Influence of quark masses and strangeness degrees of freedom on inhomogeneous chiral phases

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Introduction

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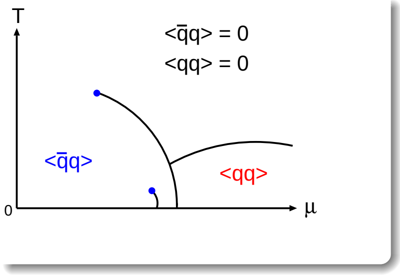
- QCD phase diagram (standard picture):



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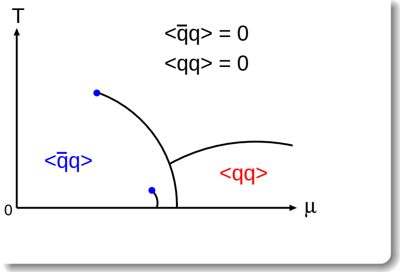

- assumption: $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space

Introduction

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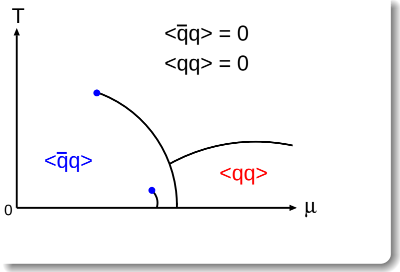
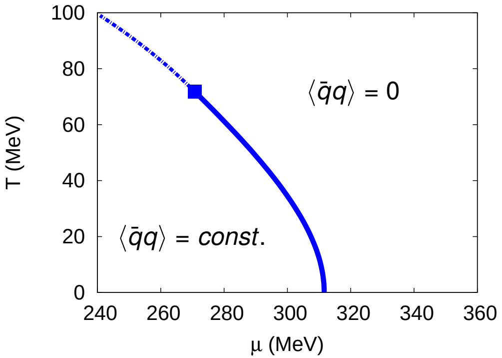


- assumption: $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space
- How about non-uniform phases ?

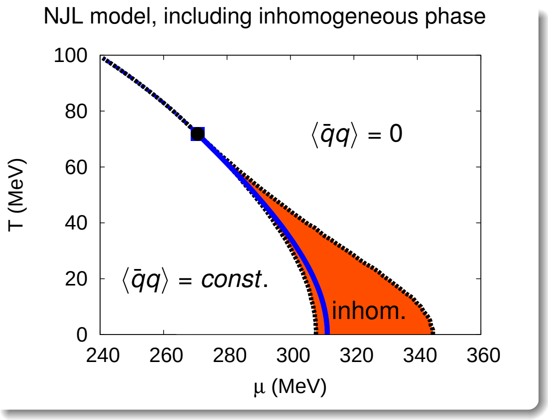
Introduction

NJL model, homogeneous phases only



[D. Nickel, PRD (2009)]

Introduction



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Introduction

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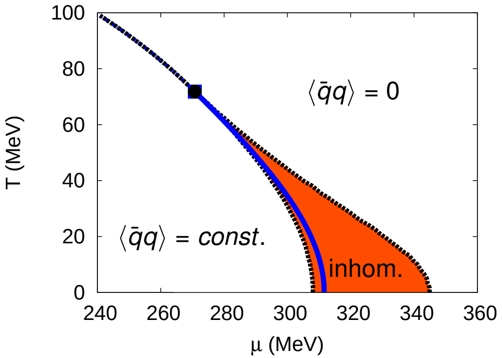
NJL model, including inhomogeneous phase



[D. Nickel, PRD (2009)]

- 1st-order phase boundary completely covered by the inhomogeneous phase!
- Critical point $\rightarrow$ Lifshitz point
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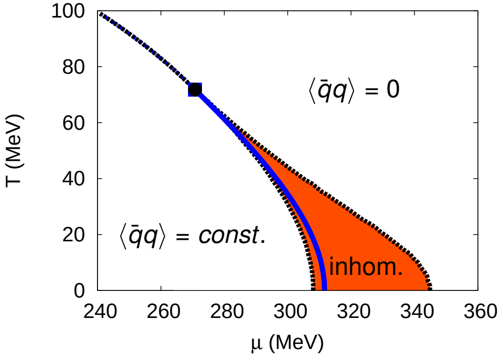

[D. Nickel, PRD (2009)]

- 1st-order phase boundary completely covered by the inhomogeneous phase!
- Critical point $\rightarrow$ Lifshitz point
  [D. Nickel, PRL (2009)]
- Inhomogeneous phase rather robust under model extensions and variations
  [MB, S. Carignano, PPNP (2015)]

Questions addressed in this talk:

- What is the effect of nonzero bare quark masses?

