Applications of the NJL model to the partonic structure of the pion

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40th Max Born Symposium
Three Days on Strong Correlations in Dense Matter

9-12 October 2019, University of Wrocław

Dedication to David

Where and when we met?

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Hadron Structure 88
14-18 Nov 1988, Pešťany, Czechoslovakia

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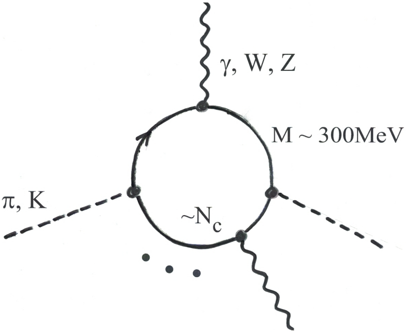
NJL *1961

As young as the NJL model!

NJL in high-energy processes

Chiral quark models

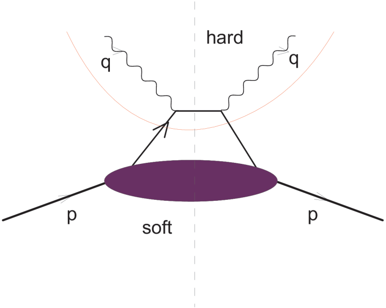


- χSB breaking $\to$ massive quarks
- Point-like interactions
- Soft matrix elements with pions (and photons, W, Z)
- Large-N_c $\to$ one-quark loop
- Regularization

pion – Goldstone boson of χSB, fully relativistic $q\bar{q}$ bound state of the Bethe-Salpeter equation

Simplest covariant field-theoretic model for the pion!

Parton distributions



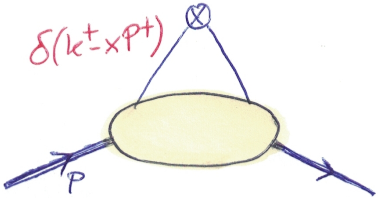
$Q^2 = -q^2, \;\; x = \frac{Q^2}{2p\cdot q}, \;\; Q^2 \to \infty$

Factorization of soft and hard processes, Wilson's OPE

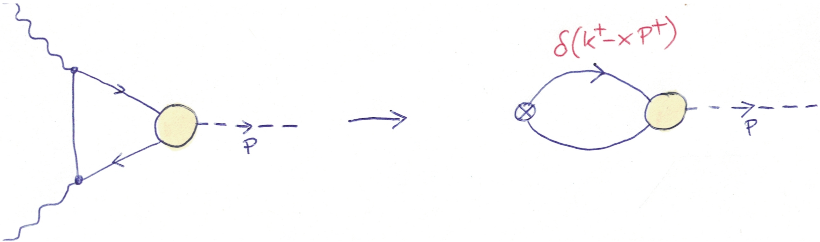
$$\langle J(q)J(-q)\rangle = \sum_i C_i(Q^2;\mu)\langle \mathcal{O}_i(\mu)\rangle$$

Twist expansion $\to$ $F(x,Q) = F_0(x,\alpha(Q)) + \frac{F_2(x,\alpha(Q))}{Q^2} + \ldots$

Bjorken limit $\to$ light-cone momentum is constrained:
$k^+ \equiv k^0 + k^3 = xP^+ \quad x \in [0,1]$



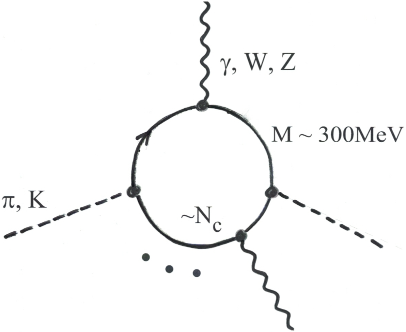
Distribution amplitude (DA) of the pion



Enters various measures of exclusive processes,
e.g., pion-photon transition form factor

[Anikin, Dorokhov, Tomio 2000]
[Praszałowicz, Rostworowski 2001, + Bzdak 2003, + Kotko 2009]
[ERA, WB 2002]

