I had a dream *)

From Holographic EoS & Hadron Freeze-out for HICs to Viscosities, Diffusion Coeff. etc.

B. Kämpfer

Helmholtz-Zentrum Dresden-Rossendorf
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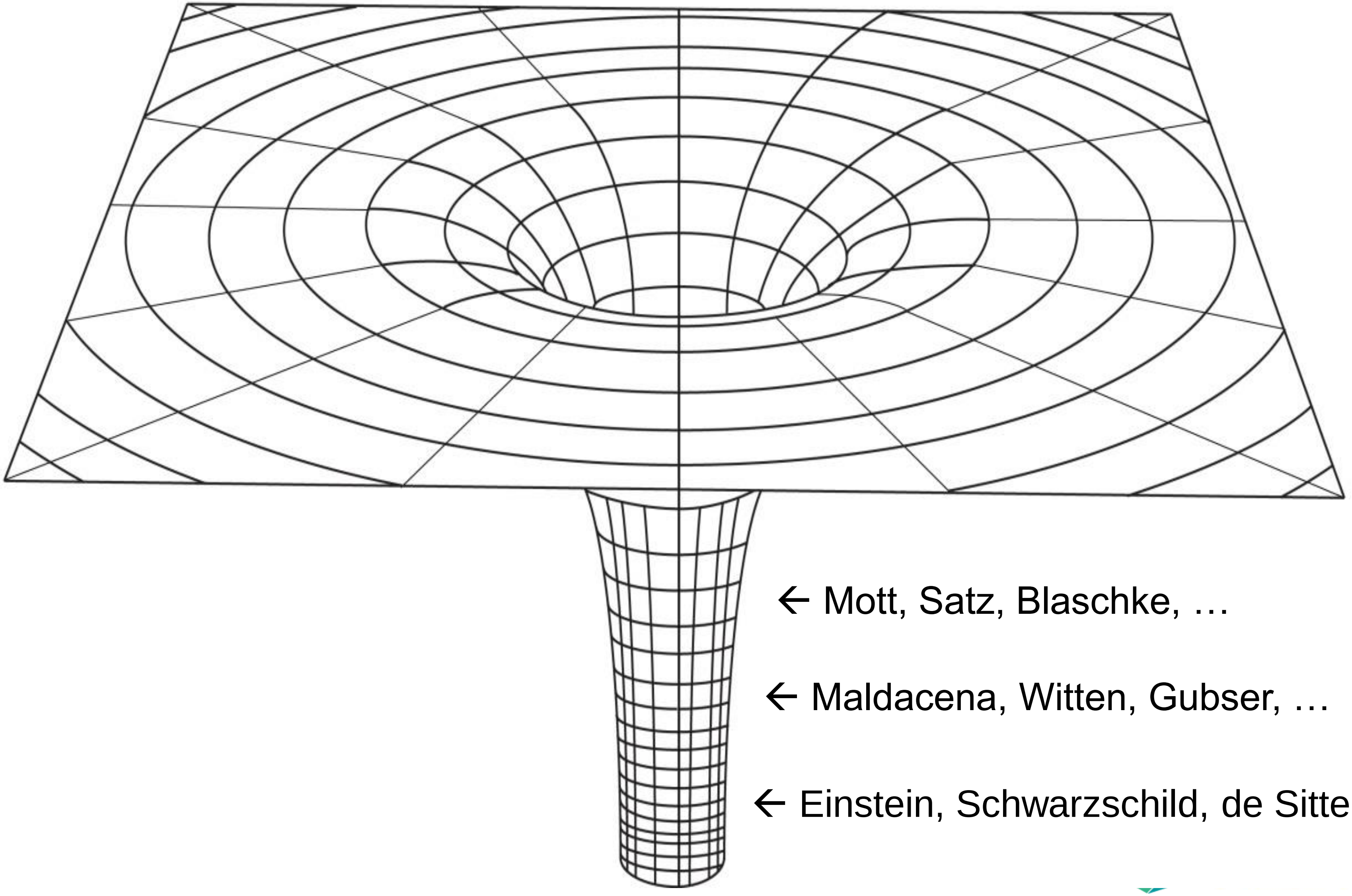
*) with J. Knaute, PLB (2018), PRD (2017)
R. Zollner, EPJA (2017), PRC (2016)
R. Yaresko, EPJC (2015), PLB (2015)

„Very deep is the well of the past“

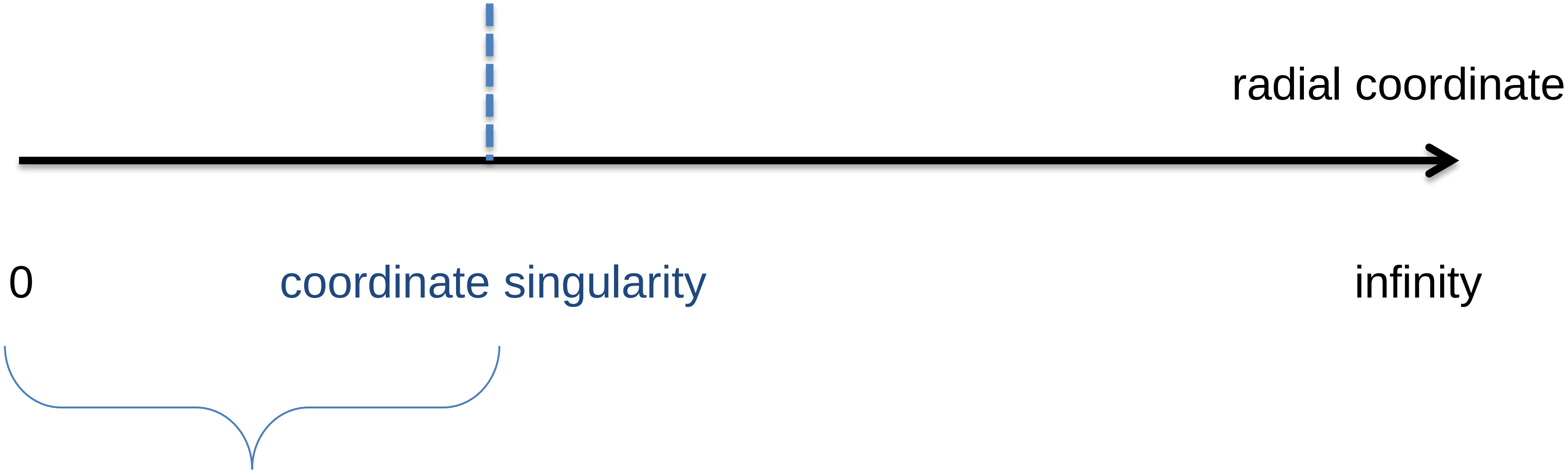


← Mott, Satz, Blaschke, …

← Maldacena, Witten, Gubser, …

← Einstein, Schwarzschild, de Sitter

Einstein, Schwarzschild, de Sitter, Finkelstein, Hawking, Bekenstein



radial coordinate

0

coordinate singularity

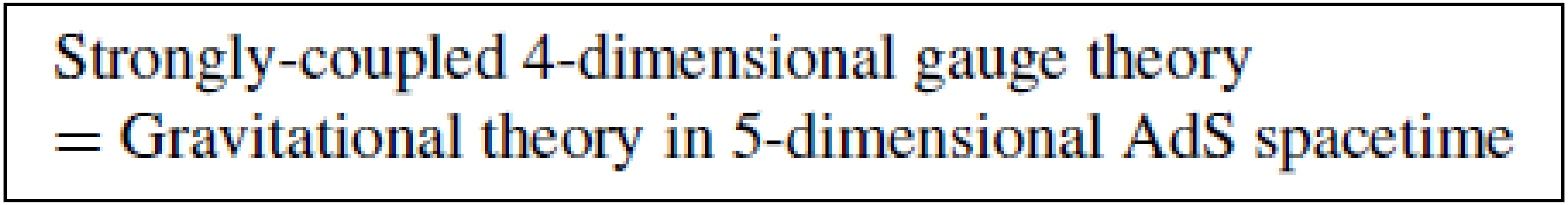
infinity

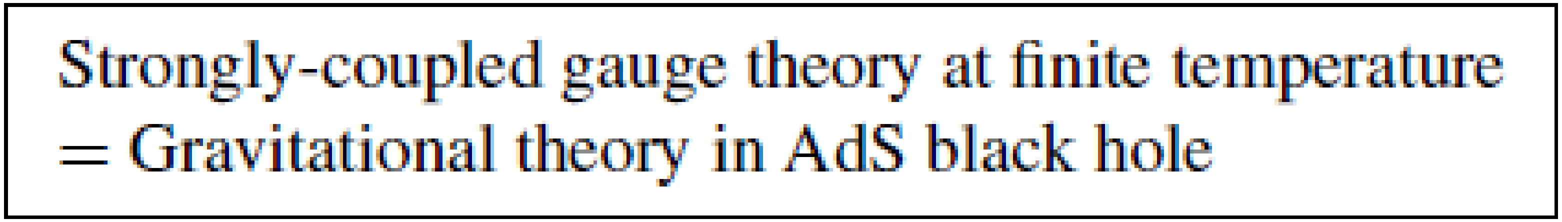
Wheeler: Black Hole

holds also in 5D

´t Hooft, Susskind, Maldacena, Witten, Gubser …

AdS/CFT, Holography


Strongly-coupled 4-dimensional gauge theory
= Gravitational theory in 5-dimensional AdS spacetime