  [MB, S. Carignano, PLB (2019); arxiv:1809.10066 [hep-ph]]

- What is the influence of strange quarks?

  [S. Carignano, MB, arxiv:1910.03604 [hep-ph]]

Nonzero bare masses

- What is the effect of going away from the chiral limit?

Nonzero bare masses

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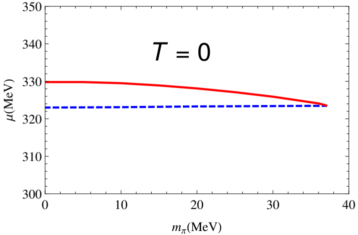
- What is the effect of going away from the chiral limit?
- Quark-meson model
  [Andersen, Kneschke, PRD (2018)]:



No inhomogeneous phase for
$m_\pi > 37.1$ MeV

Nonzero bare masses

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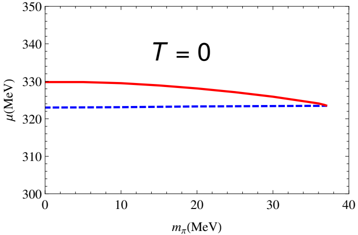

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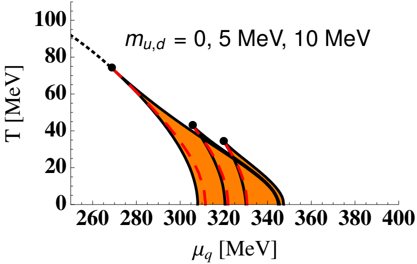

Inhomogeneous phase gets smaller but still reaches the CEP

Nonzero bare masses

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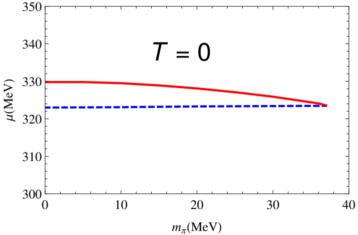

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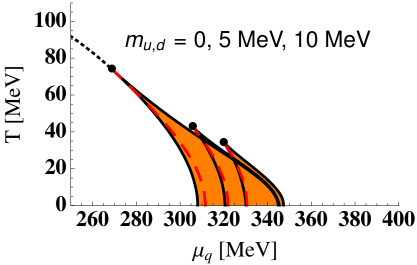

Inhomogeneous phase gets smaller but still reaches the CEP

- Can we investigate this more systematically?

NJL Model

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- Lagrangian: $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$

NJL Model

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$$\Rightarrow \quad \mathcal{L} = \bar\psi\left(i\partial\!\!\!/ - m + 2G_S(\sigma + i\gamma_5\vec\tau\cdot\vec\pi)\right)\psi - G\left(\sigma^2 + \vec\pi^{\,2}\right)$$

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$$\sigma(x) \to \langle\sigma(x)\rangle \equiv \phi_S(\vec{x}), \quad \pi_a(x) \to \langle\pi_a(x)\rangle \equiv \phi_P(\vec{x})\,\delta_{a3}$$

  - $\phi_S(\vec{x})$, $\phi_P(\vec{x})$ time independent classical fields
  - retain space dependence !

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(towards studying inhomogeneous phases beyond mean-field approximation:
$\rightarrow$ Martin Steil's talk after the coffee break)

Mean-field thermodynamic potential

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- Mean-field thermodynamic potential:

$$\Omega_{MF}(T,\mu) = -\frac{T}{V}\log\mathcal{Z}(T,\mu) = -\frac{T}{V}\,\mathbf{Tr}\log\left(\frac{S^{-1}}{T}\right) + G\,\frac{1}{V}\int d^3x\,\left(\phi_S^2(\vec{x}) + \phi_P^2(\vec{x})\right)$$

  - **Tr**: functional trace over Euclidean $V_4 = [0, \frac{1}{T}] \times V$, Dirac, color, and flavor
  - inverse dressed propagator:

    $\mathcal{S}^{-1}(x) = i\partial\!\!\!/ + \mu\gamma^0 - m + 2G_S\left(\phi_S(\vec{x}) + i\gamma_5\tau_3\phi_P(\vec{x})\right)$

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$\Rightarrow \quad \Omega_{MF} = \Omega_{MF}[\phi_S(\vec{x}), \phi_P(\vec{x})]$ minimization extremly difficult !

