40th Max Born Symposium, Wroclaw

October 11, 2019

Bound states in many-fermion systems

Gerd Röpke, Rostock

Structure of matter

energy scale	*fermions*	*interaction*	*bound states*	*density effects*	*condensed phase*
$1 \ldots 10$ meV	electrons, holes	Coulomb	excitons	screening	electron-hole liquid
$1 \ldots 10$ eV	electrons, nuclei	Coulomb	ions, atoms	screening	liquid metal
$1 \ldots 10$ MeV	protons, neutrons	$N - N$ int.	nuclei	Pauli blocking	nuclear matter
$0.1 \ldots 1$ GeV	quarks	QCD	hadrons	deconfinement	quark-gluon plasma

Fermion systems: ideal Fermi gases

Interaction - correlations

Low densities: bound states, quantum condensates
High densities: condensed phase

- Plasma physics: Ionization potential depression (IPD) [PRE (2019)]
- Nuclear physics: Weakly bound nuclei in stellar matter [in preparation]
- QCD: Deconfinement, Quark – Gluon phase transition in neutron-star mergers [Bauswein et al., Hadron-quark phase transition, PRL 122, 061102 (2019)]

In-medium Schrödinger equation

Consistent treatment of the two-particle problem:
in-medium wave equation

$$\frac{p^2}{2m_e}\psi_n(p) + \sum_q V(q)\psi_n(p+q) - E_n\psi_n(p) = \sum_q V(q)\left[\psi_n(p+q)f_e(p) - \psi_n(p)f_e(p+q)\right]$$

Pauli blocking, Fock self-energy shift, Fermi fct. f_e

V(q) --> dynamically screened Coulomb interaction

$$V^{\mathrm{s}}_{ab}(q,\omega) = V_{\mathrm{ab}}(q)\cdot\left\{1+\int\frac{d\bar{\omega}}{\pi}\cdot\frac{\mathrm{Im}\,\varepsilon^{-1}(q,\bar{\omega}-i\eta)}{\omega-\bar{\omega}}\right\}$$

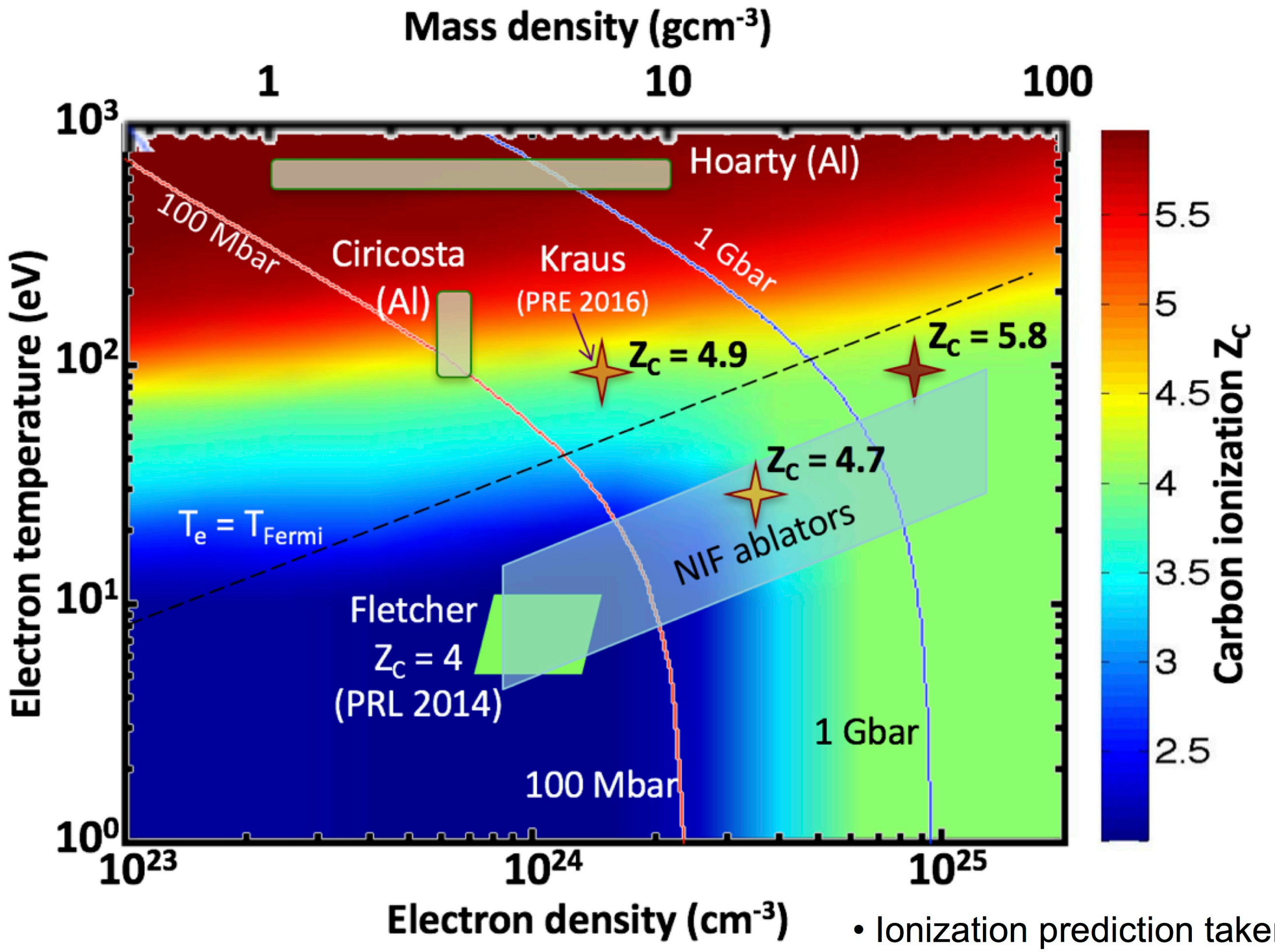
dynamical screening, dynamical self-energy

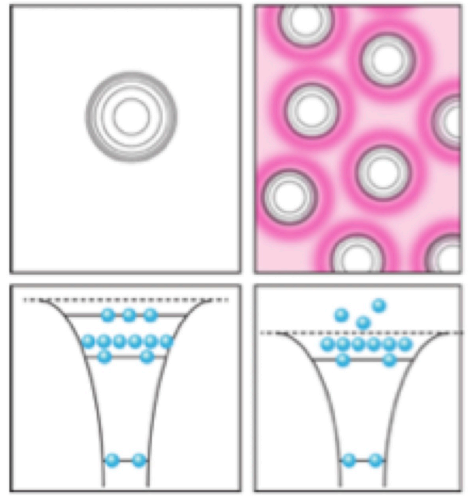
$\epsilon(\mathbf{q},\omega+i0)$ dielectric function

R. Zimmermann, K. Kilimann,
W. D. Kraeft, D. Kremp and G. Röpke
Phys. Stat. sol. (b) **90**, 175 (1978)

W.-D. Kraeft, D. Kremp, W. Ebeling, G. Röpke
Quantum Statistics of Charged Particle Systems,
Akademie-Verlag, Berlin 1986

NIF XRTS experiments find higher carbon K-shell ionization than predicted by widely used IPD models (Stewart & Pyatt, OPAL)





NIF data point

Hoarty et al., PRL **110**, 265003 (2013)

Ciricosta et al., PRL **109**, 065002 (2012)

Fletcher et al., PRL **112**, 145004 (2014)

Kraus et al., PRE **94**, 011202(R) (2016)
[C. Lin et al., PRE 96, 013202 (2017)]

- Ionization prediction taken from OPAL

Rogers et al., APJ **456**, 902 (1996)

Preliminary, from Tilo Döppner et al., LLNL

Ionization potential depression (IPD)

Degenerate plasmas: Carbon

$$\Theta = \frac{T}{T_{\rm Fermi}} = \frac{2m_e k_B T}{\hbar^2}(3\pi^2 n_e)^{-2/3} < 1$$

In-medium Schroedinger equation for $C^{5+} = C^{6+} + e$

$$[E_e(p) + \Delta_e(p)]\,\phi_{\hat n}(\mathbf{p}) + [1 - f_e(p)]\sum_{\mathbf{q}} V^{\rm scr}_{{\rm C}^{6+},e}(\mathbf{q})\phi_{\hat n}(\mathbf{p}+\mathbf{q}) = E^{5+}_{\hat n,{\rm rel}}\phi_{\hat n}(\mathbf{p})$$

Effects of degeneracy on energy level shifts

Fock shift

$$\Delta_e^{\rm Fock}(p) = -\sum_q \frac{e^2}{\epsilon_0 q^2} f_e(\mathbf{p}+\mathbf{q})$$