Strongly-coupled gauge theory at finite temperature
= Gravitational theory in AdS black hole

this talk: reckless use of the dictionary = bottom-up approach

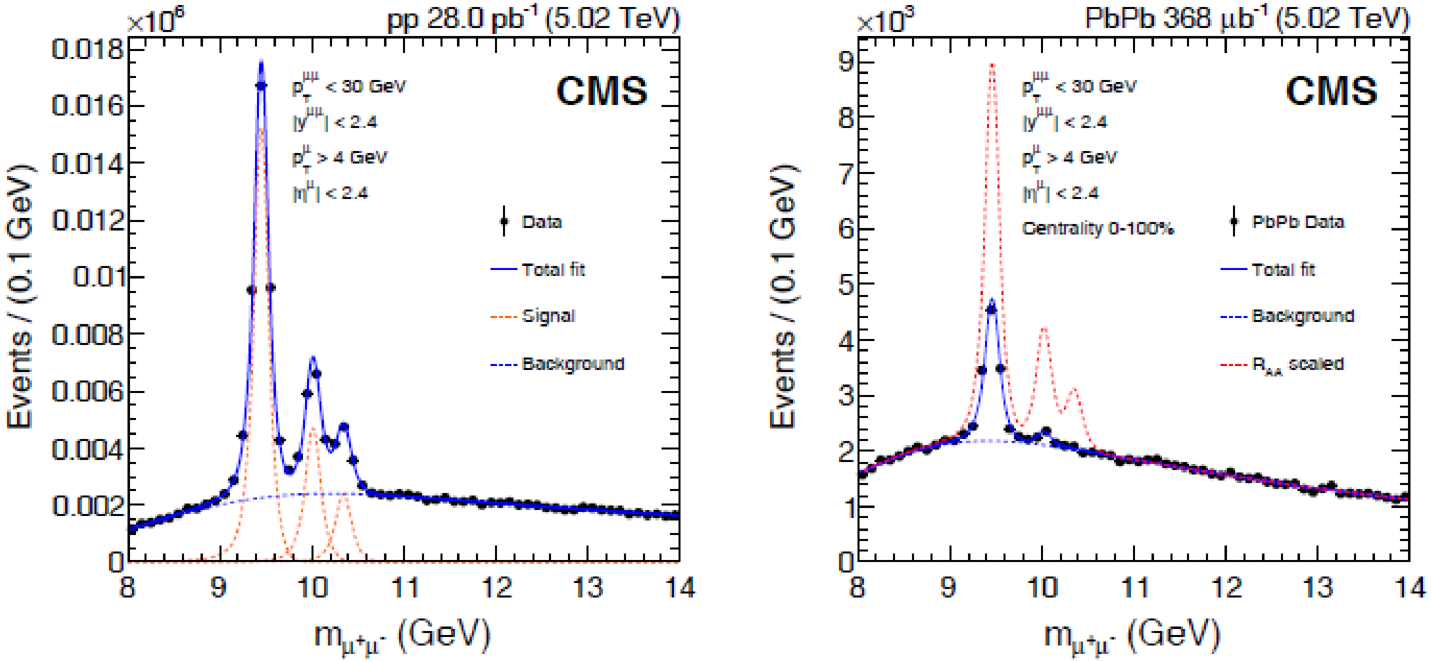
Matsui-Satz, Mott → Röpke, Blaschke ….

Mott Mechanism and the Hadronic to Quark Matter Phase Transition
D. Blaschke, F. Reinholz, G. Ropke, D. Kremp
Phys.Lett. 151B (1985) 439-443

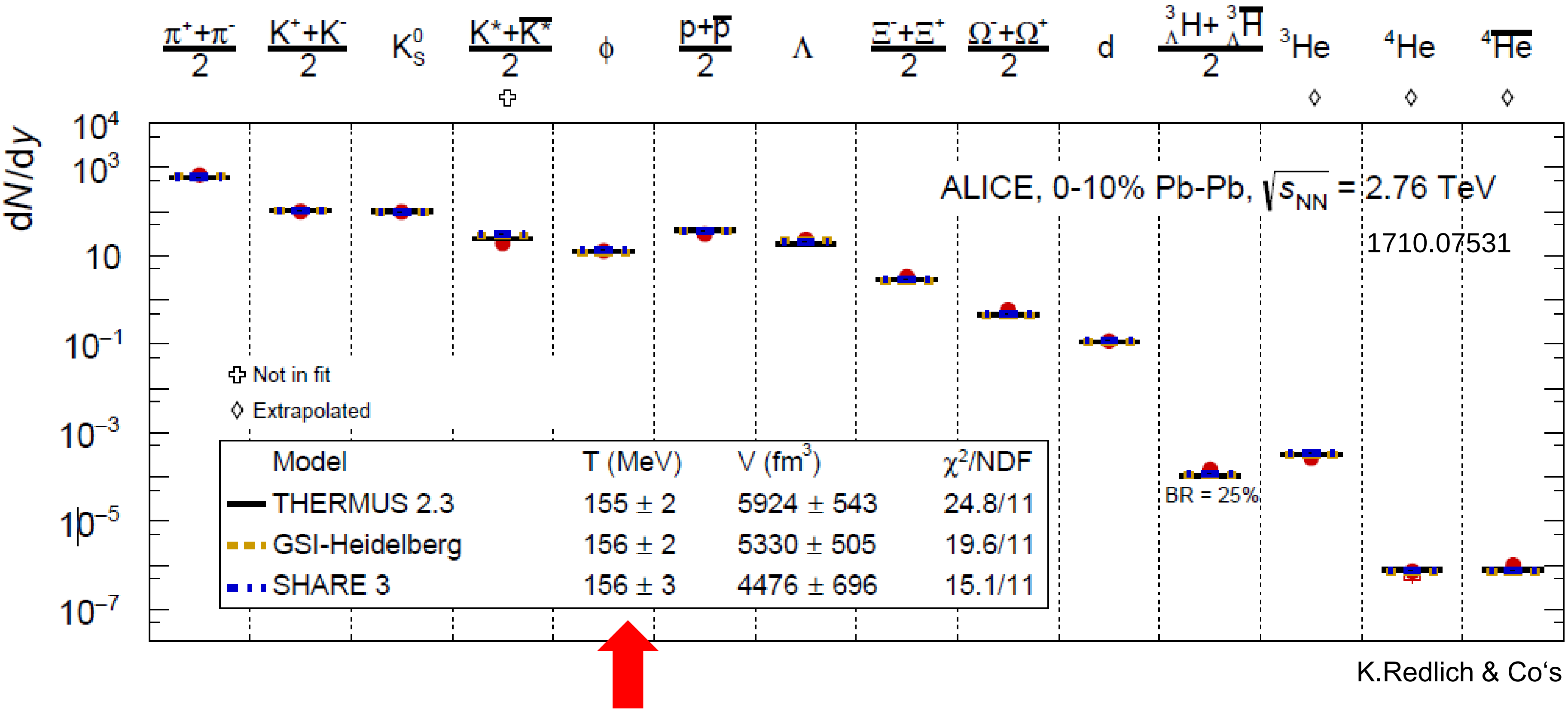
J/ψ Suppression by Quark-Gluon Plasma Formation
T. Matsui, H. Satz
Phys.Lett. B178 (1986) 416-422
Zitiert von 2913 Datensätzen

Dissociation Kinetics and Momentum Dependent J/ψ Suppression in a Quark - Gluon Plasma
G. Ropke, D. Blaschke, H. Schulz
Phys.Rev. D38 (1988) 3589-3592

LHC: sequential melting of meson excitations?



freeze-out & SHM: hadron(ic) states at T <= 155 MeV



K.Redlich & Co's

problem in given EdM model:
hadrons (vector mesons) in probe limit
melt at T(dis) << T(f.o.)

dream → night mare

Einstein-dilaton-Maxwell in 5D (holographic QCD w/o gluons & quarks)

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(\underbrace{R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)}_{\mu = 0} - \frac{\hat{f}(\phi)}{4}\underset{\uparrow}{F_{\mu\nu}^2}\right) + \text{GH}$$