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- Ginzburg-Landau expansion:

  = expansion in small amplitudes and gradients of the order parameter function

  ☹ valid only near the LP

  ☺ no ansatz functions for $\phi_S(\vec{x})$ and $\phi_P(\vec{x})$ needed

Tricritical and Lifshitz point in the chiral limit

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- GL expansion: $\Omega[M] = \Omega[0] + \frac{1}{V}\int_V d^3x \left\{ \alpha_2 |M|^2 + \alpha_{4,a}|M|^4 + \alpha_{4,b}|\vec{\nabla} M|^2 + ... \right\}$
  - order-parameter function: $M(\vec{x}) \propto \phi_S(\vec{x}) + i\phi_P(\vec{x})$

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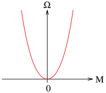
case 1.1: $\alpha_{4,a} > 0$

- $\alpha_2 > 0 \;\Rightarrow$ restored phase



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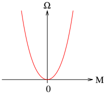
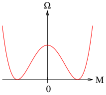
  - $\alpha_2 < 0 \Rightarrow$ hom. broken phase





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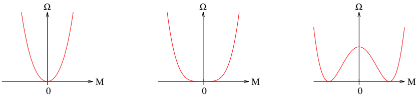
- 2nd-order p.t. at $\alpha_2 = 0$



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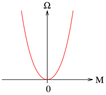
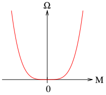
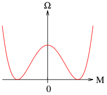
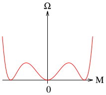






case 1.2: $\alpha_{4,a} < 0$

- 1st-order phase trans. at $\alpha_2 > 0$



Tricritical and Lifshitz point in the chiral limit

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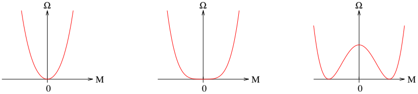

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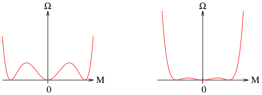

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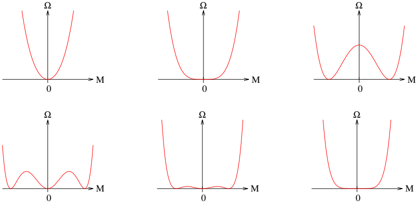

$\Rightarrow$ tricritical point (TCP): $\alpha_2 = \alpha_{4,a} = 0$

Tricritical and Lifshitz point in the chiral limit

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- case 2: $\alpha_{4,b} < 0$
  - inhomogeneous phase possible

Tricritical and Lifshitz point in the chiral limit

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  $\Rightarrow$ tricritical point (TCP): $\alpha_2 = \alpha_{4,a} = 0$
- case 2: $\alpha_{4,b} < 0$
  - inhomogeneous phase possible

  Lifshitz point (LP): $\alpha_2 = \alpha_{4,b} = 0$

Away from the chiral limit

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- $m \neq 0$: no chirally restored solution $M = 0$

  $\rightarrow$ expand about a priory unknown spatially constant mass $M_0(T, \mu)$:

$$\Omega[M] = \Omega[M_0] + \frac{1}{V}\int d^3x \left(\alpha_1 \delta M + \alpha_2 \delta M^2 + \alpha_3 \delta M^3 + \alpha_{4,a} \delta M^4 + \alpha_{4,b} (\nabla \delta M)^2 + \ldots\right)$$

  - small parameters: $\delta M(\vec{x}) \equiv M(\vec{x}) - M_0, \quad |\nabla \delta M(\vec{x})|$
  - GL coefficients: $\alpha_j = \alpha_j(T, \mu, M_0)$
  - odd powers allowed
  - require M_0 = extremum of Ω at given T and μ

    $\Rightarrow \quad \alpha_1(T, \mu, M_0) = 0 \quad \rightarrow \quad M_0 = M_0(T, \mu)$ (= homogeneous gap equation)

CEP and pseudo Lifshitz point

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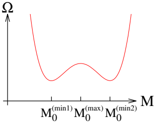
- GL expansion:

$$\Omega[M] = \Omega[M_0] + \frac{1}{V}\int d^3x \left(\alpha_2 \delta M^2 + \alpha_3 \delta M^3 + \alpha_{4,a}\delta M^4 + \alpha_{4,b}(\nabla \delta M)^2 + ...\right)$$