Chiral quark models



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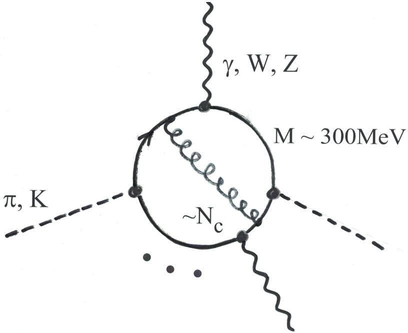
Quantities evaluated at the quark model scale
(where constituent quarks are the only degrees of freedom)

Chiral quark models



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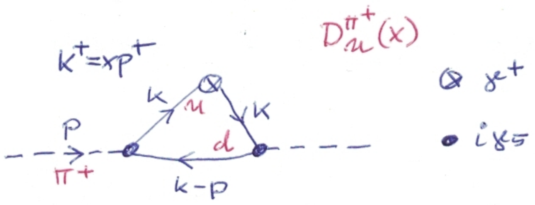
Need for QCD evolution

Gluon dressing, renorm-group improved

PDF in NJL

[Davidson, Arriola, 1995]



$$q_{\rm val}(x;Q_0) = 1 \times \theta[x(1-x)]$$

(proper treatment of symmetries with regularization)

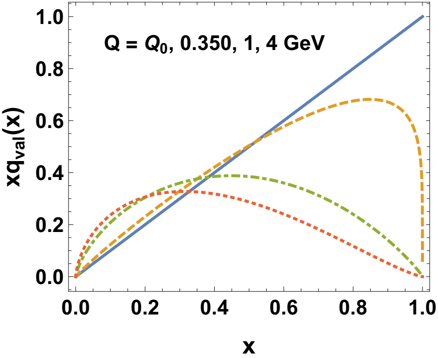
Quarks are the only degrees of freedom, hence saturate the PDF sum rules: $\int_0^1 dx\, q_{\rm val}(x;Q_0) = 1$ (valence), $2\int_0^1 dx\, x q_{\rm val}(x;Q_0) = 1$ (momentum)

Scale and evolution

QM provide non-perturbative result at a low scale Q_0

$$F(x,Q_0)|_{\rm model} = F(x,Q_0)|_{\rm QCD}\,, \quad Q_0 - \text{the matching scale}$$

Quarks carry 100% of momentum at Q_0, adjusted such that when evolved to $Q=2$ GeV, they carry the experimental value of 47% (radiative generation of gluons and sea quarks)



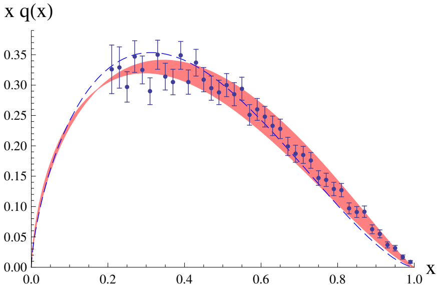
LO DGLAP evolution

$$Q_0 = 313^{+20}_{-10}\ \text{MeV}$$

NLO close to LO

$$\sim (1-x)^{p+\frac{4C_F}{\beta_0}\log\frac{\alpha(Q_0)}{\alpha(Q)}}$$

Pion valence quark PDF, NJL vs E615



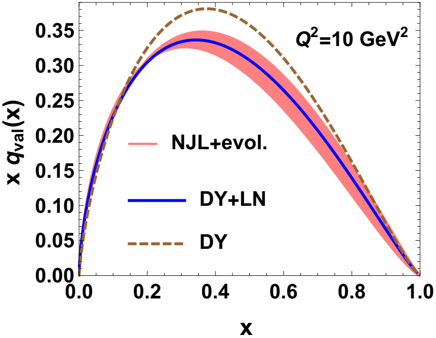
points: Fermilab E615
Drell-Yan, $\pi^{\pm}W \to \mu^{+}\mu^{-}X$

dashed line: 2005 NLO reanalysis [Wijesoorija et al.]

band: QM + LO DGLAP from $Q_0 = 313^{+20}_{-10}$ MeV to $Q = 4$ GeV

Pion valence quark DF, QM vs JAM analysis

[P. C. Barry, N. Sato, W. Melnitchouk, C.-R. Ji, PRL 121 (2018) 152001, arXiv:1804.01965]

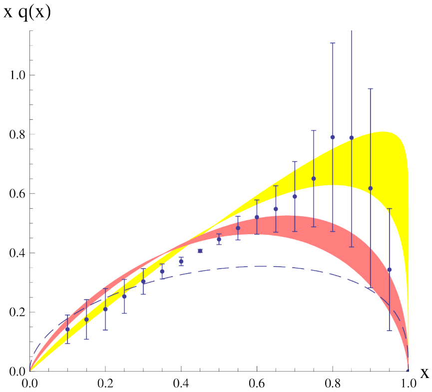


curves: JAM data analysis

band: QM + LO DGLAP from $Q_0 = 313^{+20}_{-10}$ MeV to $Q^2 = 10\ \mathrm{GeV}^2$

Many predictions for related quantities: DA, GPD, TDA, TMD, quasi/pseudo DA/PDF...