U(1)
charge → μ

→ Einstein eqs. + EoM

dictionary: 5D Riemann → 4D Minkowski

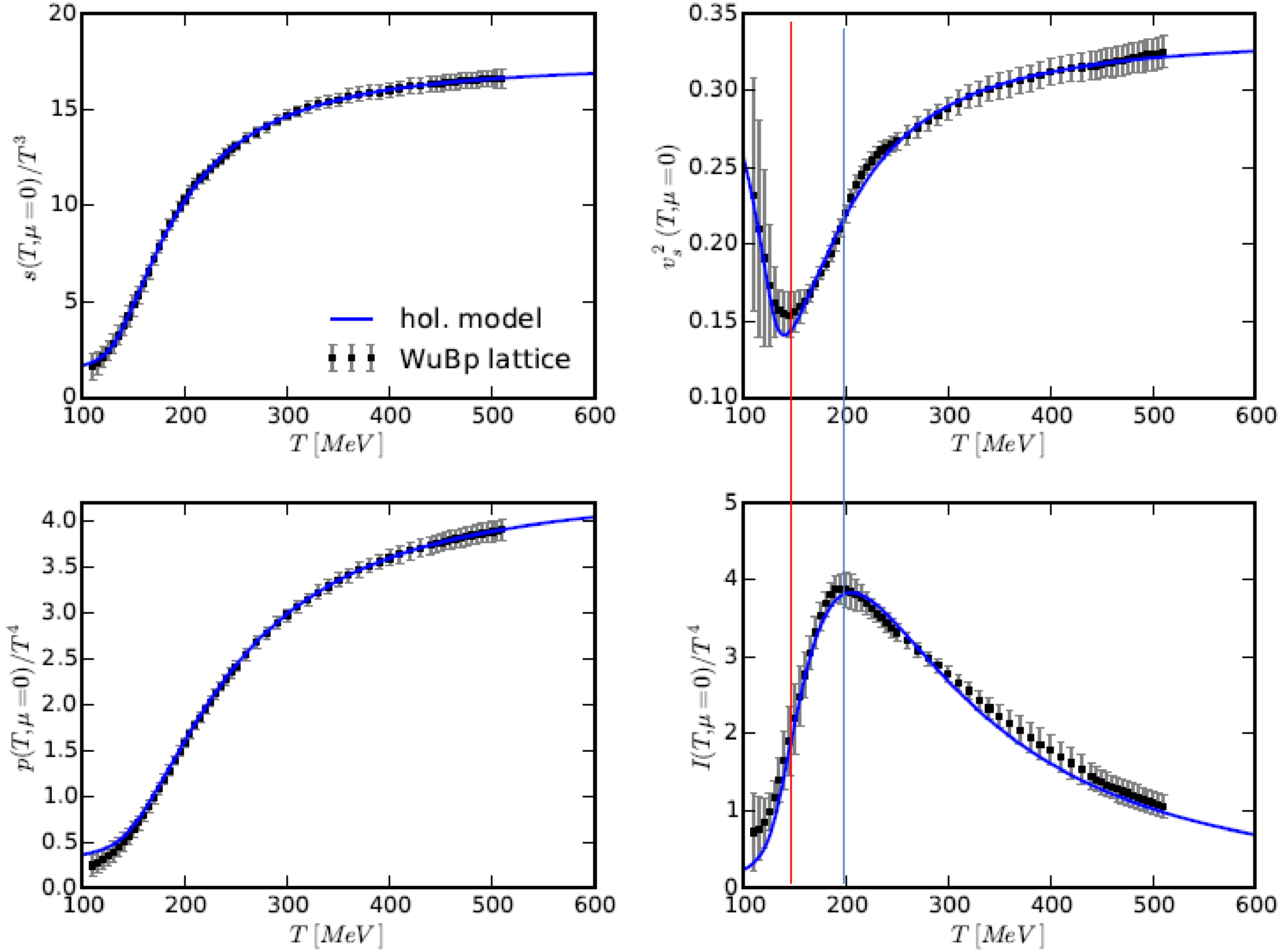
AdS + Schwarzschild BH

black brane/horizon

space-time with
constant curvature & negative cosmolog. constant

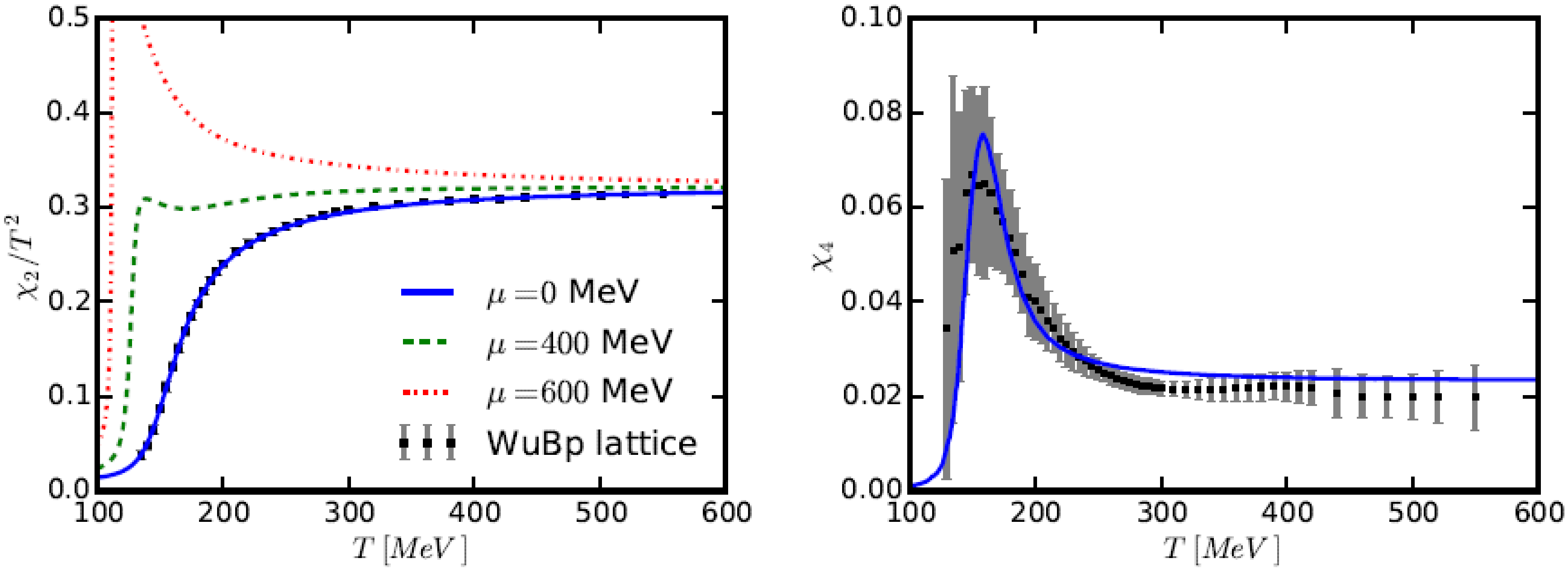
1) Phase Diagram: CEP, FOPT, HEE

strategy: encode 2+1 QCD gluons & quarks in V(phi), $\hat{f}$(phi)



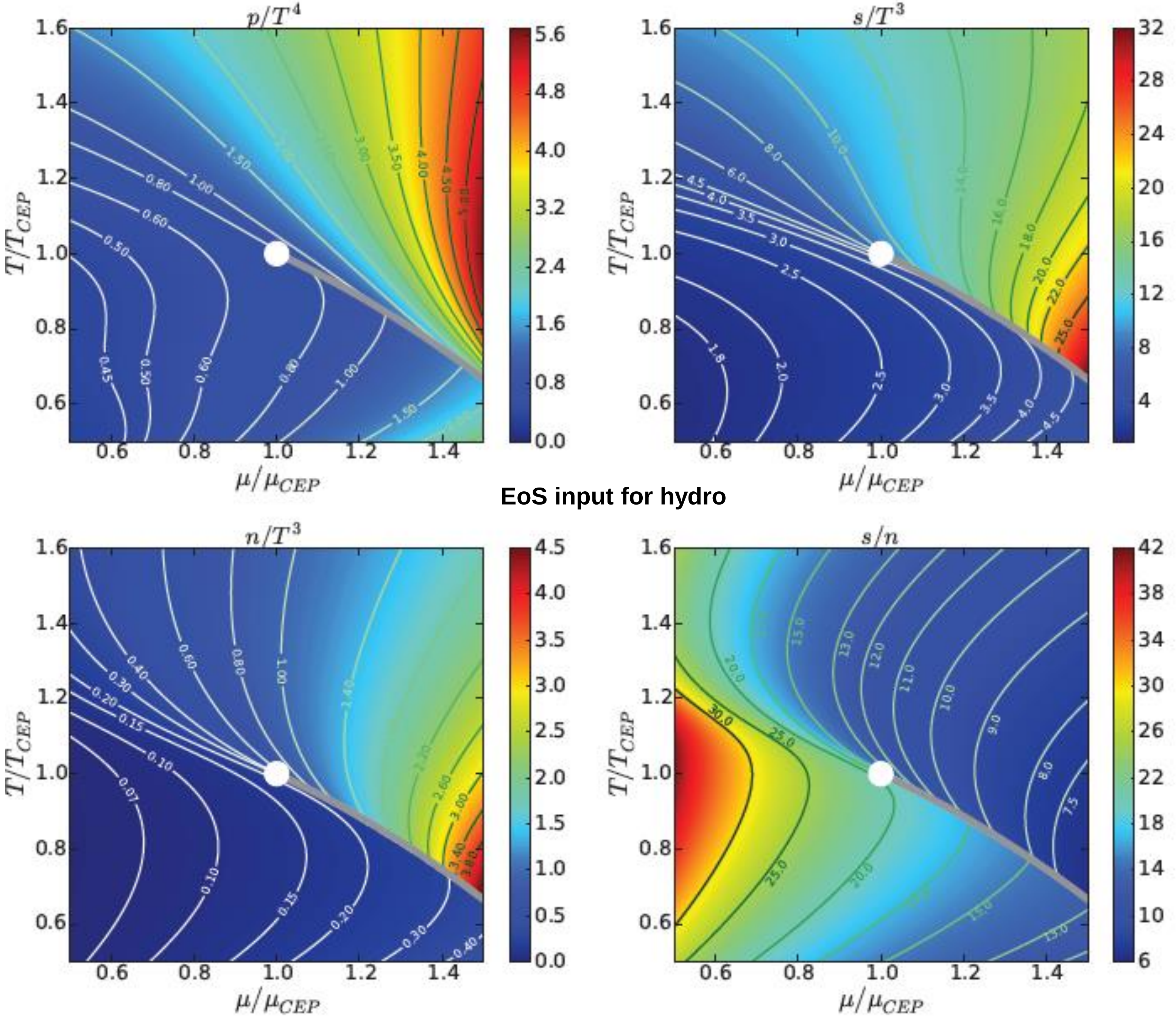
$$L^2 V(\phi) = \begin{cases} -12 \exp\left\{\frac{a_1}{2}\phi^2 + \frac{a_2}{4}\phi^4\right\} & : \phi < \phi_m \\ a_{10} \cosh\left[a_4(\phi - a_5)\right]^{a_3/a_4} \exp\left\{a_6\phi + \frac{a_7}{a_8} \tanh\left[a_8(\phi - a_9)\right]\right\} & : \phi \geq \phi_m \end{cases}$$

$$\hat{f}(\phi) = c_0 + c_1 \tanh\left[c_2(\phi - c_3)\right] + c_4 \exp\left[-c_5\phi\right]$$