- <u>case 1:</u> $\alpha_{4,b} > 0 \Rightarrow$ homogeneous



  - no restored phase, but 1st-order ph. trans. between different minima possible

CEP and pseudo Lifshitz point

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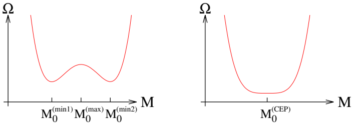

  - no restored phase, but 1st-order ph. trans. between different minima possible
  - 2 minima + 1 maximum $\rightarrow$ 1 minimum

$\Rightarrow$ critical endpoint (CEP): $\alpha_2 = \alpha_3 = 0$

CEP and pseudo Lifshitz point

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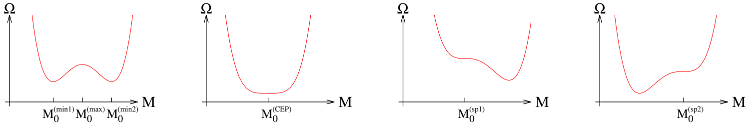

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    $\Rightarrow$ critical endpoint (CEP): $\alpha_2 = \alpha_3 = 0$
  - spinodals: left: $\alpha_2 = 0,\ \alpha_3 < 0$, right: $\alpha_2 = 0,\ \alpha_3 > 0$,

CEP and pseudo Lifshitz point

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  - strictly: only two phases – homogeneous and inhomogeneous $\Rightarrow$ no LP
  - 2nd-order phase boundary between inhom. and hom. phase: $\delta M(\vec{x}) \to 0$
  - pseudo Lifshitz point (PLP): $\delta M(\vec{x}) \to 0\,,\ \nabla \delta M(\vec{x}) \to 0$

    $\Rightarrow$ PLP: $\alpha_2 = \alpha_{4,b} = 0$

Summarizing:
GL analysis of critical and Lifshitz points

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- chiral limit ($m = 0$):
  - expansion about $M = 0$
  - TCP: $\alpha_2 = \alpha_{4,a} = 0$
  - LP: $\alpha_2 = \alpha_{4,b} = 0$
- away from the chiral limit ($m \neq 0$):
  - expansion about $M_0(T, \mu)$ solving $\alpha_1(T, \mu, M_0) = 0$
  - CEP: $\alpha_2 = \alpha_3 = 0$
  - PLP: $\alpha_2 = \alpha_{4,b} = 0$

GL coefficients

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$$\alpha_1 = \frac{M_0 - m}{2G} + M_0 F_1 \,, \qquad \alpha_2 = \frac{1}{4G} + \frac{1}{2} F_1 + M_0^2 F_2 \,, \qquad \alpha_3 = M_0 \left(F_2 + \frac{4}{3} M_0^2 F_3 \right) \,,$$

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- $F_n = 8 N_c \int \frac{d^3p}{(2\pi)^3} \, T \sum_j \frac{1}{[(i\omega_j + \mu)^2 - \vec{p}^{\,2} - M_0^2]^n}$, $\quad \omega_j = (2j+1)\pi T$

GL coefficients

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- chiral limit:
  - $m = 0 \;\Rightarrow\; M_0 = 0$ solves gap equation $\alpha_1 = 0$
  - $M_0 = 0 \;\Rightarrow\; \alpha_3 = 0$ (no odd powers)
  - $M_0 = 0 \;\Rightarrow\; \alpha_{4,a} = \alpha_{4,b} \;\Rightarrow$ TCP = LP [Nickel, PRL (2009)]

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- towards the chiral limit:
  - $M_0 \to 0 \;\Rightarrow\; \alpha_3, \alpha_{4ba}, \alpha_{4,b} \propto F_2 \;\Rightarrow\;$ CEP $\to$ TCP = LP

GL coefficients

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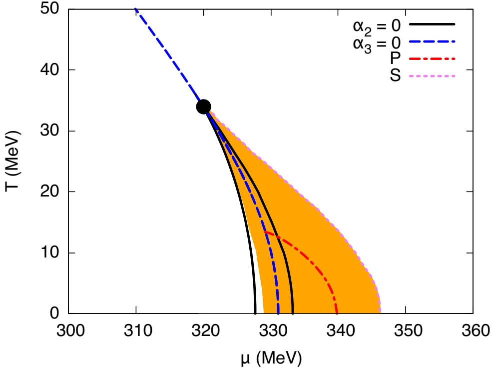
    The CEP coincides with the PLP!