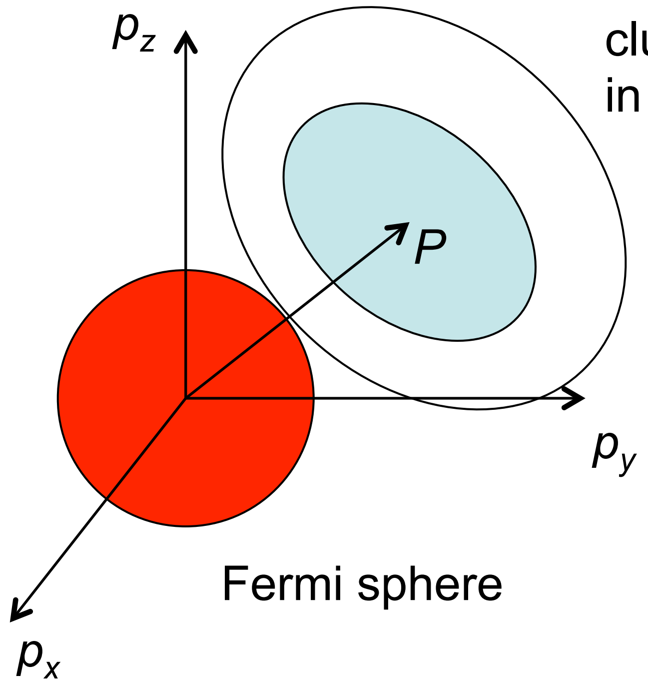
"Fermi hole"

$$\Delta_0^{\rm bound,\,Fock} = -\sum_{p,q} \phi_0^2(p)\frac{e^2}{\epsilon_0 q^2} f_e(\mathbf{p}+\mathbf{q})$$

Pauli blocking

$$\Delta_0^{\rm bound,\,Pauli} = -\sum_{p,q} \phi_0(p) f_e(p) V_{{\rm C}^{6+},e}(q)\phi_0(\mathbf{p}+\mathbf{q})$$

Pauli blocking – phase space occupation



cluster wave function (atom, ions,…)
in momentum space

P - center of mass momentum

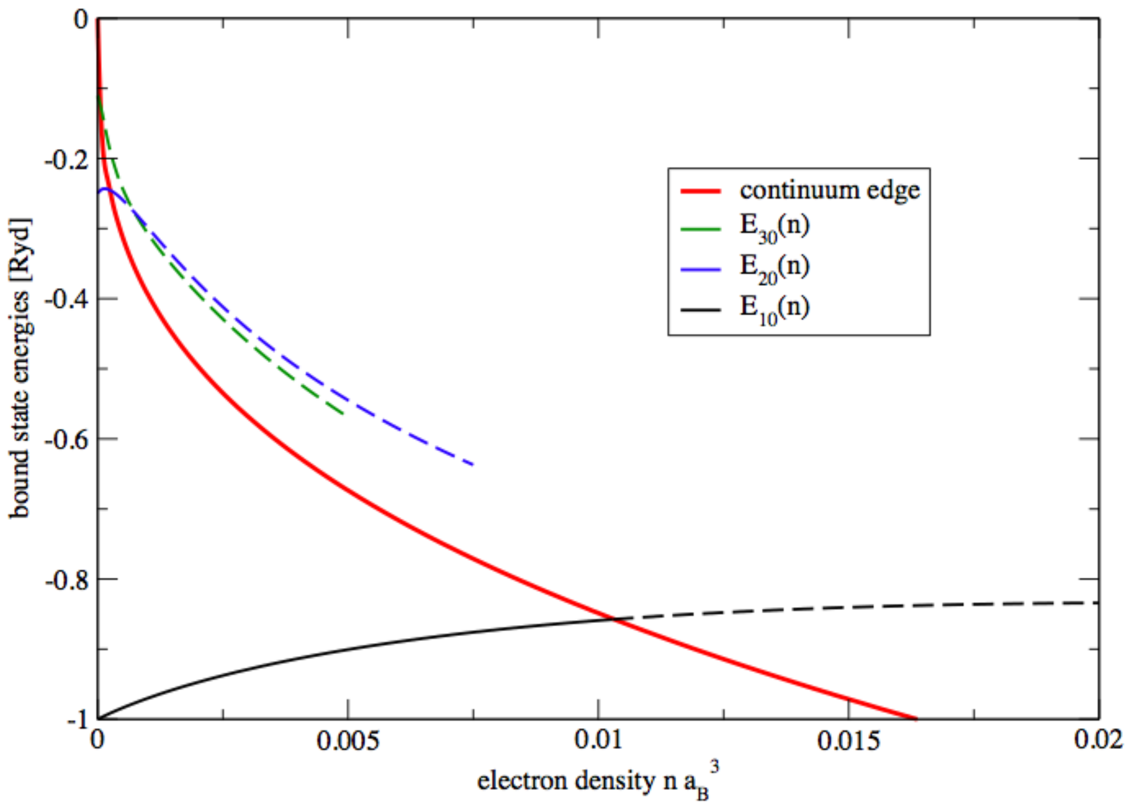
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of binding energies

$$\lim_{n\to 0} \Delta E_{10}^{\rm PF} = \frac{n}{2}\sum_q \frac{4\pi e^2}{q^2}\phi_{10}(q)[\phi_{10}(0)-\phi_{10}(q)] = 32\pi n' - 20\pi n' = 12\pi n'$$



H-Plasma:
Shift of the ground state and two excited states, Pauli-Fock-approximation, T=0

$$\lim_{n\to 0} \Delta E_{20}^{\rm PF} = 48\pi n'$$

$$\lim_{n\to 0} \Delta E_{30}^{\rm PF} = 108\pi n'$$

$$\Delta E^{\rm Fock}(p=0) = -\sum_q V(q) f_e(q) = -\frac{4p_F}{\pi} = -4\left(\frac{3n'}{\pi}\right)^{1/3}$$

W. Ebeling, D. Blaschke, R. Redmer, H. Reinholz, G.Roepke, J. Phys. A: Math. Theor. **42**, 214033 (2009)

W. Ebeling, W-D. Kraeft, G.Roepke, Contr. Plasma Phys. **52**, 7 (2012)

IPD and dissolution of bound states

Quantum statistical approach
(Green functions, spectral functions, self-energy,
quasiparticle shifts of free states and bound clusters)

Low density limit: Chemical picture, Debye screening

Increasing density: ions are strongly correlated but classical
(dynamical ionic structure factor, fluctuation-dissipation theorem)
electrons become degenerated, Pauli principle

PHYSICAL REVIEW E **99**, 033201 (2019)

Ionization potential depression and Pauli blocking in degenerate plasmas at extreme densities

Gerd Röpke,[1,2] David Blaschke,[2,3,4] Tilo Döppner,[5] Chengliang Lin,[1] Wolf-Dietrich Kraeft,[1]
Ronald Redmer,[1] and Heidi Reinholz[1,6]
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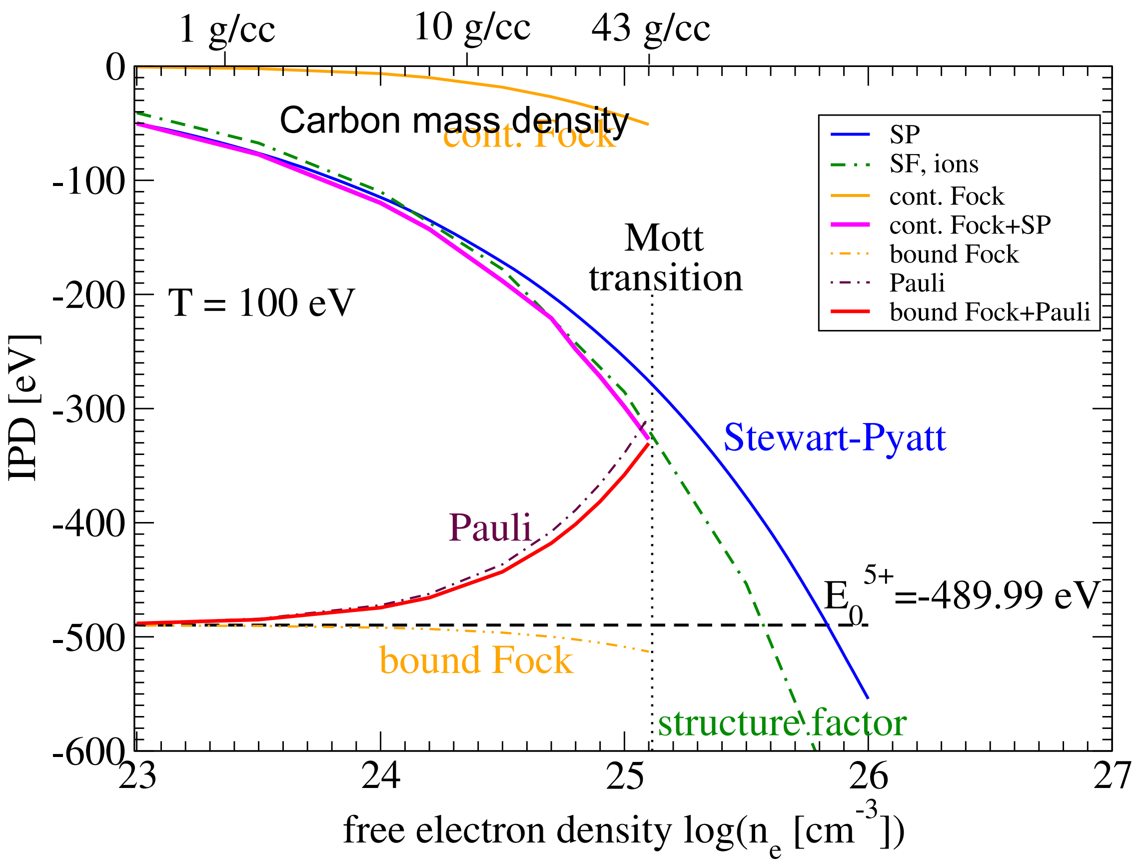
[2]*Department of Theoretical Nuclear Physics, National Research Nuclear University (MEPhI), 115409 Moscow, Russia*
[3]*Institute of Theoretical Physics, University of Wroclaw, 50-204 Wroclaw, Poland*
[4]*Joint Institute for Nuclear Research, 141980 Dubna, Russia*
[5]*Lawrence Livermore National Laboratory, Livermore, California 94550, USA*
[6]*School of Physics, University of Western Australia, WA 6009 Crawley, Australia*