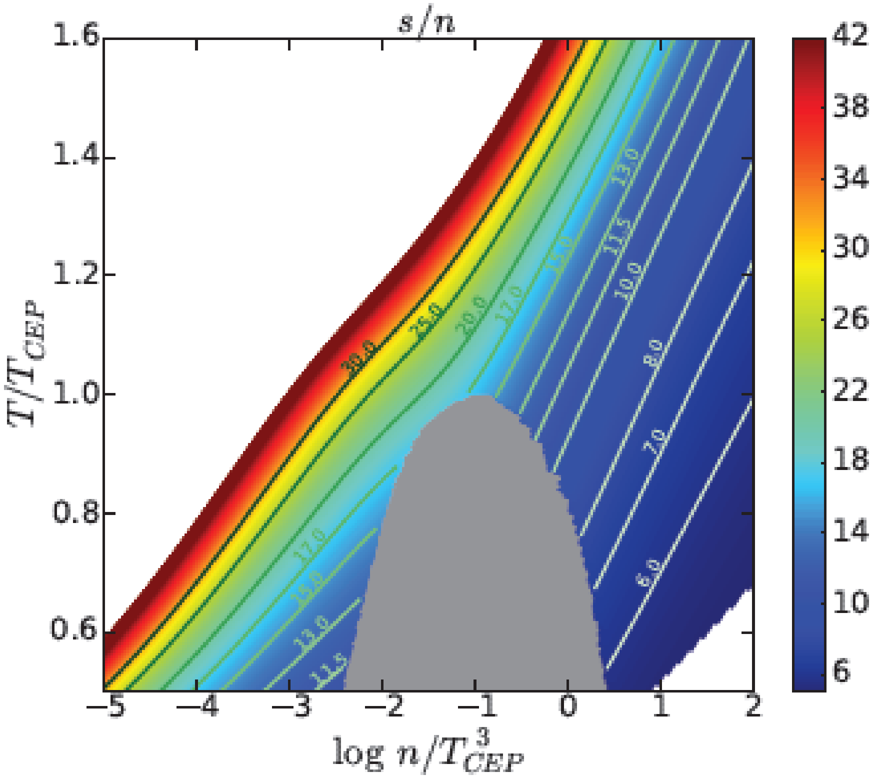
CEP (T, μ) = (112, 612) MeV

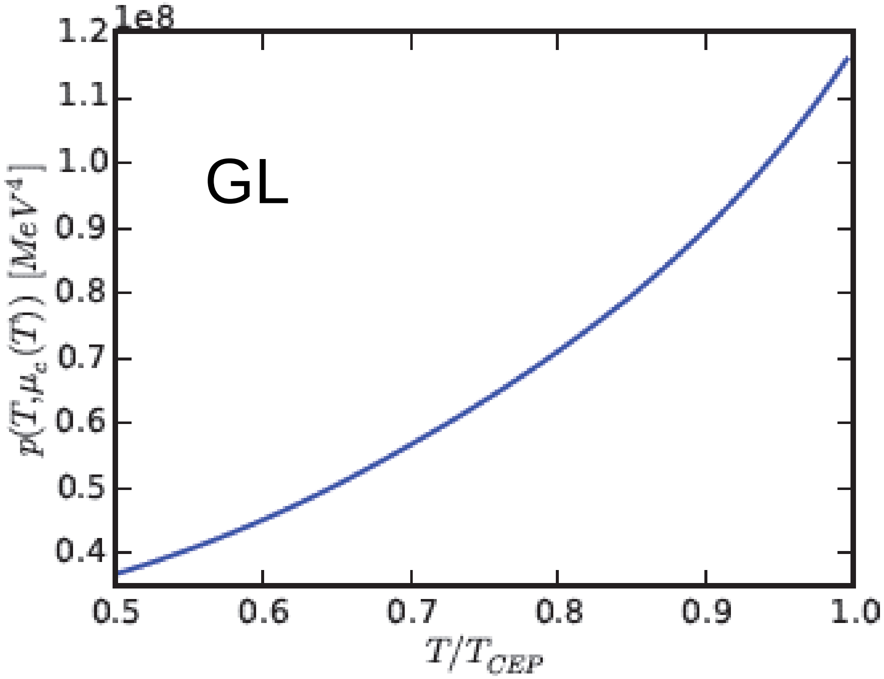
vs. (89, 723) MeV in 1706.00445



EoS input for hydro

important: pattern of isentropic curves





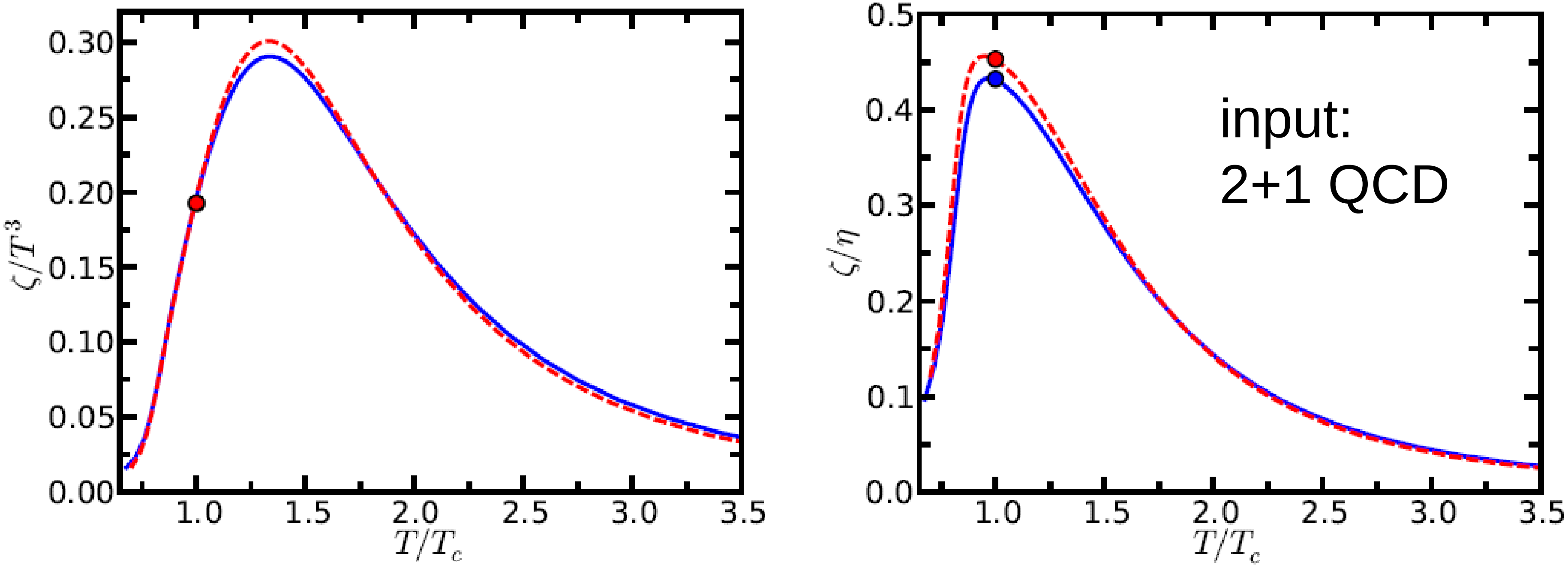
graceful exit

Clausius-Clapeyron

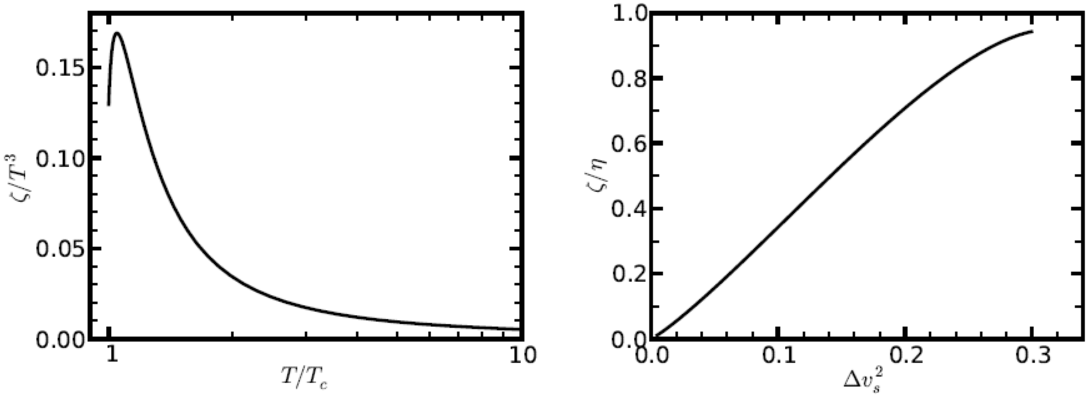
2) Bulk viscosity in LHC wedge

Eling-Oz formula:

$$\left.\frac{\zeta}{\eta}\right|_{\phi_H} = \left(\frac{d\log s}{d\phi_H}\right)^{-2} = \left(\frac{1}{v_s^2}\frac{d\log T}{d\phi_H}\right)^{-2};$$