Results

[MB, S. Carignano, PLB (2019)]

- phase diagram for m = 10 MeV:



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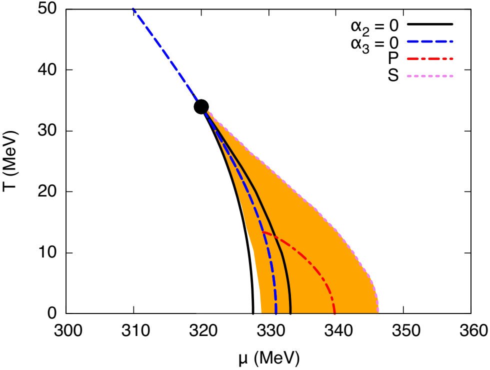
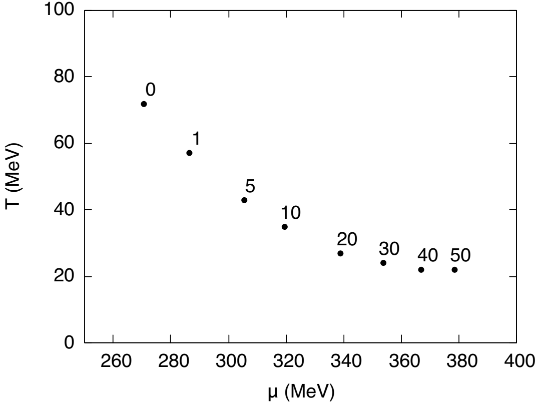


- dominant instability in the scalar channel

Mass dependence

- position of the CEP=PLP for different *m*:



m/MeV	m_π/MeV
0.	0.
1.	43.
5.	96.
10.	135.
20.	191.
30.	235.
40.	271.
50.	303.

Quark-meson model

[L. Kurth, Master's theis project, ongoing]

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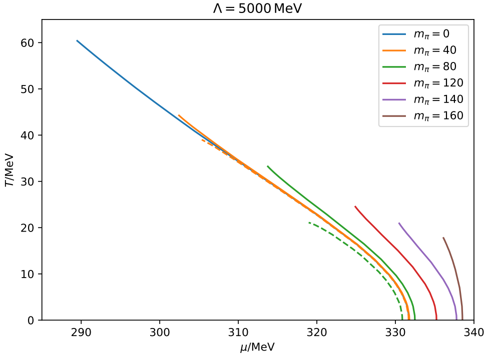

- Instability in the scalar channel remains well beyond physical masses.

Including strange quarks

Motivation

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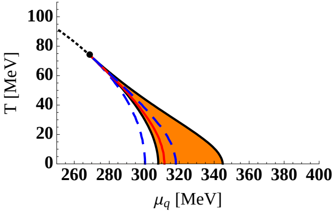
- 2-flavor NJL: TCP $\rightarrow$ LP, CEP $\rightarrow$ PLP



[D. Nickel, PRD (2009)]

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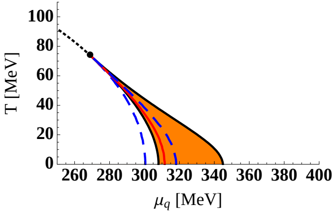
- Is this also true in QCD?



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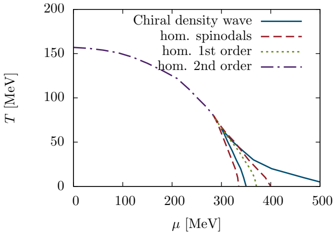

[D. Müller et al. PLB (2013)]

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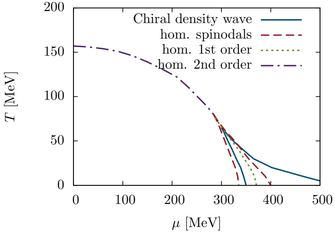

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  - CEP reaches T-axis

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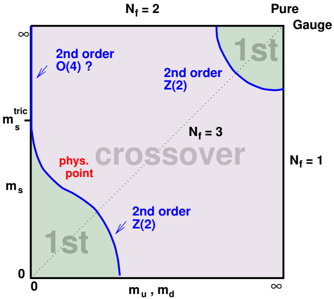
  - chance to study the inhomogeneous phase on the lattice!