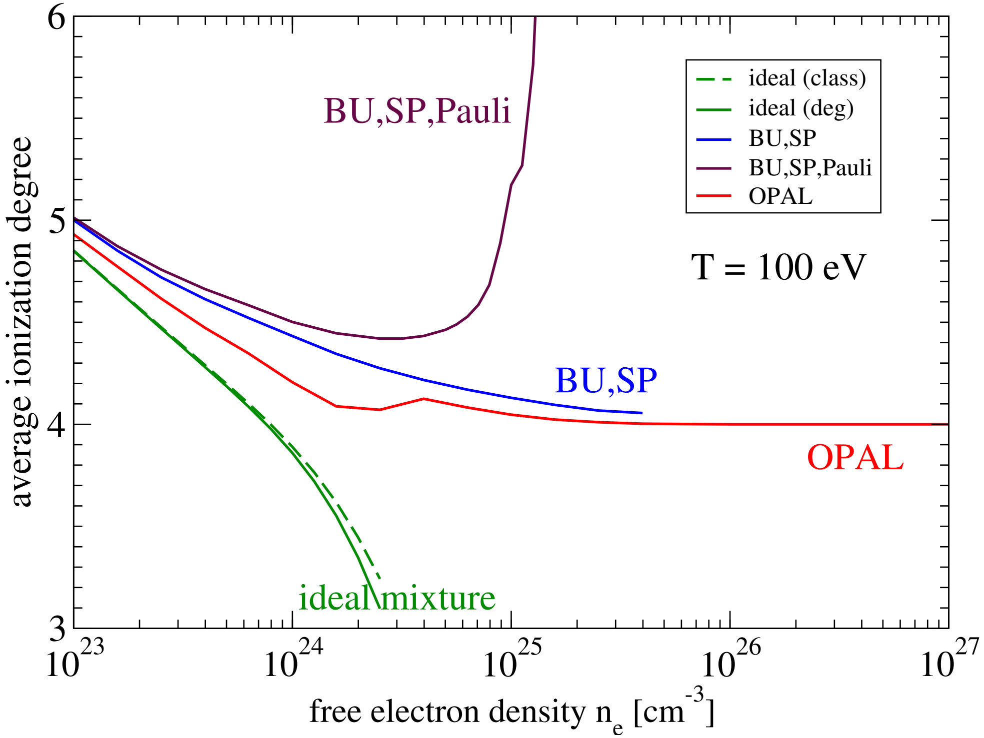
IPD of C^{5+} at T=100 eV



Stewart-Pyatt, structure factor, Fock, and Pauli shifts

Improve: shift of the bound state, broadening of bound states, ionic structure factor and band formation

Ionization degree of C plasmas



Present work:
DFT-MD simulations
Density of states,
Conductivity
(R. Redmer,
M. Bethkenhagen)

partial densities

$$\sigma_{C^{5+}}^{\text{bound}}(T) = \sum_{\gamma,\nu}^{\text{bound}} \left[e^{\beta I_{\gamma,\nu}^{5+}} - 1 \right] \theta\left(I_{\gamma,\nu}^{5+}\right), \qquad \theta(x) = \left\{ \begin{array}{ll} 1 & \text{if } x > 0, \\ 0 & \text{else,} \end{array} \right\}$$

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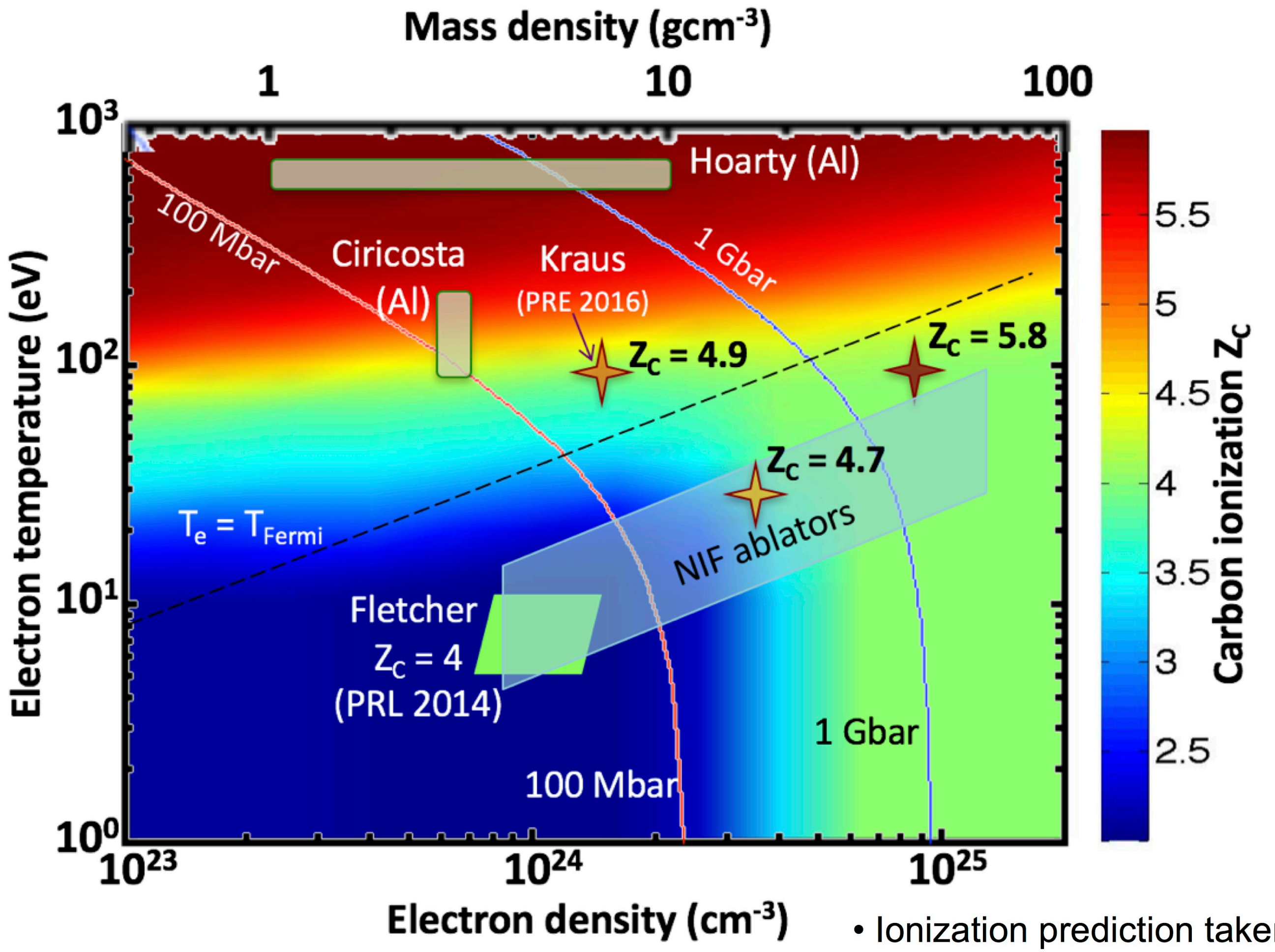