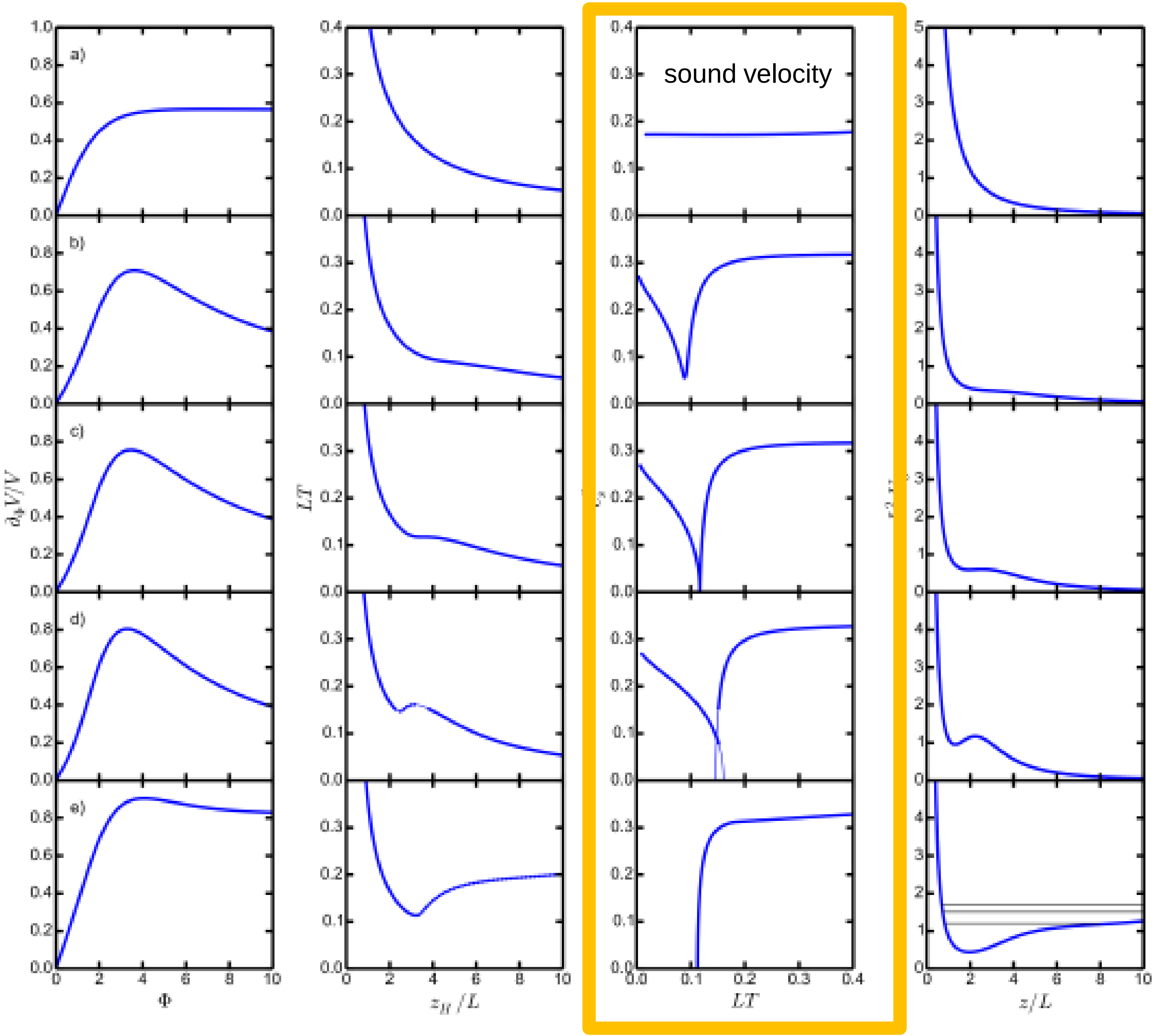
input:
2+1 QCD



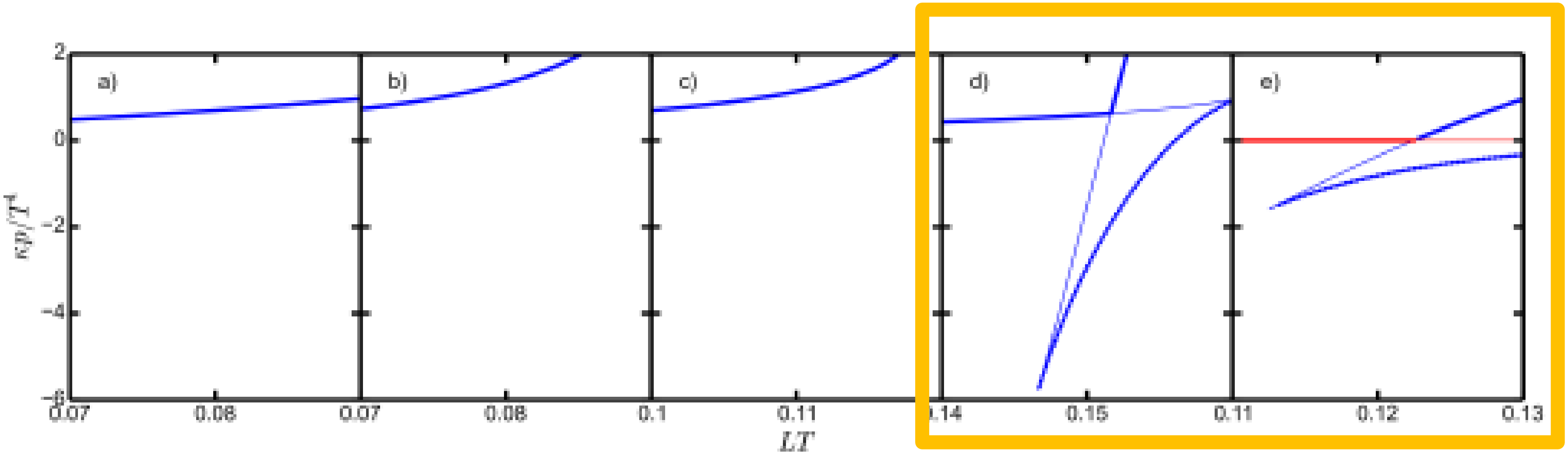
input:
SU(3) YM

3) phase richness from dilaton potential ansatz

$$-L^2V(\Phi) = 12\cosh(\gamma\Phi) + a\Phi^2 + b\Phi^4$$

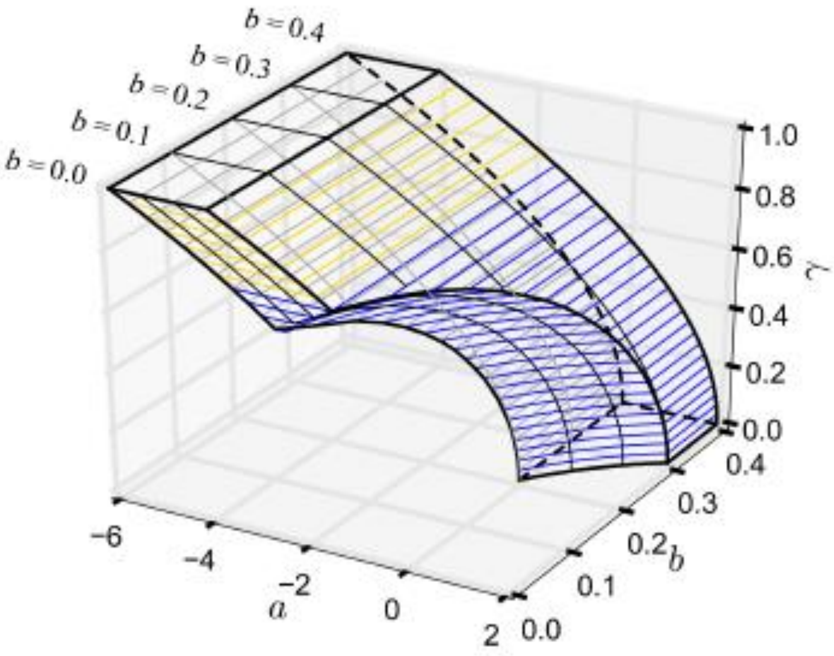


scaled pressure

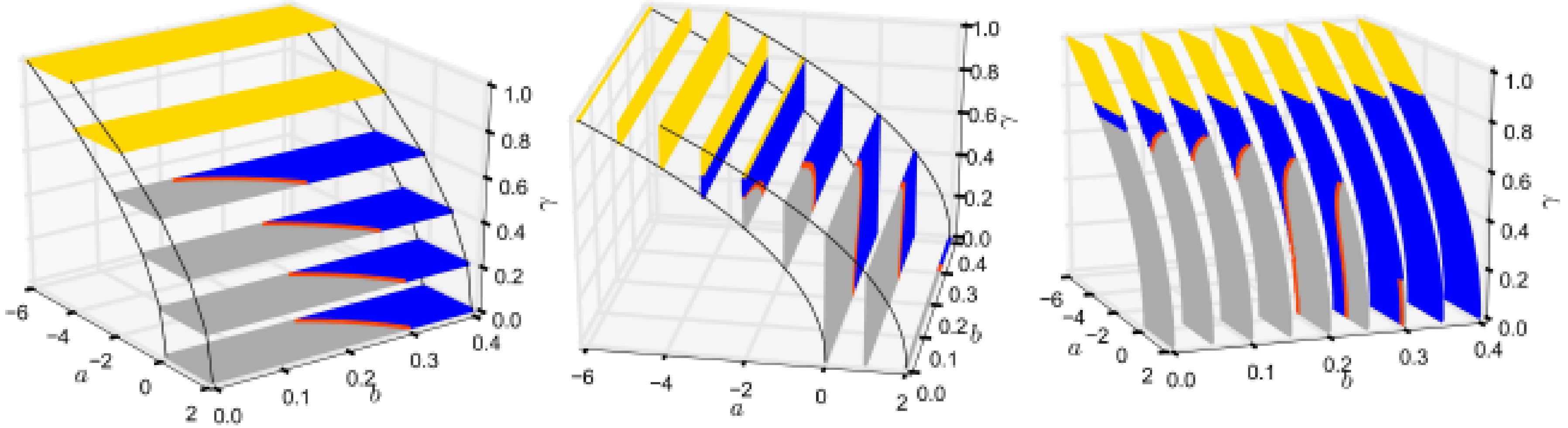


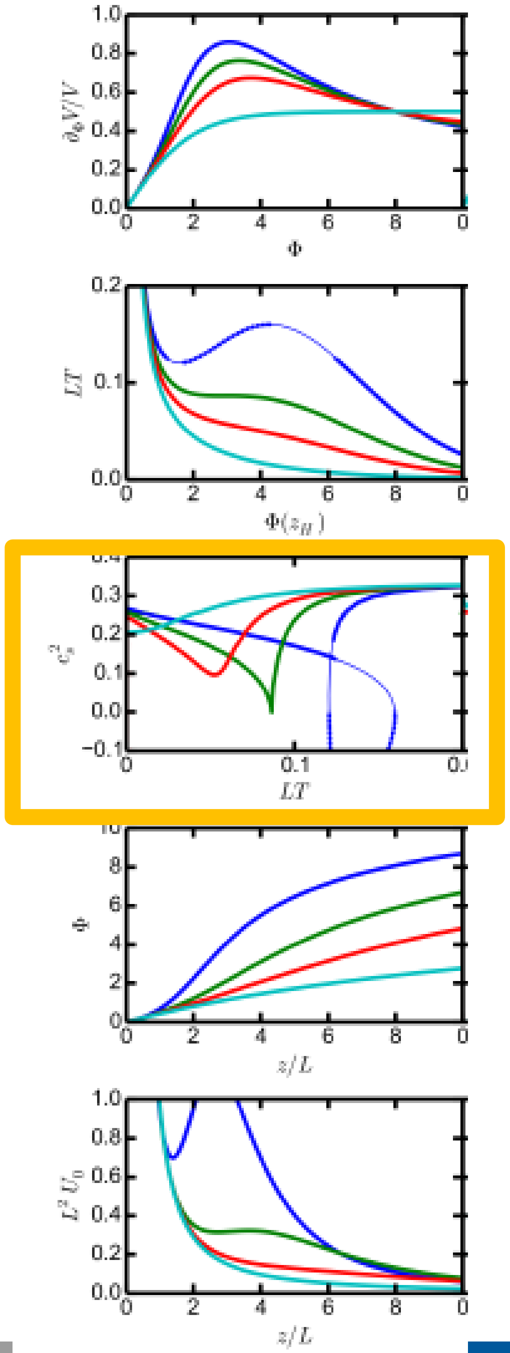
Example	γ	a	b	Δ	Transition
a)	0.56	-0.077	0	2.63	none
b)	0	1.155	0.18	3.3	cross-over
c)	0	1.155	0.20	3.3	second-order
d)	0	1.155	0.25	3.3	first-order
e)	0.83	-2.69	0	3.06	Hawking-Page

exploring the parameter space



yellow: HP
blue: FOPT
red: SOPT





$$-L^2V(\Phi) = 12\cosh(\gamma\Phi) + a\Phi^2 + b\Phi^4$$

Einstein eqs. + EoM
+ boundary conds.