[from de Forcrand et al., POSLAT 2007]

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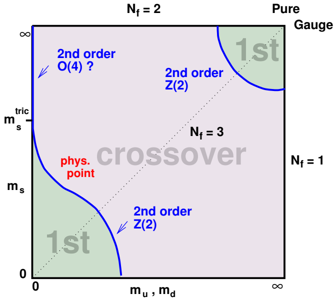
- Here: Ginzburg-Landau study for 3-flavor NJL



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3-flavor NJL model

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- Lagrangian: $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + \mathcal{L}_4 + \mathcal{L}_6$
  - fields and bare masses: $\psi = (u, d, s)^T$, $\hat{m} = \text{diag}_f(m_u, m_d, m_s)$
  - 4-point interaction: $\mathcal{L}_4 = G \sum_{a=0}^{8} \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2 \right]$
  - 6-point ('t Hooft) interaction: $\mathcal{L}_6 = -K \left[\text{det}_f \bar{\psi}(1+\gamma_5)\psi + \text{det}_f \bar{\psi}(1-\gamma_5)\psi \right]$

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- Mean fields:
  - light sector: $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \equiv \sigma_\ell$, $\langle \bar{u}i\gamma_5 u \rangle = -\langle \bar{d}i\gamma_5 d \rangle \equiv \pi_\ell$
  - strange sector: $\langle \bar{s}s \rangle \equiv \sigma_s$, $\langle \bar{s}i\gamma_5 s \rangle = 0$
  - no flavor-nondiagonal mean fields
  - allow for inhomogeneities: $\sigma_\ell = \sigma_\ell(\vec{x})$, $\pi_\ell = \pi_\ell(\vec{x})$, $\sigma_s = \sigma_s(\vec{x})$

Mean-field Thermodynamic Potential

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- $\Omega_{MF}(T,\mu) = -\frac{T}{V}\text{Tr}\,\log\left(i\partial\!\!\!/ + \mu\gamma^0 - \hat{M}\right) + \frac{1}{V}\int d^3x\,\mathcal{V}(\vec{x})$
  - $\hat{M}_{u,d}(\vec{x}) = m_\ell - \left[4G - 2K\sigma_s(\vec{x})\right]\left(\sigma_\ell(\vec{x}) \pm i\gamma^5\pi_\ell(\vec{x})\right)$
  - $\hat{M}_s(\vec{x}) = m_s - 4G\sigma_s(\vec{x}) + 2K\left(\sigma_\ell^2(\vec{x}) + \pi_\ell^2(\vec{x})\right)$
  - $\mathcal{V}(\vec{x}) = 2G\left[2(\sigma_\ell^2(\vec{x}) + \pi_\ell^2(\vec{x})) + \sigma_s^2(\vec{x})\right] - 4K(\sigma_\ell^2(\vec{x}) + \pi_\ell^2(\vec{x}))\sigma_s(\vec{x})$

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- $K = 0$: light and strange sectors decouple!

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- $K = 0$: light and strange sectors decouple!
- Chiral density wave ansatz for the light sector:

$\sigma_\ell(\vec{x}) = \sigma_0\cos(\vec{q}\cdot\vec{x}), \quad \pi_\ell(\vec{x}) = \sigma_0\sin(\vec{q}\cdot\vec{x}), \quad m_\ell = 0$

$\sigma_s = const.$

$$\Rightarrow \quad \hat{M}_\ell = M_0\exp(i\gamma^5\tau^3\vec{q}\cdot\vec{x}), \quad M_0 = -(4G - 2K\sigma_s)\sigma_0,$$
$$M_s = const.,$$

consistent with the literature [Moreira et al., PRD (2014)]

Ginzburg-Landau expansion

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- Difficulty for $\hat{m} \neq 0$: no restored solution

Ginzburg-Landau expansion

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- Order parameters: $\Delta_\ell(\vec{x}) = -4G\left(\sigma_\ell(\vec{x}) + i\pi_\ell(\vec{x})\right)$, $\Delta_s(\vec{x}) = -4G\,\delta\sigma_s(\vec{x})$

$$\rightarrow \qquad \Omega_{MF}[\sigma_\ell, \pi_\ell, \sigma_s] = \Omega[0, 0, \sigma_s^{(0)}] + \frac{1}{V}\int d^3x\, \omega_{GL}(\Delta_\ell, \Delta_s)\,,$$

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  $\sigma_s^{(0)}$: homogeneous solution of the gap equation for $\sigma_\ell = \pi_\ell = 0$.
- Order parameters: $\Delta_\ell(\vec{x}) = -4G\left(\sigma_\ell(\vec{x}) + i\pi_\ell(\vec{x})\right)$, $\Delta_s(\vec{x}) = -4G\,\delta\sigma_s(\vec{x})$

$$\rightarrow \qquad \Omega_{MF}[\sigma_\ell, \pi_\ell, \sigma_s] = \Omega[0, 0, \sigma_s^{(0)}] + \frac{1}{V}\int d^3x\, \omega_{GL}(\Delta_\ell, \Delta_s)\,,$$