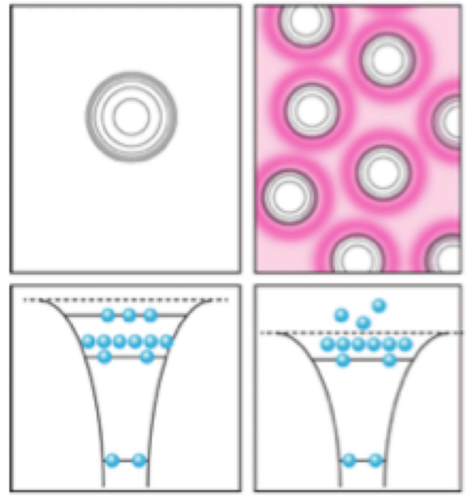

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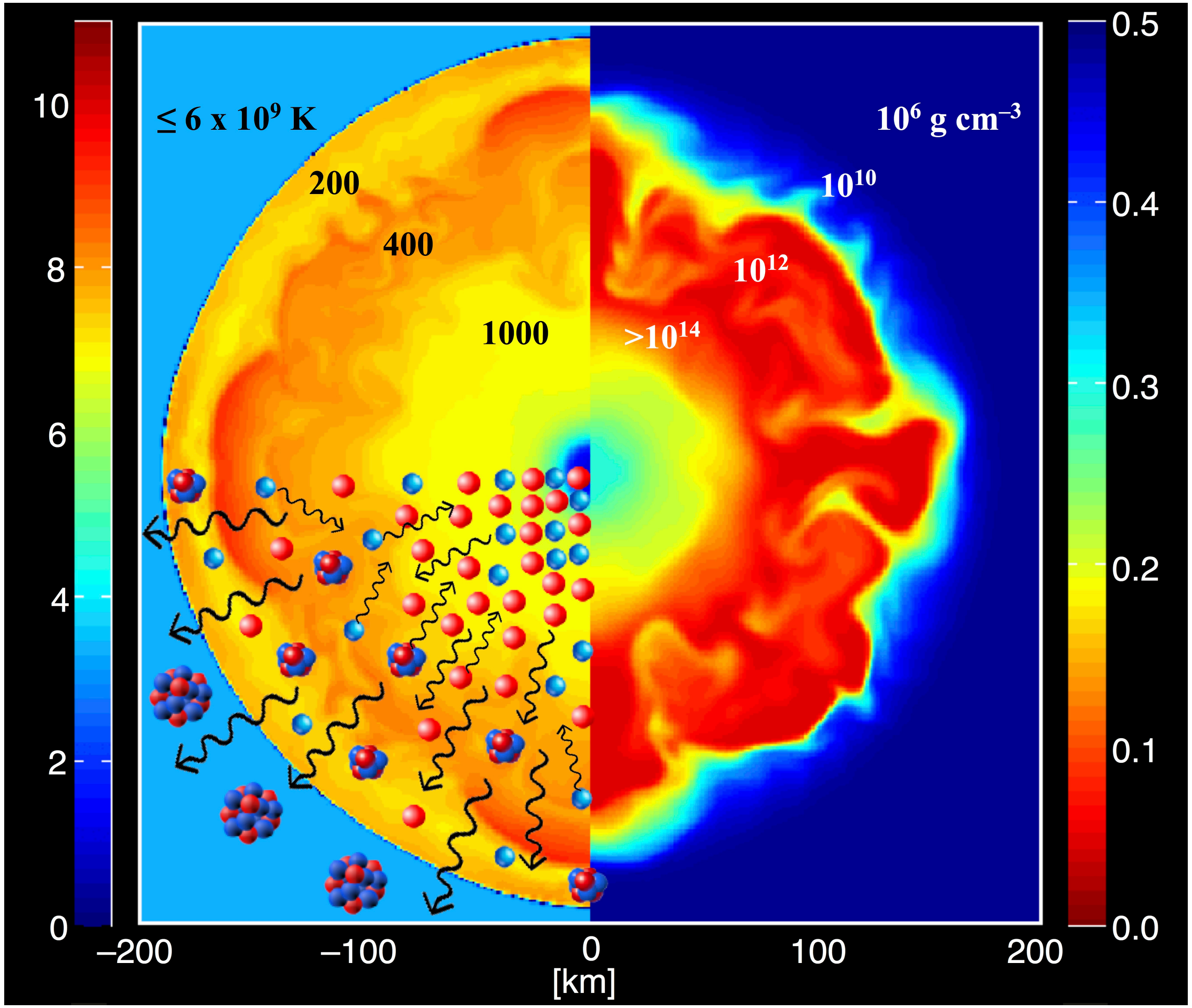
Nuclear Systems

- strongly correlated quantum systems
- nuclear structure
- heavy ion collisions
- astrophysics: compact objects

Stellar matter: Supernova explosion



Snapshot:
Temperature,
Density,
Proton fraction,
Entropy,
Neutrino flux
Cluster formation

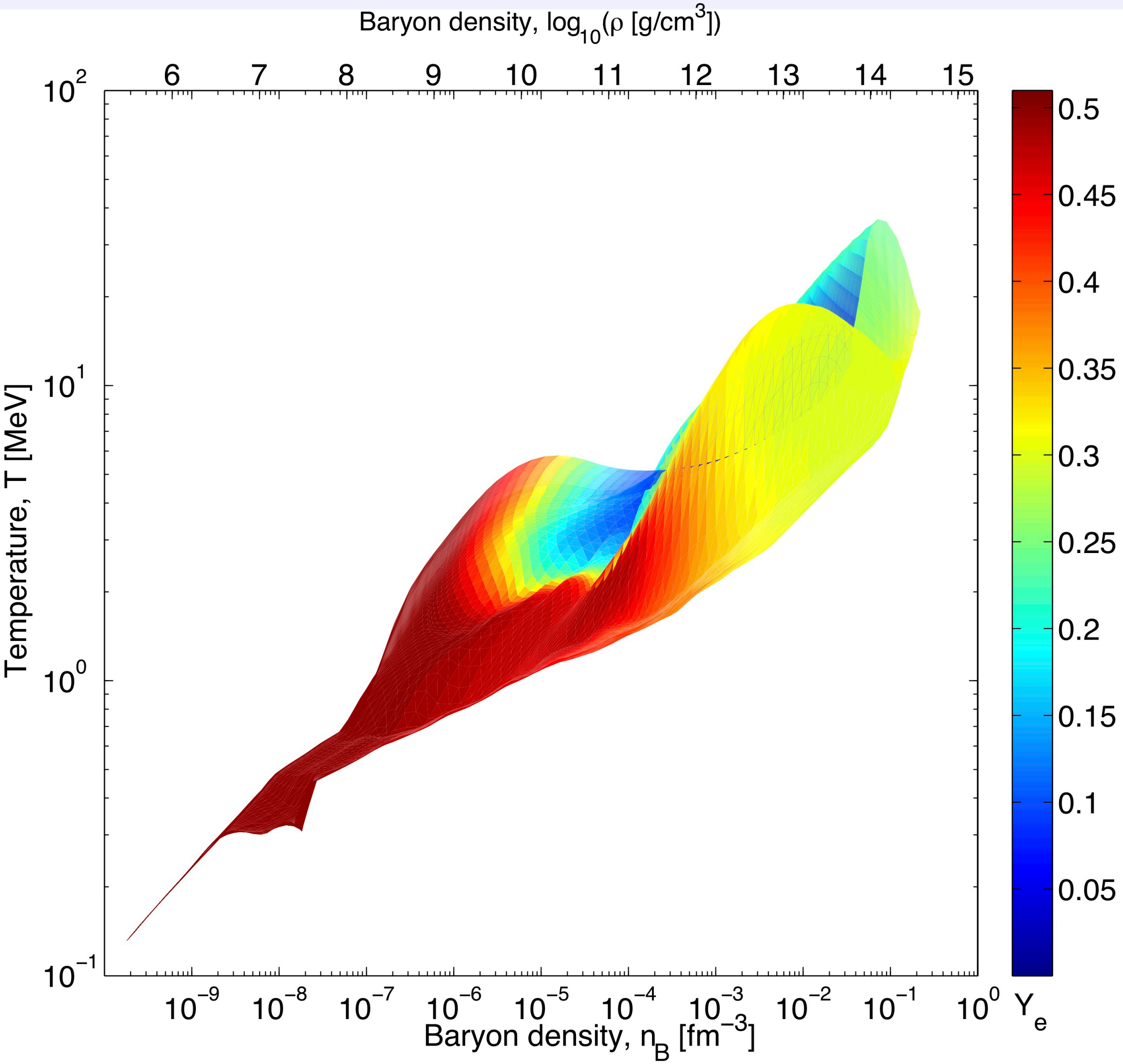
Simulation by
Tobias Fischer

Nuclear matter phase diagram

Core collapse supernovae

Relevant Parameters:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



T. Fischer, GSI Darmstadt

Composition? Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
K center of mass momentum

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

Chemical equilibrium, mass action law (Saha),
Nuclear Statistical Equilibrium (NSE)

Excited states? Scattering states?