Pion quark DF, NJL vs. transverse lattice

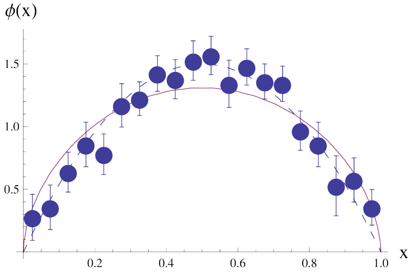


points: transverse lattice
[Dalley, van de Sande 2003]

yellow: QM evolved to 0.35 GeV

pink: QM evolved to 0.5 GeV

dashed: GRS param. at 0.5 GeV

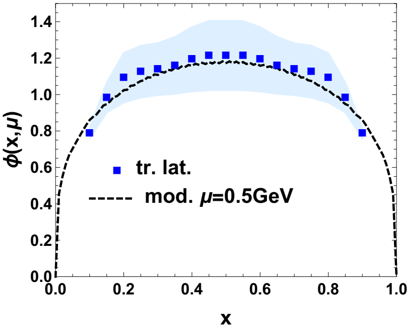


points: E791 data from dijet production in $\pi + A$

solid line: QM at $Q = 2$ GeV

dashed line: asymptotic form $6x(1-x)$ at $Q \to \infty$

Pion DA, NJL vs. transverse lattice



points: transverse lattice data [Dalley, van de Sande 2003]

Double parton distributions

(new stuff)

WB at LC2019

Courtoy, Noguera, Scopetta, arXiv:1909.09530

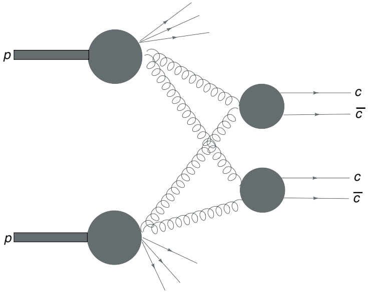
WB, ERA, arXiv:1910.03707

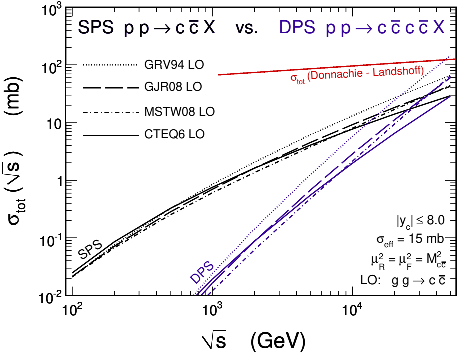
Motivation for multi-parton distributions

- Old story (Fermilab), renewed interest (e.g., ATLAS measurement for pp→ W+2 jets 2013) [Kuti, Weiskopf 1971, Konishi, Ukwa, Veneziano 1979, Gaunt, Stirling 2010, Diehl, Ostermeier, Schäfer 2012, . . . , reviews: Bartalani et al. 2011, Snigirev 2011, Rinaldi,Ceccopieri 2018]
- Model exploration [MIT bag: Chang, Manohar, Waalewijn 2013], valon [WB+ERA 2013], constituent quarks: Rinaldi, Scopetta, Vento 2013, Rinaldi, Scopetta, Traini, Vento 2018]
- Questions: factorization, interference of single- and double-parton scattering [Manohar, Waalewijn 2012, Gaunt 2013], longitudinal-transverse decoupling, positivity bounds [Diehl, Casemets 2013], transverse momentum dependence [Casemets, Gaunt 2019]
- Gaunt-Stirling sum rules [Gaunt, Stirling 2010, WB+ERA 2013, Diehl, Plöß, Schäfer 2019]

Double parton scattering

[example from Łuszczak, Maciuła, Szczurek 2011]