- Expand ω_{GL} in Δ_ℓ, Δ_s and their gradients.
- $[\Delta_i]$ = (mass) $\rightarrow$ counting scheme: $\mathcal{O}(\vec{\nabla}) = \mathcal{O}(\Delta_i)$

Ginzburg-Landau potential

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- Resulting structure:

$$\omega_{GL} = \alpha_2|\Delta_\ell|^2 + \alpha_{4,a}|\Delta_\ell|^4 + \alpha_{4,b}|\vec{\nabla}\Delta_\ell|^2 + \beta_1\Delta_s + \beta_2\Delta_s^2 + \beta_3\Delta_s^3 + \beta_{4,a}\Delta_s^4 + \beta_{4,b}(\vec{\nabla}\Delta_s)^2 + \gamma_3|\Delta_\ell|^2\Delta_s + \gamma_4|\Delta_\ell|^2\Delta_s^2 + \mathcal{O}(\Delta_i^5)$$

Ginzburg-Landau potential

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- Resulting structure:

$$\begin{aligned}\omega_{GL} = &\ \alpha_2|\Delta_\ell|^2 + \alpha_{4,a}|\Delta_\ell|^4 + \alpha_{4,b}|\vec{\nabla}\Delta_\ell|^2 \\ &+ \beta_1\Delta_s + \beta_2\Delta_s^2 + \beta_3\Delta_s^3 + \beta_{4,a}\Delta_s^4 + \beta_{4,b}(\vec{\nabla}\Delta_s)^2 \\ &+ \gamma_3|\Delta_\ell|^2\Delta_s + \gamma_4|\Delta_\ell|^2\Delta_s^2 \qquad + \mathcal{O}(\Delta_i^5)\end{aligned}$$

- Stationarity condition: $\frac{\partial\Omega_{GL}}{\partial\Delta_s}\Big|_{\Delta_\ell=\Delta_s=0} = 0 \quad \Leftrightarrow \quad \beta_1 = 0$

Ginzburg-Landau potential

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$$\Rightarrow \quad \Delta_s = -\frac{\gamma_3}{2\beta_2}|\Delta_\ell|^2 + \mathcal{O}(|\Delta_\ell|^4)$$

Ginzburg-Landau potential

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$$\Rightarrow \quad \omega_{GL} = \alpha_2|\Delta_\ell|^2 + \left(\alpha_{4,a} - \frac{\gamma_3^2}{4\beta_2}\right)|\Delta_\ell|^4 + \alpha_{4,b}|\vec{\nabla}\Delta_\ell|^2 + \mathcal{O}(\Delta_\ell^6)$$

Ginzburg-Landau potential

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- CP: $\alpha_2 = \alpha_{4,a} - \frac{\gamma_3^2}{4\beta_2} = 0$, LP: $\alpha_2 = \alpha_{4,b} = 0$ CP and LP split for $\gamma_3 \neq 0$!

GL coefficients

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$$\alpha_2 = (1+\delta)\left[\frac{1}{4G} + \frac{1}{2}(1+\delta)F_1(0)\right],$$

$$\alpha_{4,a} = \frac{1}{4}(1+\delta)^4 F_2(0) + \frac{1}{4}\kappa^2\left(F_1(M_{s,0}) + 2M_{s,0}^2 F_2(M_{s,0})\right),$$

$$\alpha_{4,b} = \frac{1}{4}(1+\delta)^2 F_2(0),$$

$$\gamma_3 = \kappa\left\{\frac{1}{2G} + (1+\delta)F_1(0) + \frac{1}{2}\left(F_1(M_{s,0}) + 2M_{s,0}^2 F_2(M_{s,0})\right)\right\},$$

- $F_n(M) = 8N_c \int \frac{d^3p}{(2\pi)^3}\, T\sum_j \frac{1}{[(i\omega_j+\mu)^2-\vec{p}^{\,2}-M^2]^n}$, $\quad M_{s,0} = m_s - 2GM_{s,0}F_1(M_{s,0})$
- $\kappa = \frac{K}{8G^2}$, $\quad \delta = \kappa(M_{s,0} - m_s)$