Asymmetric nuclear light clusters in supernova matter

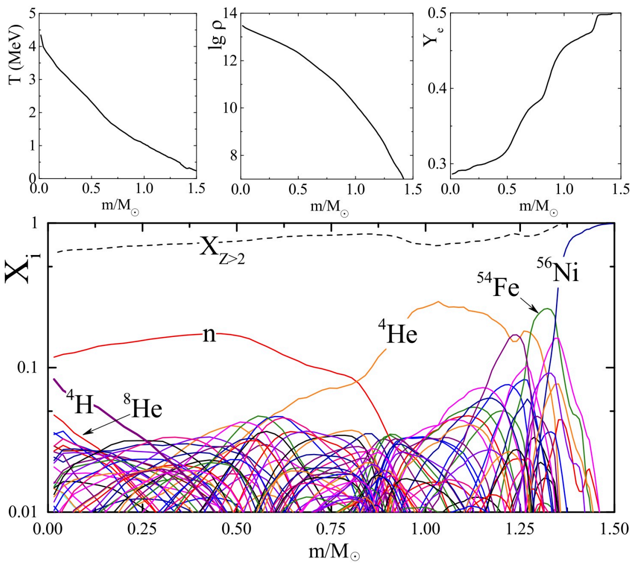


Figure 1. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $\mathrm{g\cdot cm^{-3}}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions of of nuclei X_i as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of elements with $Z>2$. EoS is pure NSE.

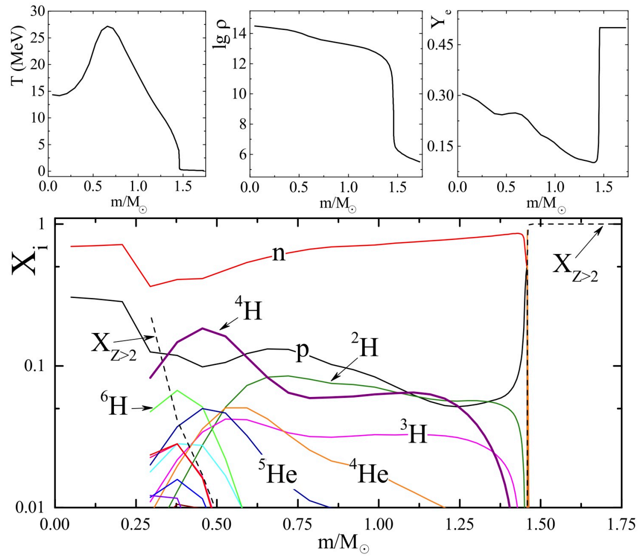


Figure 7. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $\mathrm{g\cdot cm^{-3}}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions X_i of of hydrogen and helium isotopes as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of all rest nuclei. Stellar profile corresponds to 200 ms after bounce approximately, calculations according to modified HS EoS.

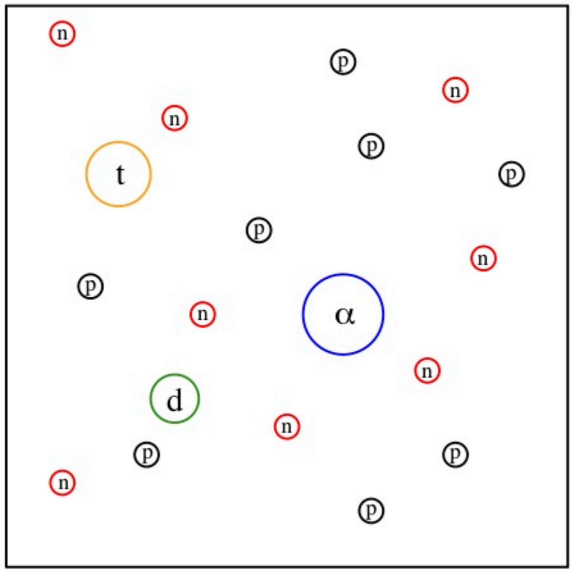
A. V. Yudin, M. Hempel, S. I. Blinnikov, D. K. Nadyozhin, I. V. Panov,
Monthly Notices of the Royal Astronomical Society **483**, 5426 (2019)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law

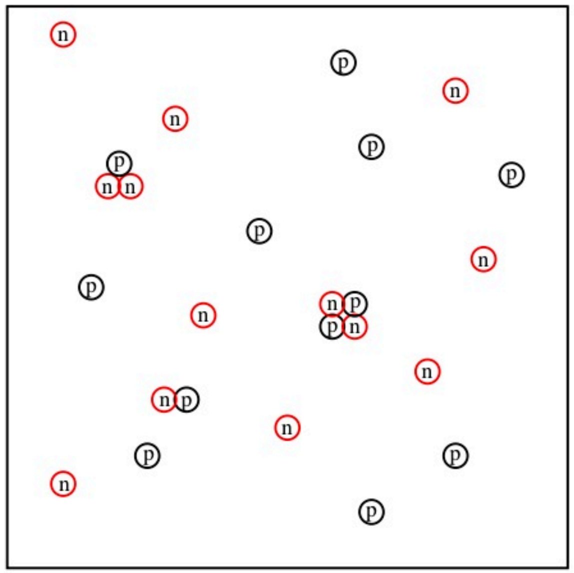


Interaction between the components
internal structure: Pauli principle

Physical picture:

"elementary" constituents

and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

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mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

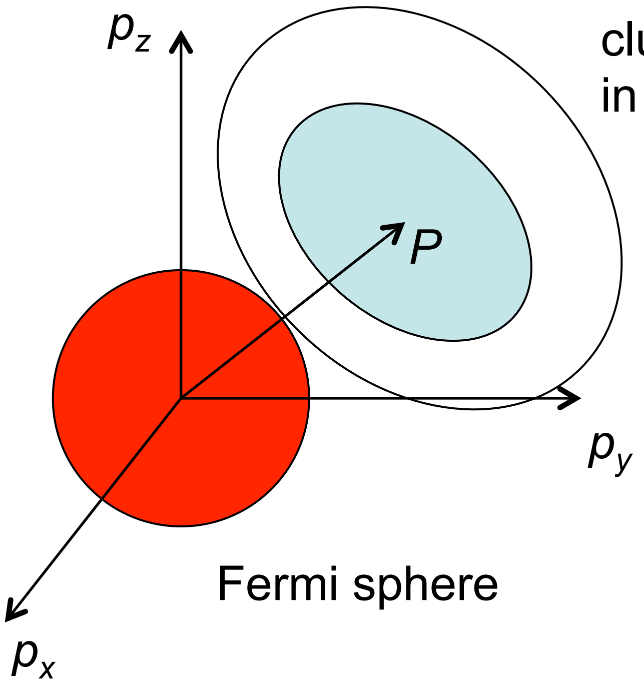
Correlated medium?

Thouless criterion

$$E_d(T,\mu)=2\mu$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (atom, ions,…)
in momentum space

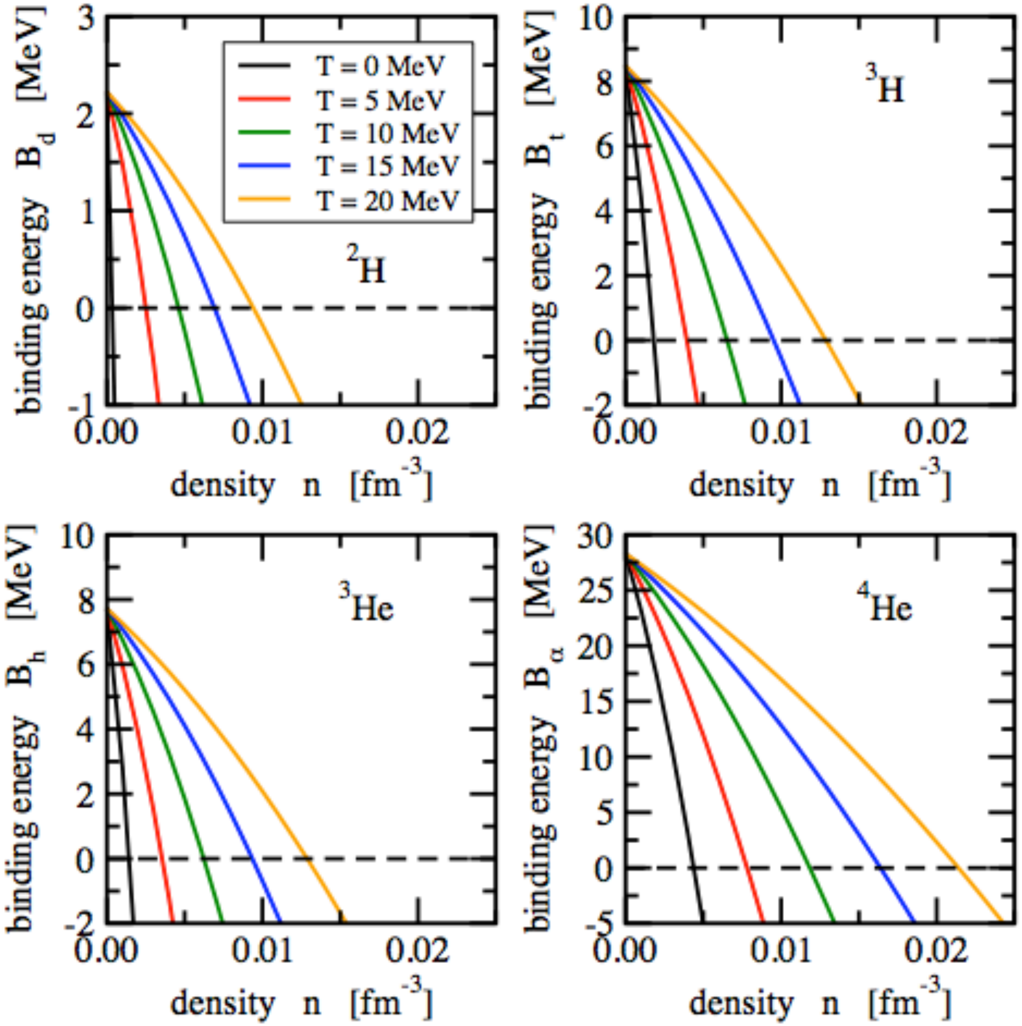
P - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum *P*