DPS can be comparable to SPS at the LHC

Assumption: $D_{gg}(x_1, x_2, \boldsymbol{b}) = g(x_1)g(x_2)F(\boldsymbol{b})$

– no correlations, transverse-longitudinal factorization

Definition

Intuitive probabilistic definition:

Multi-parton distribution = probability distribution that struck partons have LC momentum fractions x_i

Field-theoretic definition of (spin-averaged) PDF and dPDF [Diehl, Ostermeier, Schaeffer 2012] of a hadron with momentum p:

$$D_j(x) = \int \frac{\mathrm{d}z^-}{2\pi}\, e^{i x z^- p^+} \langle p|\, \mathcal{O}_j(0,z)\, |p\rangle\Big|_{z^+=0\,,\boldsymbol{z}=\mathbf{0}}$$

$$F_{j_1 j_2}(x_1,x_2,\boldsymbol{y}) = 2p^+ \int \mathrm{d}y^-\, \frac{\mathrm{d}z_1^-}{2\pi}\, \frac{\mathrm{d}z_2^-}{2\pi}\, e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle p|\, \mathcal{O}_{j_1}(y,z_1)\, \mathcal{O}_{j_2}(0,z_2)\, |p\rangle\Big|_{z_1^+=z_2^+=y^+=0\,,\boldsymbol{z}_1=\boldsymbol{z}_2=\mathbf{0}}$$

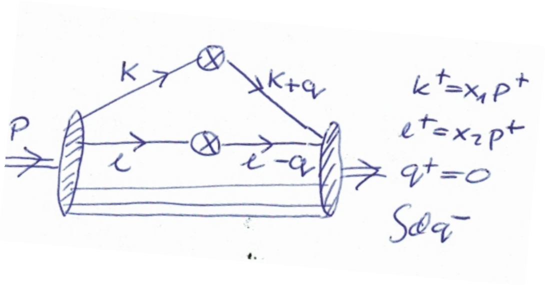
$\mathcal{O}_q(y,z) = \frac{1}{2}\,\bar{q}(y-\frac{z}{2})\gamma^+ q(y+\frac{z}{2}),\ \ldots$ (LC gauge) $\qquad v^\pm = (v^0 \pm v^3)/\sqrt{2}$

$\boldsymbol{y}$ plays the role of the transverse distance between the two quarks

dPDF in momentum space

Fourier transform in $\boldsymbol{y}$

$$F_{j_1 j_2}(x_1, x_2, \boldsymbol{y}) \to \tilde{F}_{j_1 j_2}(x_1, x_2, \boldsymbol{q})$$



Special case of $\boldsymbol{q} = \boldsymbol{0}$:

$$D_{j_1 j_2}(x_1, x_2) = \tilde{F}_{j_1 j_2}(x_1, x_2, \boldsymbol{q} = \boldsymbol{0})$$

Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition on LC + conservation laws $\rightarrow$

$$|P\rangle = \sum_N \int d[x,\boldsymbol{k}]_N \Phi(\{x_i,\boldsymbol{k}_i\})|\{x_i,\boldsymbol{k}_i\}\rangle_N$$

$$d[x,\boldsymbol{k}]_N = \prod_{i=1}^{N}\left[\frac{dx_i d^2k_i}{\sqrt{2(2\pi)^3 x_i}}\right]\delta\left(1-\sum_{i=1}^{N}x_i\right)\delta^{(2)}\left(1-\sum_{i=1}^{N}\boldsymbol{k}_i\right)$$

Gaunt-Stirling sum rules

[Gaunt, Stirling, JHEP (2010) 1003:005, Gaunt, PhD Thesis]

Fock-space decomposition on LC + conservation laws $\rightarrow$

$$\sum_i \int_0^{1-x_2} dx_1\, x_1 D_{ij}(x_1,x_2) = (1-x_2)D_j(x_2) \quad \text{(momentum)}$$

$$\int_0^{1-x_2} dx_1\, D_{i_{\rm val}j}(x_1,x_2) = (N_{i_{\rm val}} - \delta_{ij} + \delta_{\bar{i}j})D_j(x_2) \quad \text{(quark number)}$$