GL coefficients

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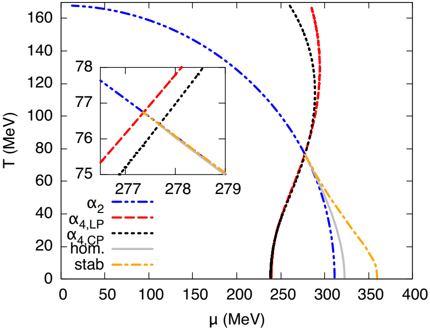
- $\kappa = \frac{K}{8G^2}$, $\quad \delta = \kappa(M_{s,0} - m_s)$
- Interesting limit: $K = 0 \;\Rightarrow\; \kappa = \delta = 0 \;\Rightarrow\; \alpha_{4,a} = \alpha_{4,b},\;\; \gamma_3 = 0 \;\Rightarrow\;$ CP=LP

Results

[S. Carignano, MB, arXiv:1910.03604]

- realistic parameters (fitted to vacuum meson spectrum):



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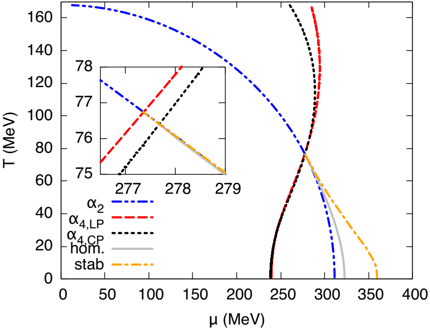

- splitting between CP and LP small

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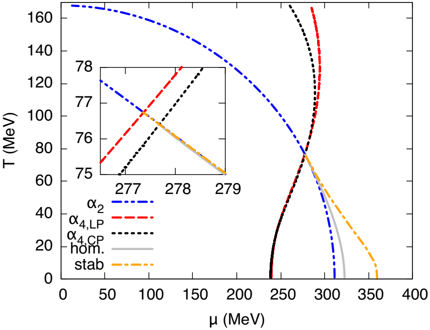
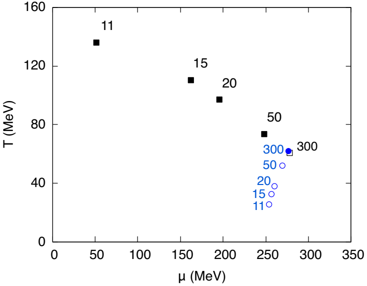


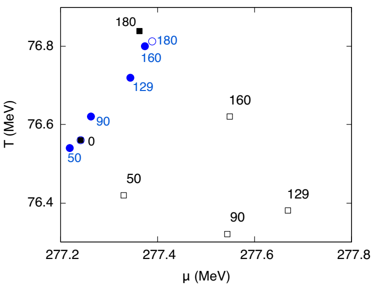
- splitting between CP and LP small
- hom. 1st-order phase boundary completely covered by inhom. phase

Parameter dependence

m_s [MeV]



$K\Lambda^5$ (G refitted)



- sizeable splitting between CP and LP at small m_s, CP $\rightarrow$ T-axis, as expected
- very weak K dependence at physical m_s

Conclusions

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- Ginzburg-Landau analysis of the effect of bare quark masses and strange quarks the inhomogeneous chiral phase in the NJL model
- nonzero $m_{u,d}$:
  - PLP coincides with CEP
  - dominant instability towards inhomogeneities in the scalar channel
  - numerical result: inhomogeneous phase survives large (higher than physical) quark masses
  - similar results for the quark-meson model
- strange quarks:
  - CP and LP no longer agree as a consequence of the axial anomaly
  - numerical result: effect small for realistic m_s
- QCD?

Inhomogeneous chiral phases in QCD?

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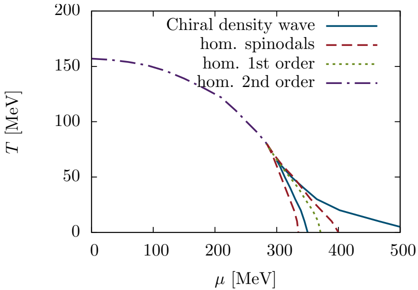
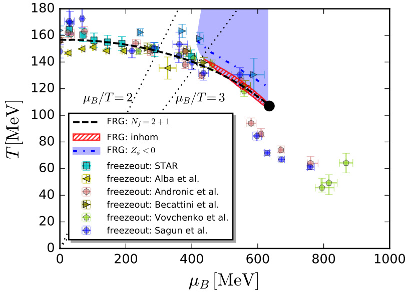
DSE (2 flavors)

[Müller et al. PLB 2013]



FRG (2+1 flavors)

[Fu et al. arXiv:1909.02991]



- DSE (simple truncation): similar to NJL
- FRG: region with $Z_\phi(0) \propto \alpha_{4,b} < 0$