→ gravity & dilaton profiles

+ AdS/CFT dictionary

→ thermodynamics

read curves: cross over
as emulation of QCD(2+1)_phys

holographic vector mesons in probe limit

action: $$S_V = \frac{1}{k}\int \mathrm{d}^4x\, \mathrm{d}z\, \sqrt{g} e^{-\Phi(z)} F^2$$

EoM: $$\left(\partial_\xi^2 - (U_T - m_n^2)\right)\psi = 0$$

$$\mathrm{d}s^2 \;=\; e^{A(z)-\frac{2}{3}\Phi(z)}\left(f(z)\mathrm{d}t^2 - \mathrm{d}\vec{x}^{\,2} - \frac{\mathrm{d}z^2}{f(z)}\right)$$

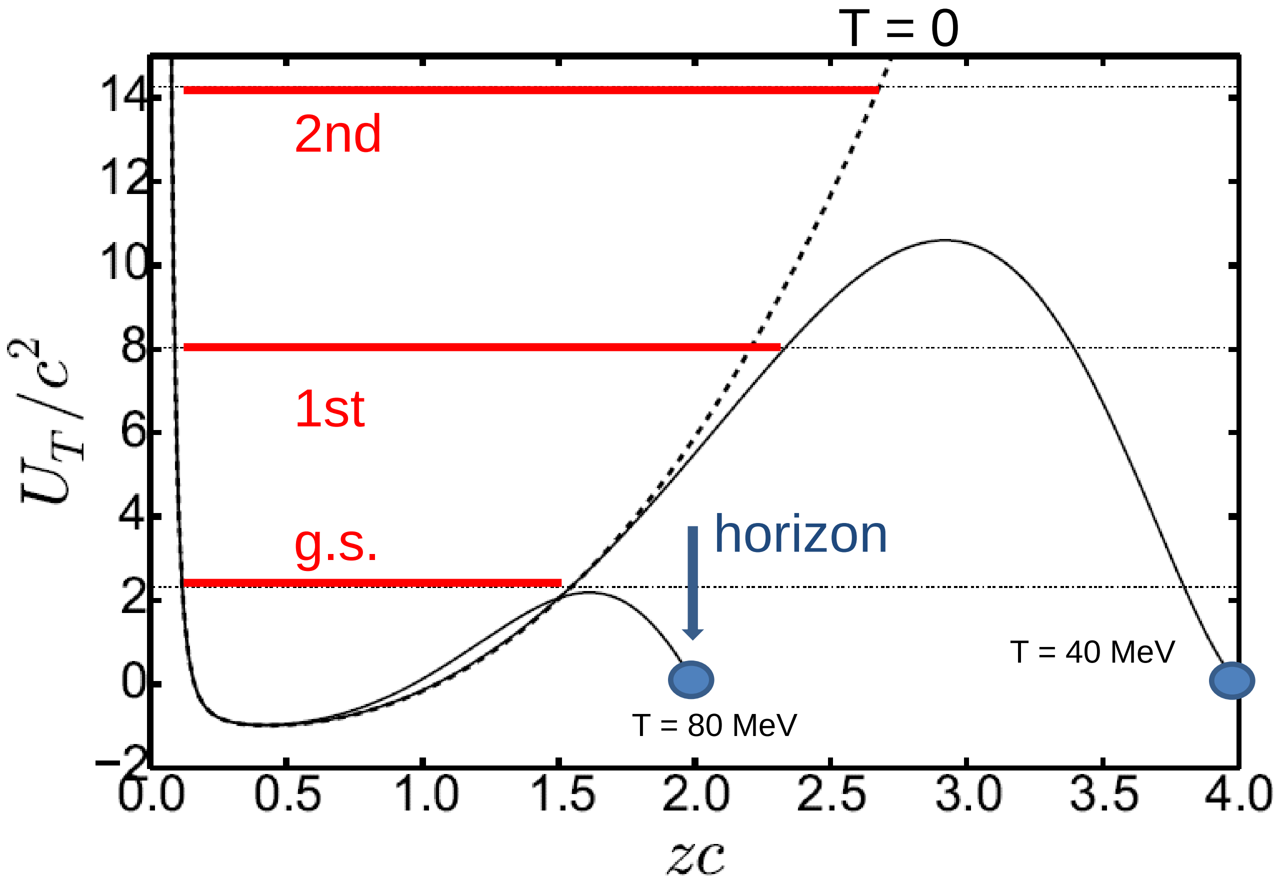
$$U_T = \left(\frac{1}{2}\left(\frac{1}{2}\partial_z^2 A - \partial_z^2\Phi\right) + \frac{1}{4}\left(\frac{1}{2}\partial_z A - \partial_z\Phi\right)^2\right) f^2 + \frac{1}{4}\underbrace{\left(\frac{1}{2}\partial_z A - \partial_z\Phi\right)}_{s'}\partial_z f^2.$$

popular requirement: U(T = 0) → Regge spectrum
(radial excitations n)

→ FOPT: 2+1 QCD in chiral limit (cf. Columbia plot)

Schrödinger equivalent potential

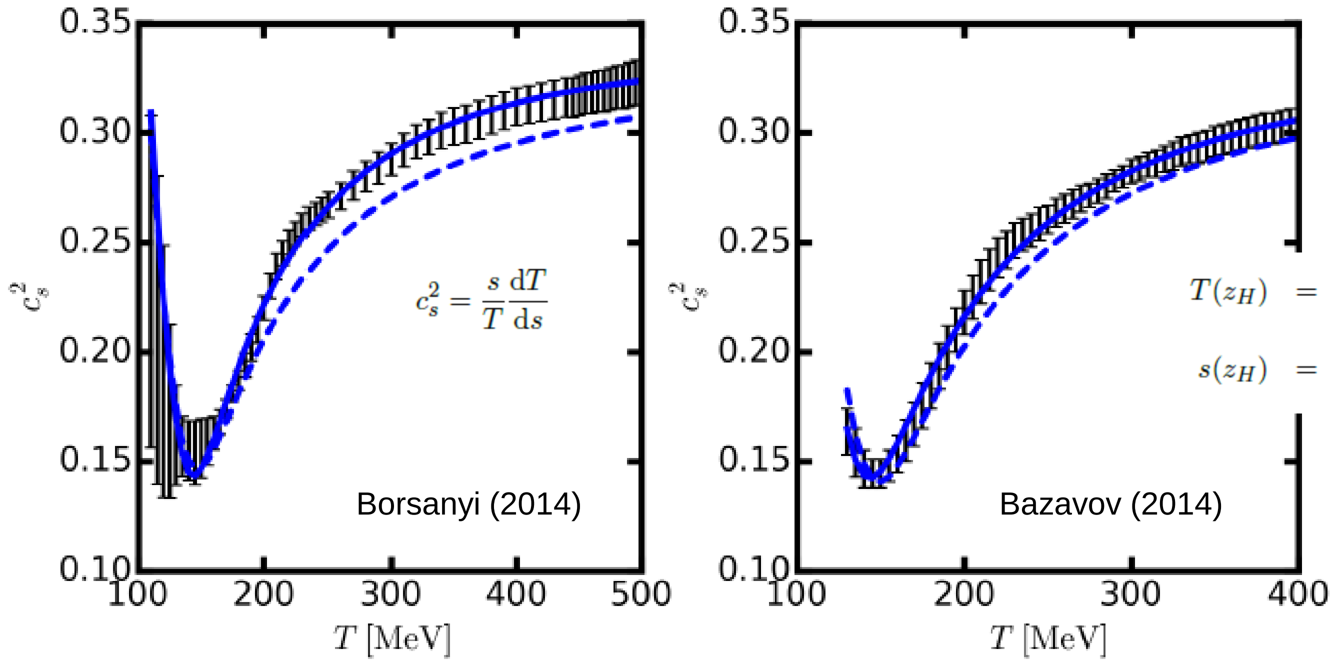
for modes in Klein-Kaluza decomposition of V in axial gauge



SW(T > 0):
Colangelo, Giannuzzi,
Nicotri, JHEP (2012)

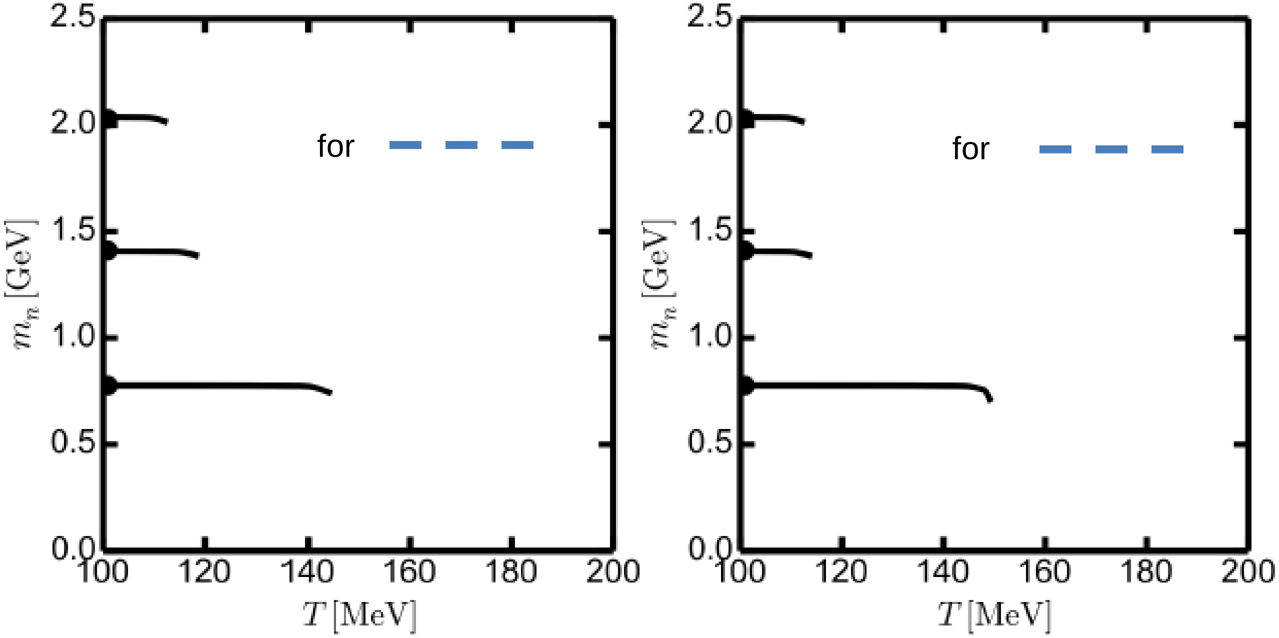
h → T(H)
needs
fine tuning

sequential disappearance upon temperature increase



an ad hoc ansatz:

$$\begin{aligned} T(z_H) &= \frac{T_x}{\theta}\left(\frac{1}{x} + \theta - 3b + 3bx - x^2\right), \\ s(z_H) &= \frac{16\pi G_5}{z_H^3} \end{aligned}$$



sequential melting of rho mesons (1s, 2s, 3s)

vector mesons as probes in gravity-dilaton background: improvement by G model

$$S_G^V = \frac{1}{k_V}\int \mathrm{d}^4x\,\mathrm{d}z\sqrt{g_5}e^{-\phi}\boxed{G(\phi)}F^2$$

$$\left[\partial_\xi^2 - (U_T(\xi) - m_i^2)\right]\psi_i = 0, \quad i = 0, 1, 2\cdots$$

$$U_T = \left(\frac{1}{2}\mathcal{S}_T' + \frac{1}{4}\mathcal{S}_T^2\right)f^2 + \frac{1}{2}\mathcal{S}_T f f'$$