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of Binding Energies of Light Clusters

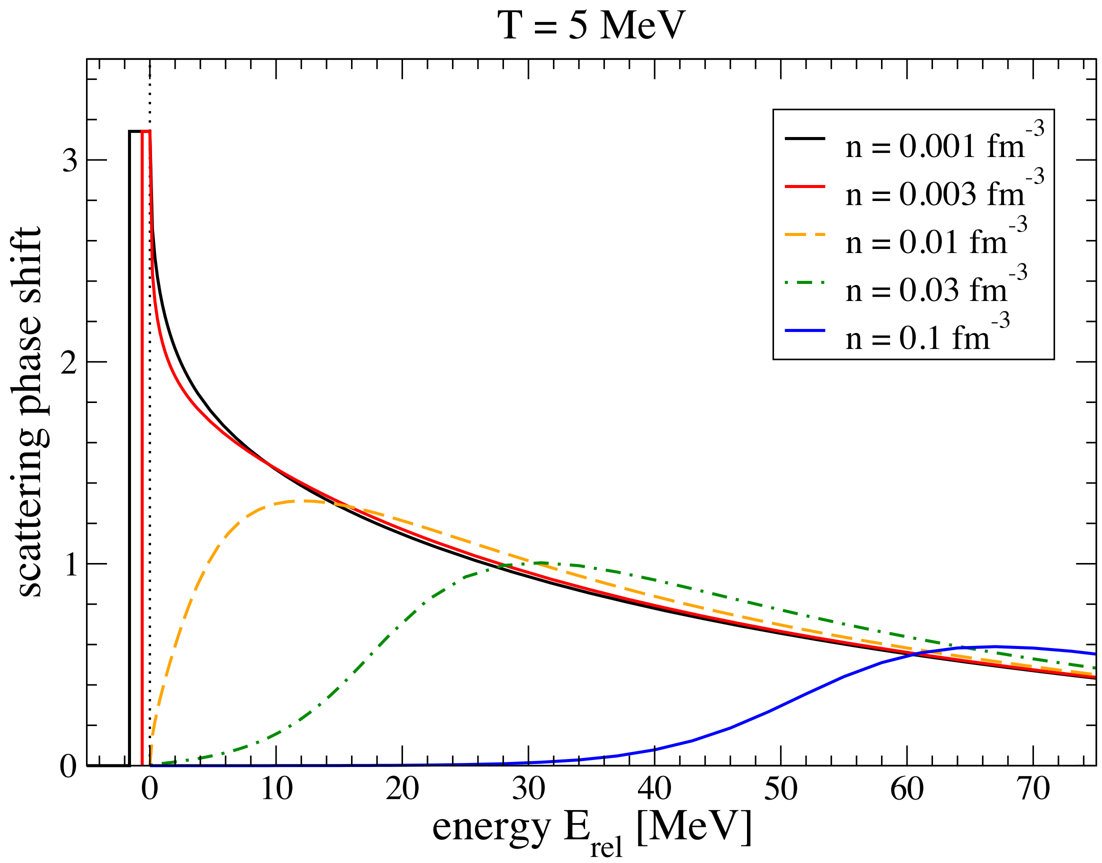


Symmetric matter

S. Typel, G. Röpke, T. Klähn, D. Blaschke, and H. H. Wolter. PRC **81**, 015803 (2010)

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



Tamm-Dancoff

deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

Example: ^{5}He

Partial density
$$n_{^5\mathrm{He}} = 8\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} b_{\alpha n}(T)e^{(-E_\alpha+3\mu_n+2\mu_p)/T}$$

virial coefficient

nuclear stat. equ.
$$b_{\alpha n}^{\mathrm{NSE}}(T) = \frac{5^{3/2}}{2}e^{(-E_{^5\mathrm{He}}+E_{^4\mathrm{He}})/T}$$

generalized Beth-Uhlenbeck
$$b_{\alpha n}^{\mathrm{gBU}}(T) = \left(\frac{5}{4}\right)^{1/2}\frac{1}{\pi T}\int_0^\infty dE_{\mathrm{lab}}\, e^{-4E_{\mathrm{lab}}/5T}\left\{\delta_{\alpha n}^{\mathrm{tot}}(E_{\mathrm{lab}}) - \frac{1}{2}\sin[2\delta_{\alpha n}^{\mathrm{tot}}(E_{\mathrm{lab}})]\right\}$$

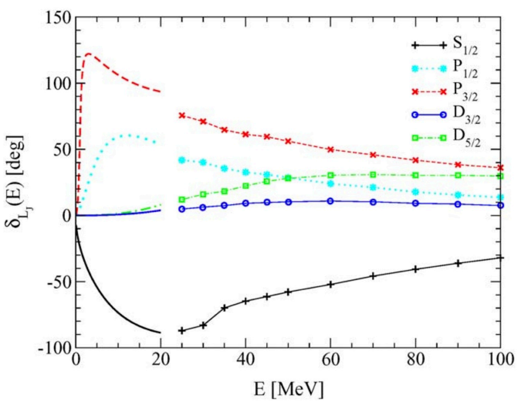
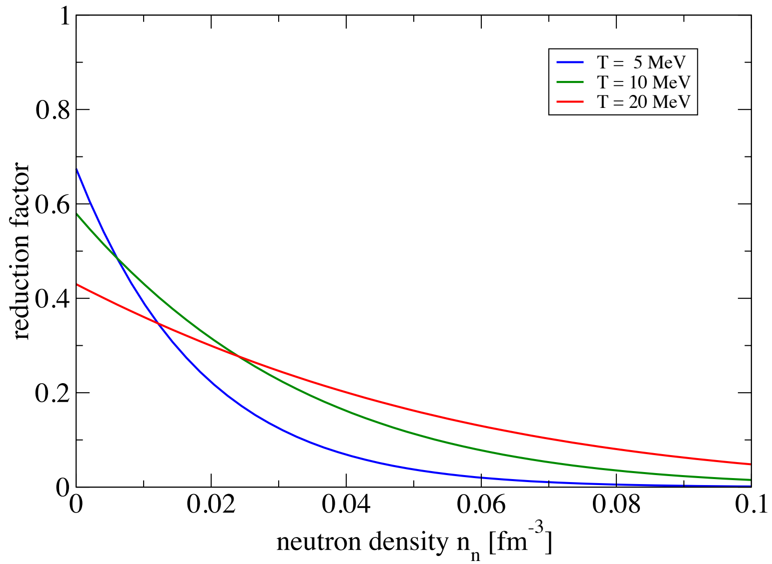


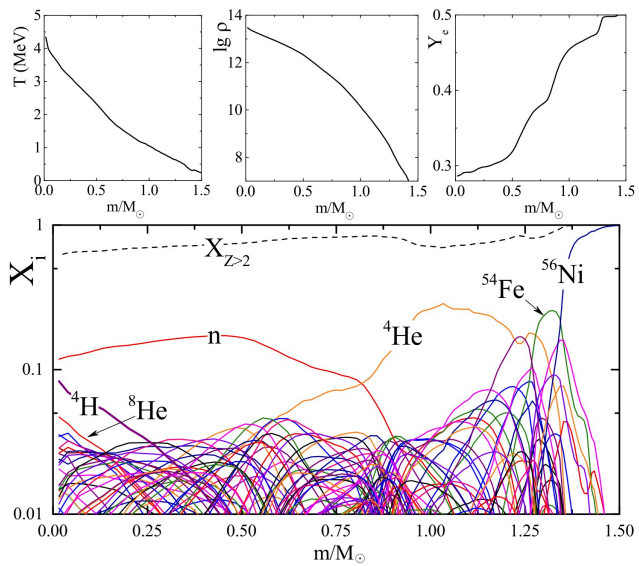
Fig. 2. (Color online.) The phase shifts for elastic neutron-alpha scattering $\delta_{L_J}(E)$ versus laboratory energy E. As discussed in the text, the solid lines are from Arndt and Roper [37] and the symbols are from Amos and Karataglidis [38]. For clarity, we do not show the F-waves included in our results for $b_{\alpha n}$.