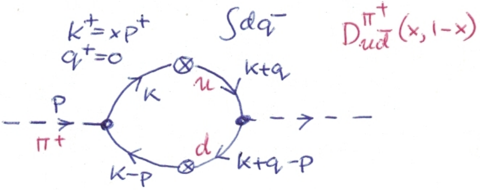
$(A_{i_{\rm val}} \equiv A_i - A_{\bar{i}})$

$$N_{i_{\rm val}} = \int_0^1 dx\, D_{i_{\rm val}}(x)$$

- Preserved by DGLAP evolution
- Non-trivial to satisfy with the (guessed) function
- Checked in light-front perturbation theory and in lowest-order covariant calculations in [Diehl, Plößl, Schäfer 2019]

Important and fundamental constraints!

dPDF of the pion in NJL model

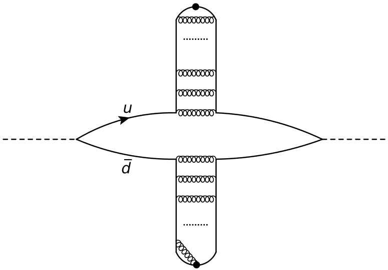


$$D_{u\bar{d}}(x_1, x_2) = 1 \times \delta(1 - x_1 - x_2)\theta[x_1(1 - x_1)]$$

- GS sum rules satisfied (preserved by the evolution)
- Results at the quark-model scale → need for evolution

DGLAP evolution in the Mellin space

[Kirschner 1979, Shelest, Snigirev, Zinovev 1982]: method of solving DGLAP based on the Mellin moments, similarly to single PDF simplification for valence distributions



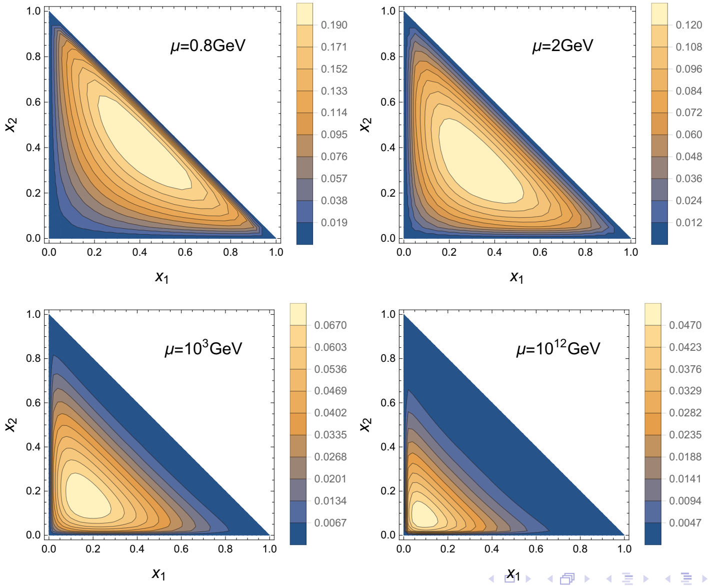
$t = \frac{1}{2\pi\beta} \log\left[1 + \alpha_s(\mu)\beta \log(\Lambda_{\rm QCD}/\mu)\right]$ (single scale for simplicity), $\beta = \frac{11N_c - 2N_f}{12\pi}$

Valence:

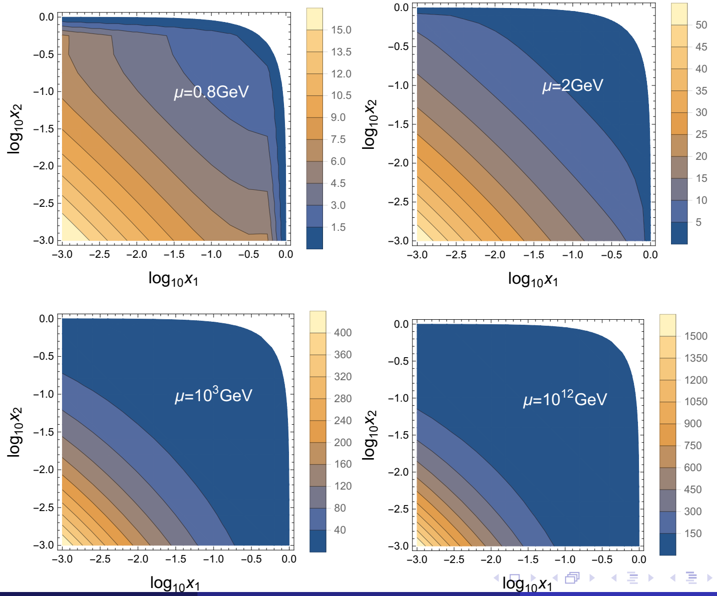
$$\text{dPDF}: \quad \frac{d}{dt} M^{n_1 n_2}_{j_1 j_2}(t) = \left(P^{n_1}_{j_1 \to j_1} + P^{n_2}_{j_2 \to j_2}\right) M^{n_1 n_2}_{j_1, j_2}(t)$$

