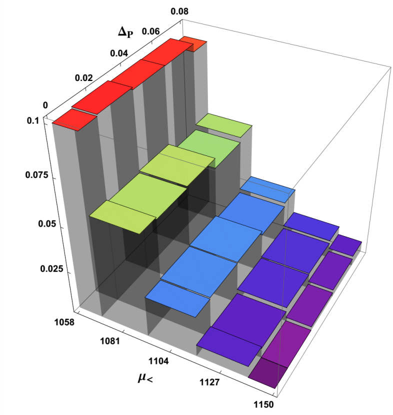
$$P\left(E \,|\, \overrightarrow{\pi}_i\right) = \prod_m P\left(E_m \,|\, \overrightarrow{\pi}_i\right).$$

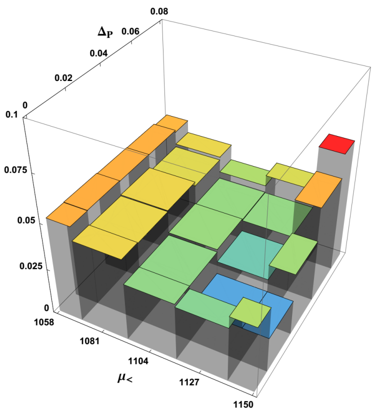
where m is index of the constraints.
The posterior distribution of models on parameter diagram is given by Bayes' theorem

$$P\left(\overrightarrow{\pi}_i \,|\, E\right) = \frac{P\left(E \,|\, \overrightarrow{\pi}_i\right) P\left(\overrightarrow{\pi}_i\right)}{\sum\limits_{j=0}^{N-1} P\left(E \,|\, \overrightarrow{\pi}_j\right) P\left(\overrightarrow{\pi}_j\right)},$$

where $P\left(\overrightarrow{\pi}_j\right)$ is a prior distribution of a models taken to be uniform: $P\left(\overrightarrow{\pi}_j\right) = 1/N$.

BA results





Ayriyan, Alvarez-Castillo, Blaschke, Grigorian. In preparation.

Conclusions

The mixed phase interpolation method is very simple and well describes quark-hadron pasta phase for any given surface tension value.

The third family survives mixed phase effects for the pasta phase for the considered EoS models.

$\Lambda_1 - \Lambda_2$ relation from GW170817 favours softer EoS and hybrid stars with strong first order phase transitions (even with no third family due to the mixed phase).

The region $\Lambda_2 < \Lambda_1$ has physical meaning in case of low-mass twins, when heavier companion belongs to the second family and the lighter one to the third family.

If NICER approves the “fictitious radius measurement” it will support late onset for the considered models.

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