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)



ratio generalized Beth-Uhlenbeck/NSE

Asymmetric nuclear light clusters in supernova matter



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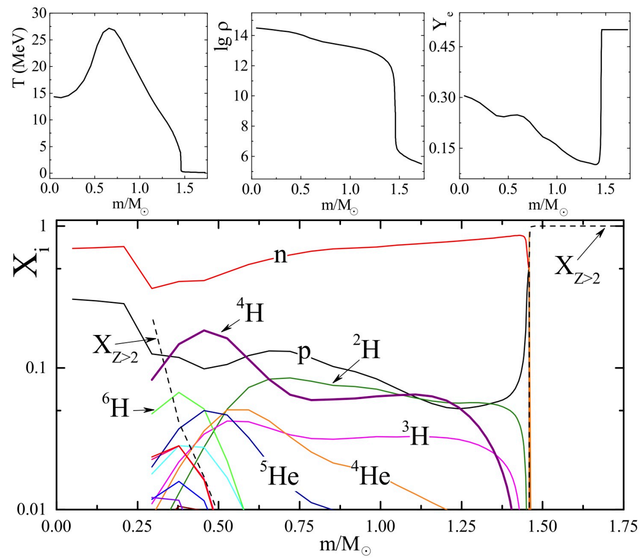

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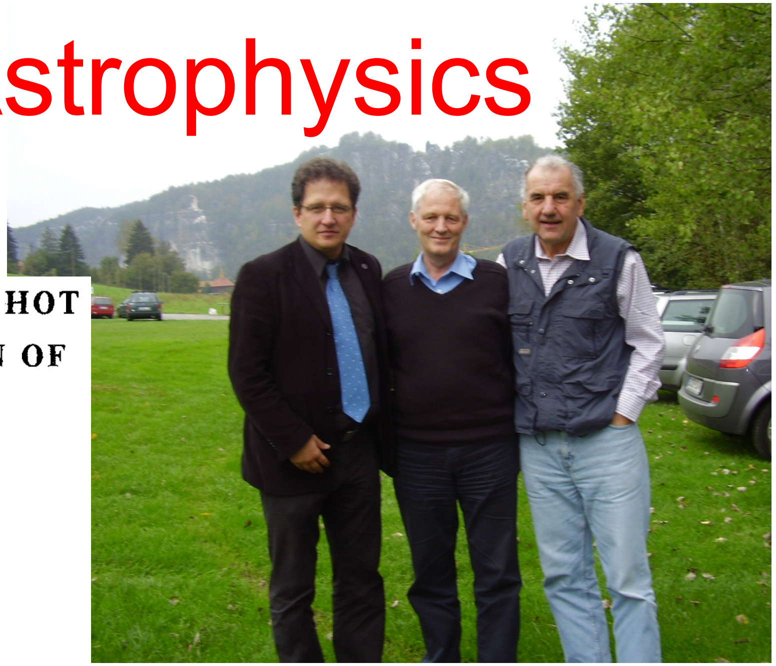
Quarks: Pauli – Mott - Astrophysics

Astrophysics and Space Science **95** (1983)

QUANTUM STATISTICAL CLUSTER ABUNDANCES IN HOT NUCLEAR MATTER AND ELEMENTAL COMPOSITION OF COSMIC-RAY SOURCES

G. RÖPKE and D. BLASCHKE
Sektion Physik, Wilhelm-Pieck-Universität, Rostock, G.D.R.
and
H. SCHULZ
Zentralinstitut für Kernphysik, Rossendorf, G.D.R.



"Mott mechanism and hadronic-to-quark matter phase transition", Phys. Lett. B (1985), Blaschke-Reinholz-Röpke-Kremp

PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke
Department of Physics, Wilhelm-Pieck-Universtiy, 2500 Rostock, German Democratic Republic

H. Schulz
Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic and The Niels Bohr Institute, 2100 Copenhagen, Denmark

(Received 16 December 1985)

1989 – falling walls

ECT*- Villa Tambosi Trento, 4. Sept. 2019

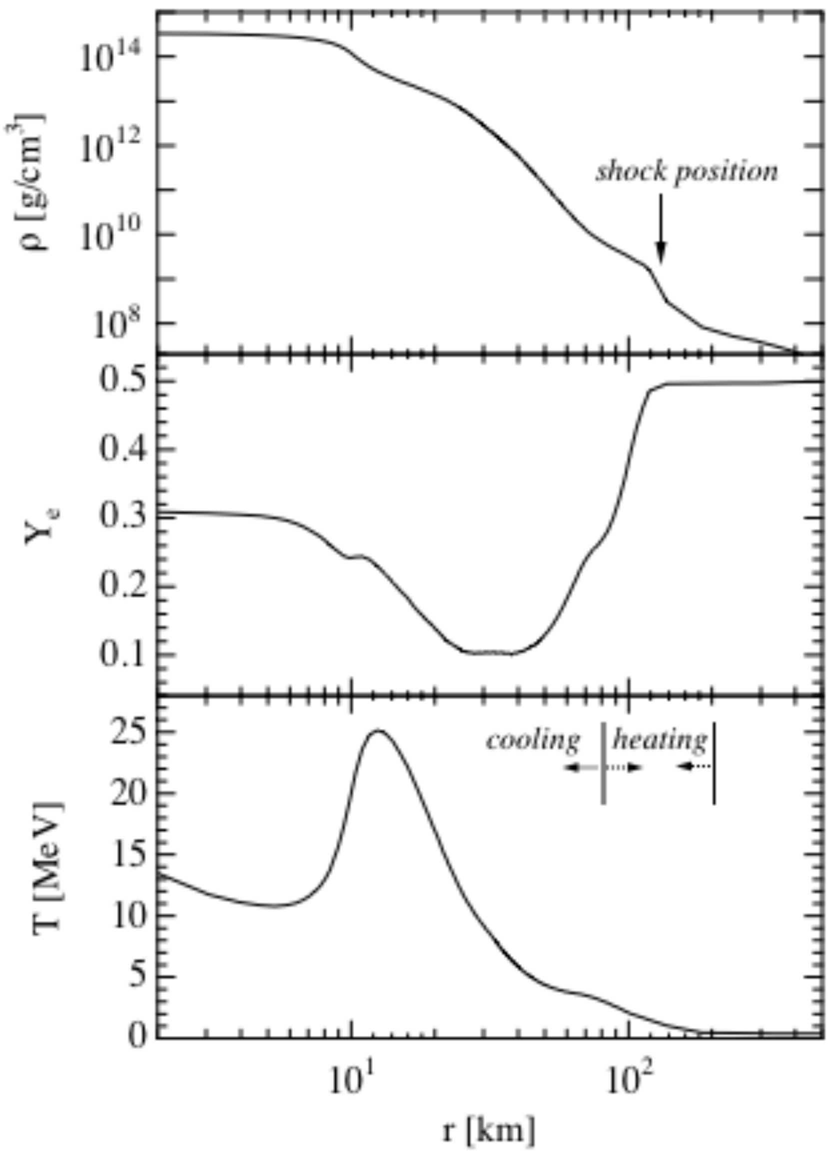
Workshop:

Light clusters in nuclei
and nuclear matter:
Nuclear structure and decay,
heavy ion collisions,
and astrophysics

One month ago:
David explains the
Mott-transition

Happy Birthday to You

Core-collapse supernovae



Density.

electron fraction, and

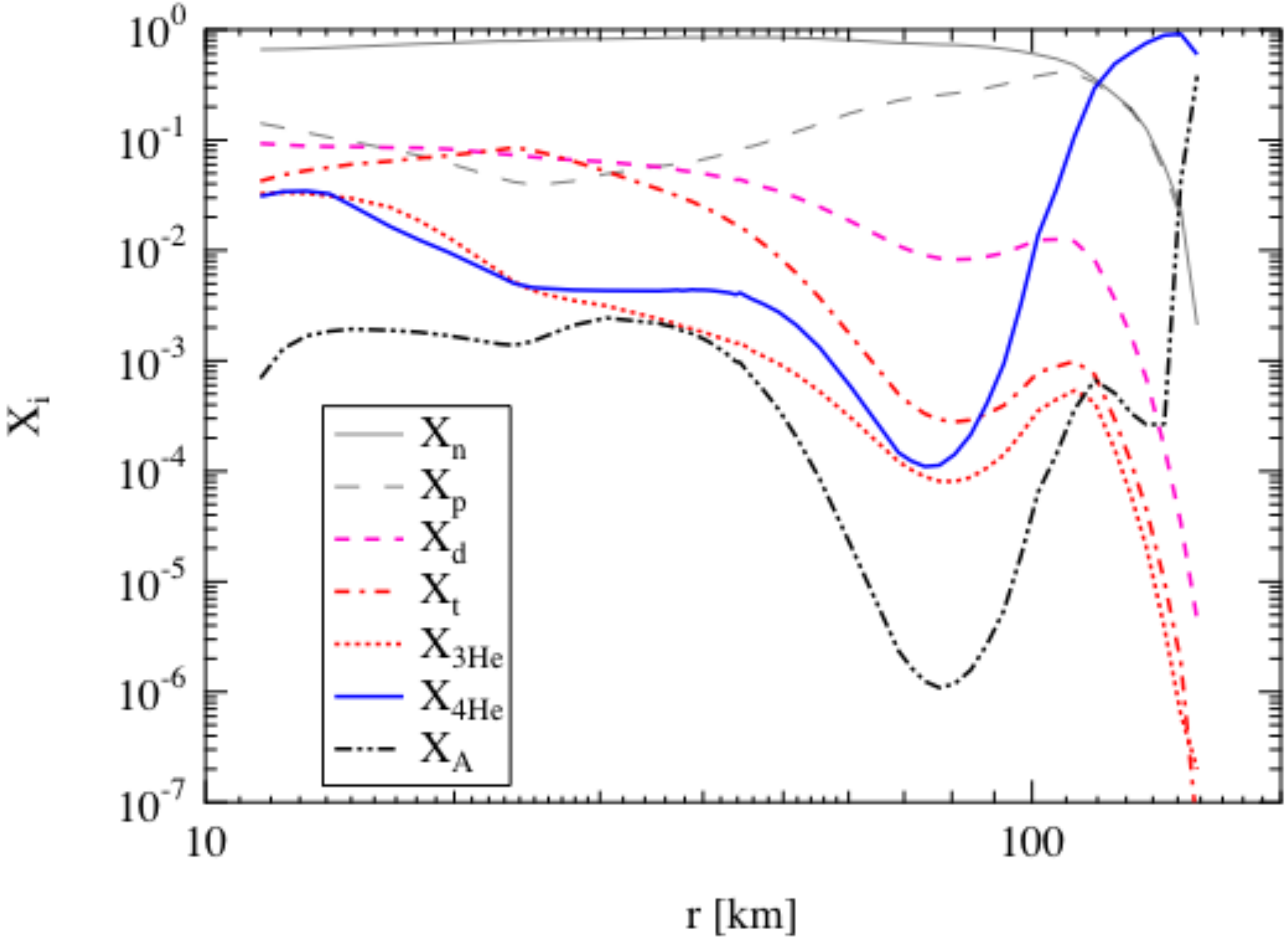
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core