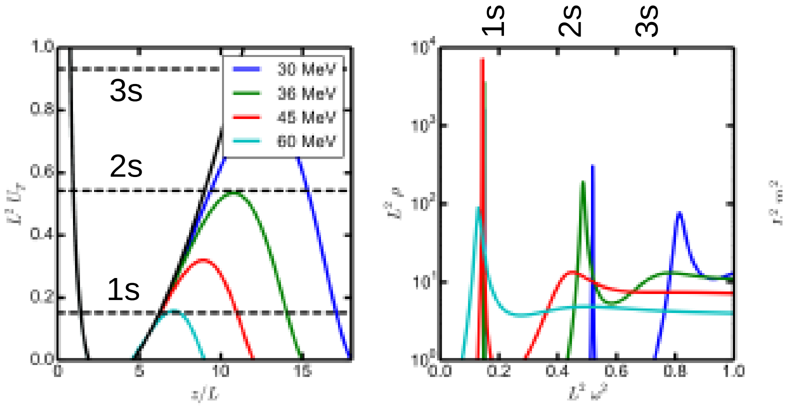
$$\mathcal{S}_T \equiv \frac{1}{2}A' - \phi' + \boxed{\partial_z \log G(\phi(z)).}$$

T = 0:

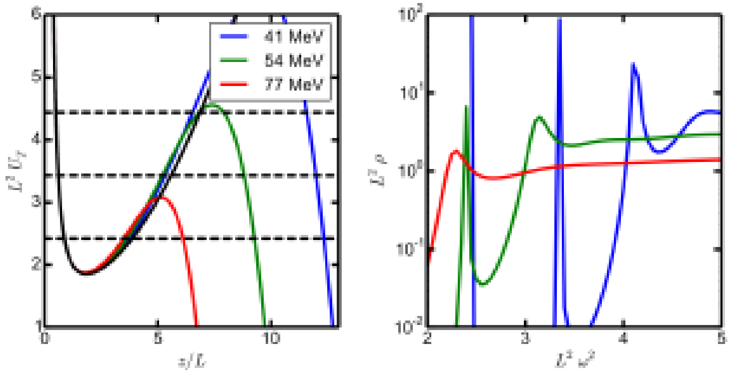
$L^2U_0 = \frac{3}{4}\left(\frac{L}{z}\right)^2 + a\left(\frac{z}{L}\right)^2 + 4b$ → Regge $L^2m_i^2 = 4ai + 4(a+b)$

flavor dependent a, b

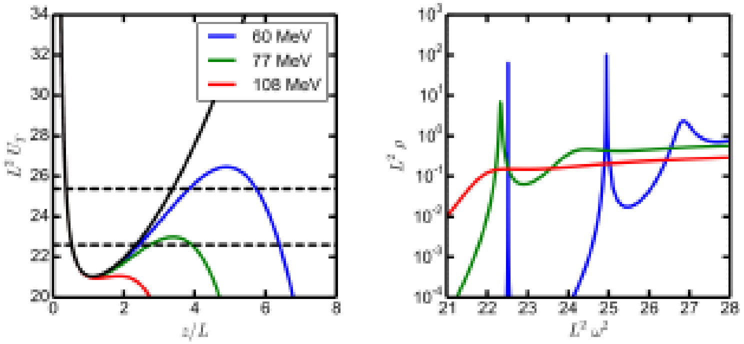
ρ-Meson



J/ψ-Meson

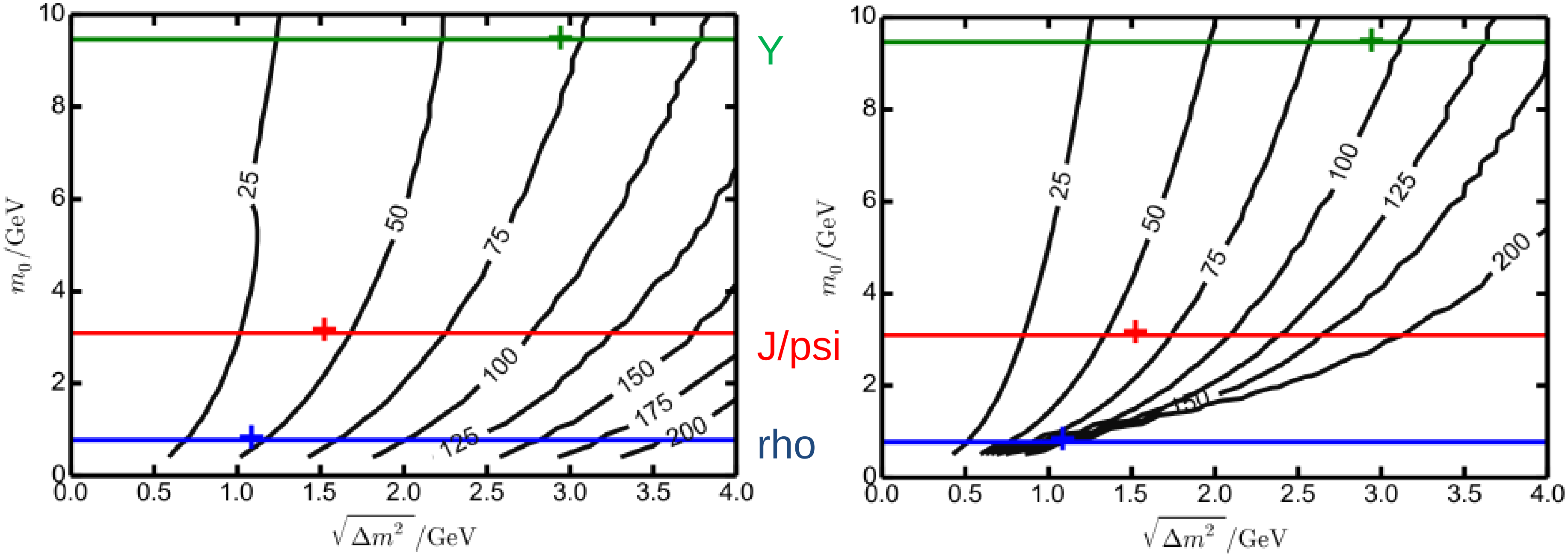


Υ-Meson



disappearance of

g.s. in Schrodinger eq.　　　　1st peak in s.f.



+: with m1 adjustment

→ U0 must be improved

Braga et al.: ad hoc gravity & dilaton + too low/high m0/m1 → high T_dis

Summary of holographic EdM model

1) phase structure & CEP coordinates
HEE (cut-off) yields same information

prediction

2) bulk viscosity = 50% shear viscosity (2+1 QCD)
100% (SU(3) YM)

predictions

3) naive vector mesons in probe limit: no-go conjecture
either
dilaton potential to match QCD thermodynamics
→ no hadrons at/below CO Tc

desaster

or
Schrodinger equivalent potential for Regge states
→ FOPT

desaster

4) G model of probe-vector mesons

improvement

questions: 1) CEP coordinates & HEE

input: p(T) & susceptibilities from lQCD

2) bulk viscosity

input: p(T) from lQCD

3) do hadrons exist at f.o. ?

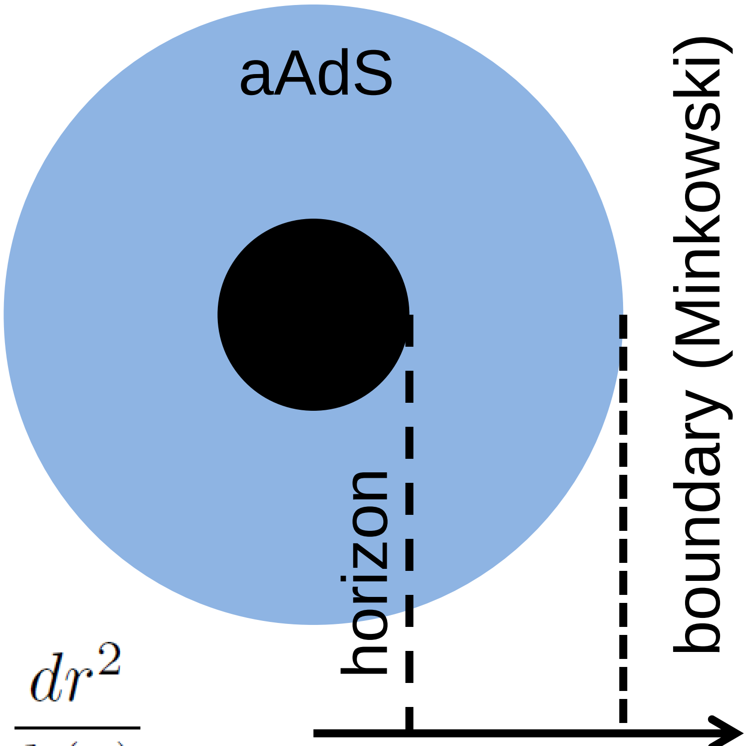
input: hadrons in vacuum

tools: holography (AdS/CFT correspondence)

bottom-up engineering

(due to missing top-down from string theory or QCD dual)

mapping out the manifold
by various coordinates:



rH = 0	infinity
zH	0
phiH	0

$$ds^2 = e^{2A(r)}\left(-h(r)dt^2 + d\vec{x}^2\right) + \frac{dr^2}{h(r)}$$

$$ds^2 \quad = \quad e^{A(z)-\frac{2}{3}\Phi(z)}\left(f(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{f(z)}\right)$$

$$ds^2 = e^{2A}(-hdt^2 + d\vec{x}^2) + e^{2B}\frac{d\phi^2}{h}$$

z, r … = holographic bulk coordinate

entanglement entropy: $S_{\mathrm{EE}} := -\mathrm{Tr}_{\mathcal{A}}\,\rho_{\mathcal{A}}\ln\rho_{\mathcal{A}}$