$$\text{PDF}: \quad \frac{d}{dt} M^{n}_{j}(t) = P^{n}_{j \to j} M^{n}_{j}(t)$$

$x_1 x_2 D^{\pi^+}_{u\bar{d}}(x_1, x_2)$

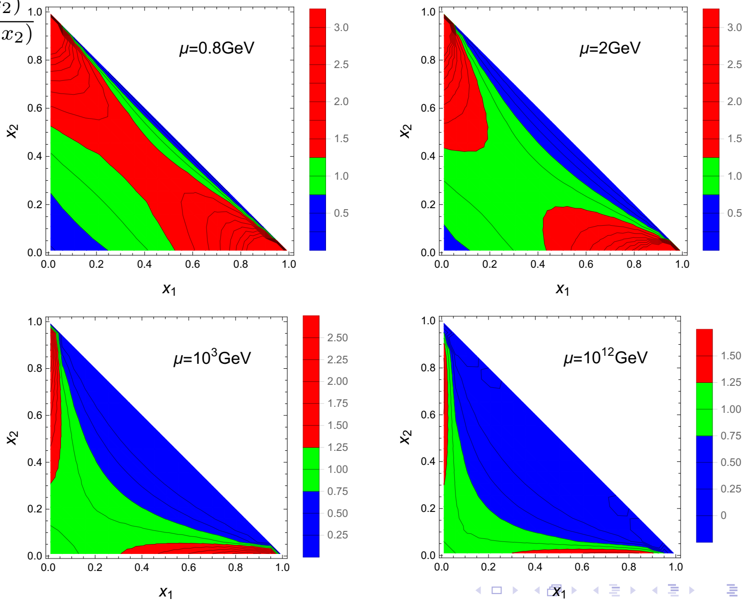


$D^{\pi^+}_{u\bar{d}}(x_1, x_2)$ – log scale



Correlation

$$\frac{D^{\pi^+}_{u\bar{d}}(x_1,x_2)}{D_u(x_1)D_{\bar{d}}(x_2)}$$

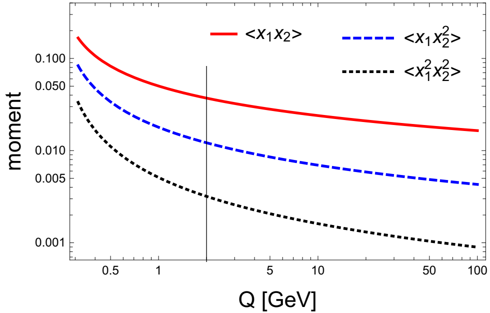


Valence moments in NJL

$$\frac{\langle x_1^n x_2^m \rangle}{\langle x_1^n \rangle \langle x_2^m \rangle} = \frac{(1+n)!(1+m)!}{(1+n+m)!} \quad \text{(NJL, any scale)}$$

(independent of the evolution scale)

	1	2	3	4	5
1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$
2	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{3}{28}$
3	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{4}{35}$	$\frac{1}{14}$	$\frac{1}{21}$
4	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{14}$	$\frac{5}{126}$	$\frac{1}{42}$
5	$\frac{2}{7}$	$\frac{3}{28}$	$\frac{1}{21}$	$\frac{1}{42}$	$\frac{1}{77}$



Double moments reduced compared to product of single moments

[lattice results coming shortly, Zimmermann et al.]

Summary

- NJL: simplest field theory of the pion in the soft regime
- Covariant calculations, all symmetries preserved $\to$ good features
- Exploration of quantities accessible on the lattice
- dPDF in NJL $= 1 \times \delta(1 - x_1 - x_2)$ + DGLAP evolution
- Moments measure the $x_1 - x_2$ factorization breaking; will be verified in forthcoming lattice calculations

120 years, David!