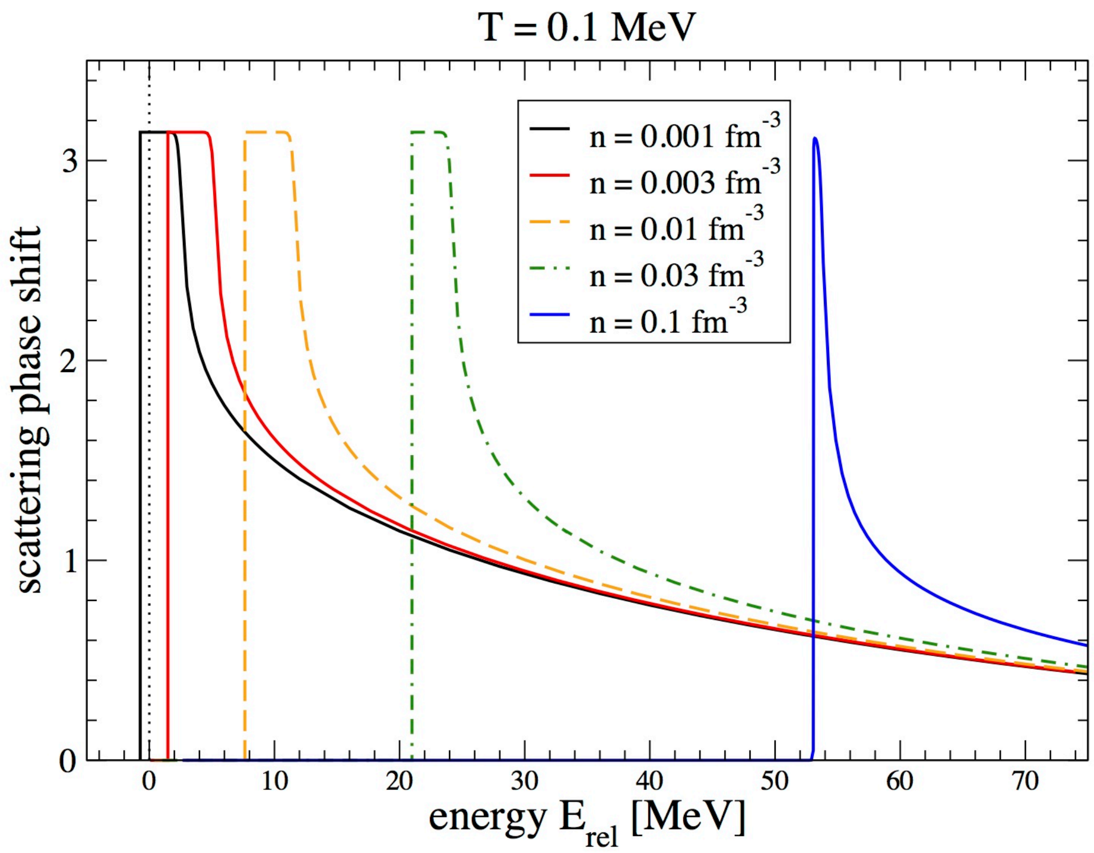


Mass fraction X
of light clusters
for a post-bounce
supernova core

K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



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EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

parametrization (d – like):

$$v_c(\mathbf{P}=0;T,n_B,Y_p) \approx \left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T) = v_{T_I=0}^0(T) \approx 0.30857+0.65327\;e^{-0.102424\;T/{\rm MeV}}$$

G. Roepke, PRC 92,054001 (2015)

Few-particle Schrödinger equation in a dense medium

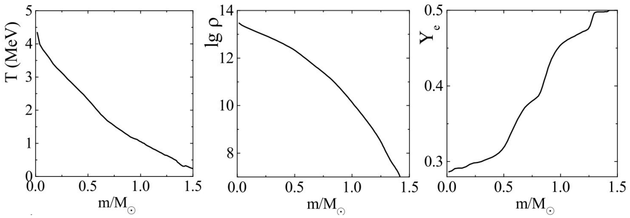
4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$
$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$
$$+\{permutations\}$$
$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

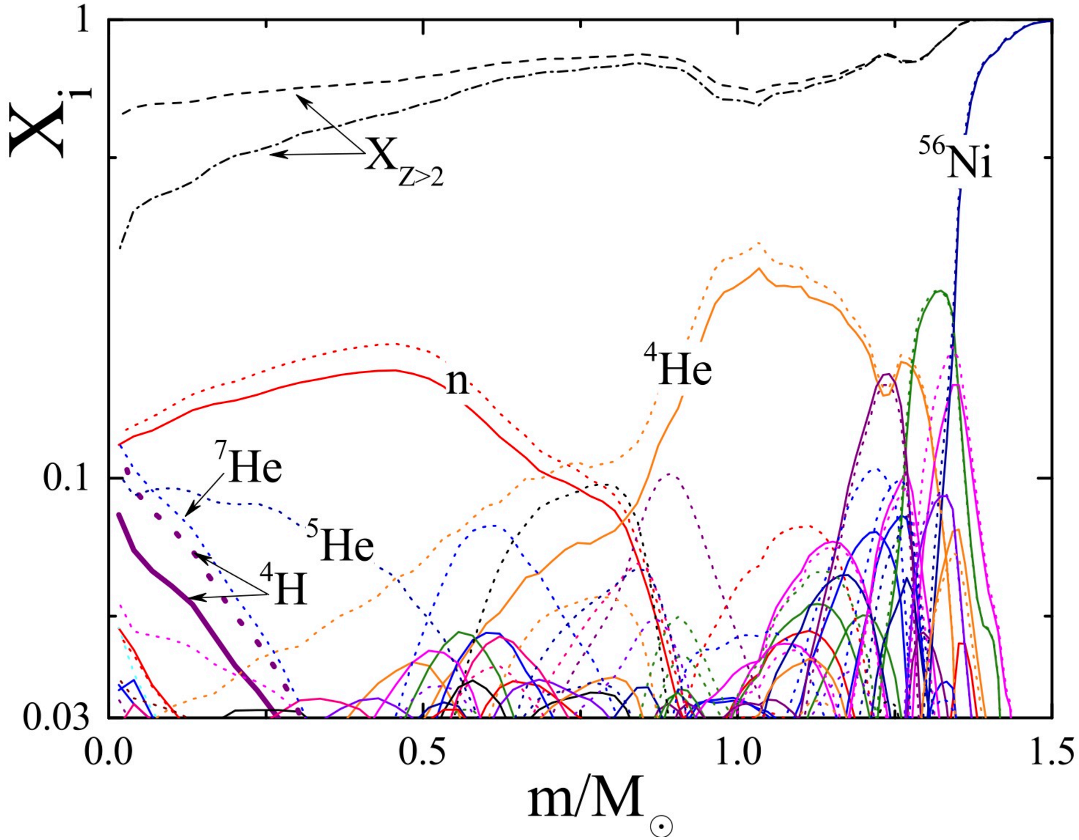
Thouless criterion
for quantum condensate:

$E_{n,P=0}(T,\mu) = 4\mu$

Light unstable clusters



arXiv:1812.09494



A. V. Yudin, M. Hempel, S. I. Blinnikov, D. K. Nadyozhin, I. V. Panov,
Monthly Notices of the Royal Astronomical Society **483**, 5426 (2019)