$$\rho_{\mathcal{A}} = \mathrm{Tr}_{\mathcal{B}}\,\rho_{tot}$$

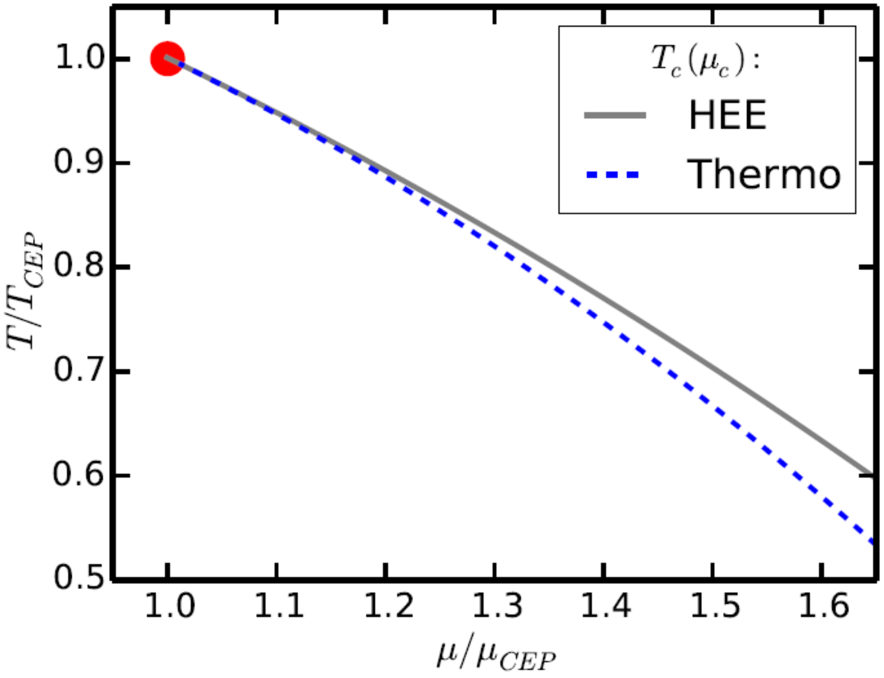
$$S_{\mathrm{HEE}} = \frac{\mathrm{Area}(\gamma_{\mathcal{A}})}{4G_N^{(d+1)}}$$

Ryu, Takayanagi (2006)

$$= \frac{1}{4}\int dx_1 dx_2 dx_3 \sqrt{\gamma}$$

$$= \frac{V_2}{2}\int_0^{l/2} dx_1 \mathrm{e}^{2A(r)}\sqrt{\mathrm{e}^{2A(r)} + \frac{r'^2}{h(r)}}$$

+ cut-off regularization



$$s^2_{\gamma_{\mathcal{A}}} = \left(\mathrm{e}^{2A} + \frac{r'^2}{h}\right) dx_1^2 + \mathrm{e}^{2A}\left(dx_2^2 + dx_3^2\right)$$

critical behavior/exponents

2+1 QCD input (cross over) → no discrete hadron states

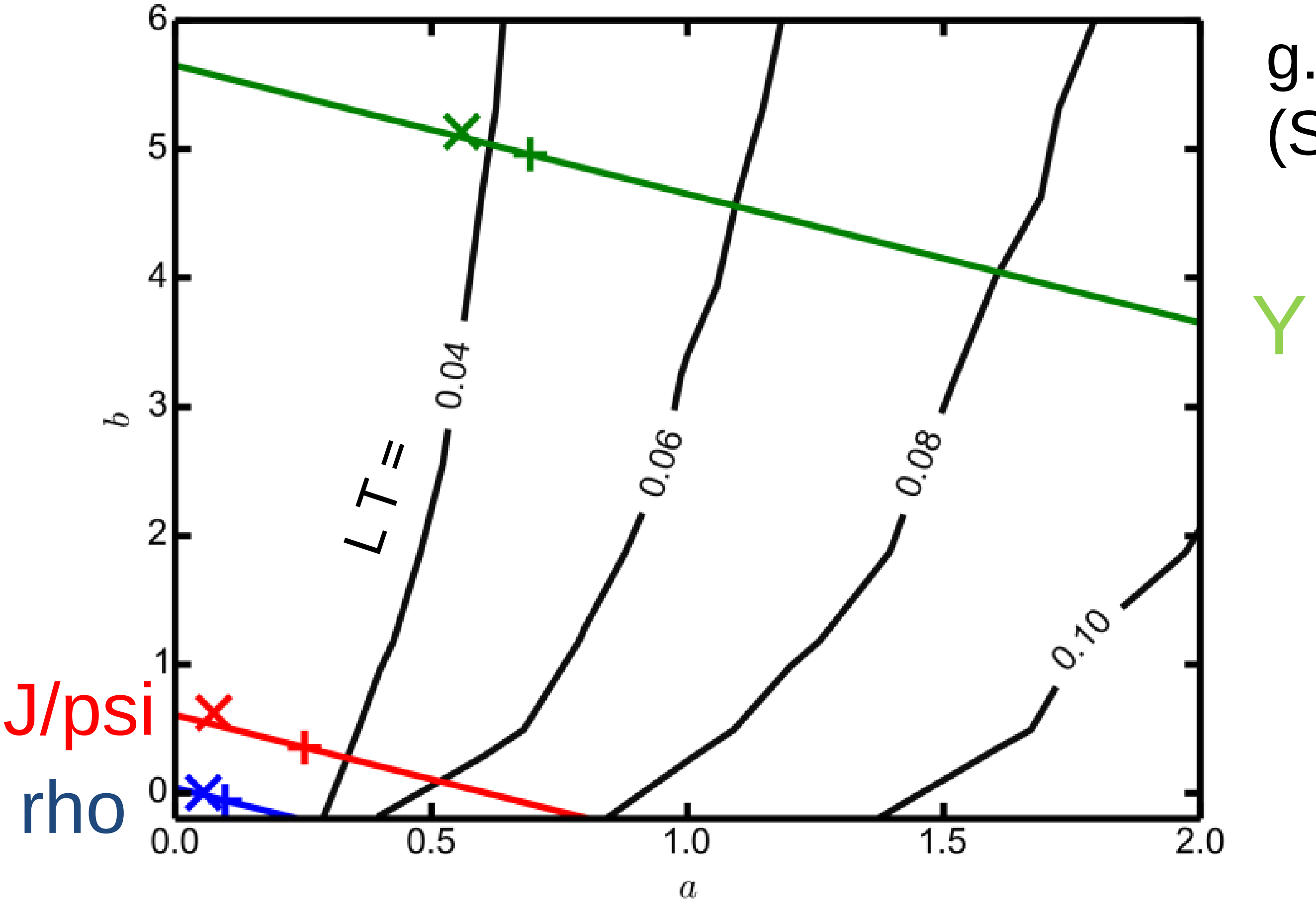
Gürsoy, Kiritsis, Mazzani, Nitti (2009), pure gluon sector:

a gapped, discrete spectrum at T = 0
facilitates a FOPT at T > 0

no-go conjecture within EdM model & probe vector mesons:

either FOPT & Regge spectrum
or cross over & meson melting sets in at T = 0

→ beyond probe limit (backreacted hadrons),
add systematically flavor to gluon dynamics,



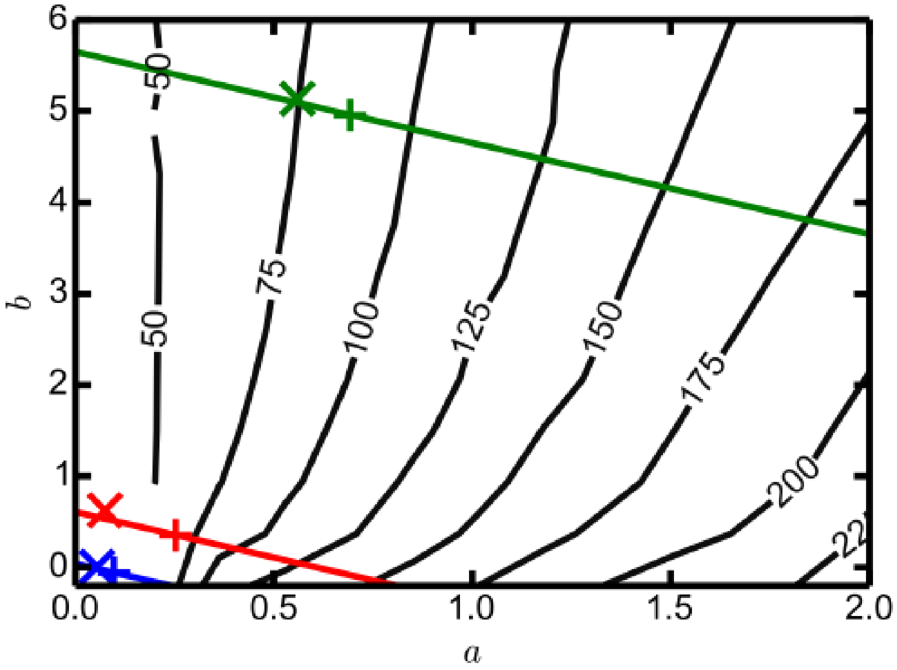
g.s. disapparance
(Schrödinger eq.)

a = L^2 Delta m^2 /4: level spacing

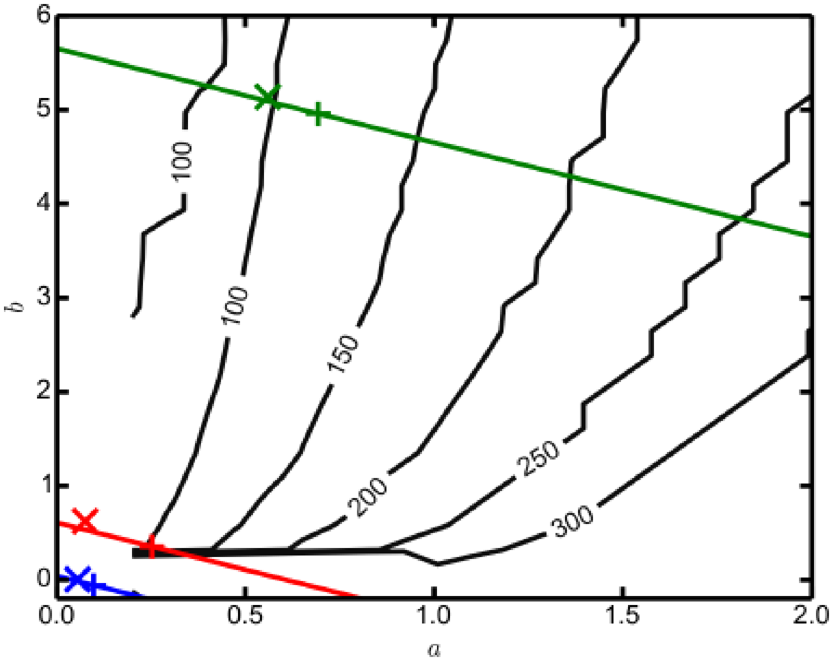
b = L^2(m^2 – Delta m^2) / 4: mass gap (for a = 0)

L= 1 / (2 GeV)

Schrödinger eq.

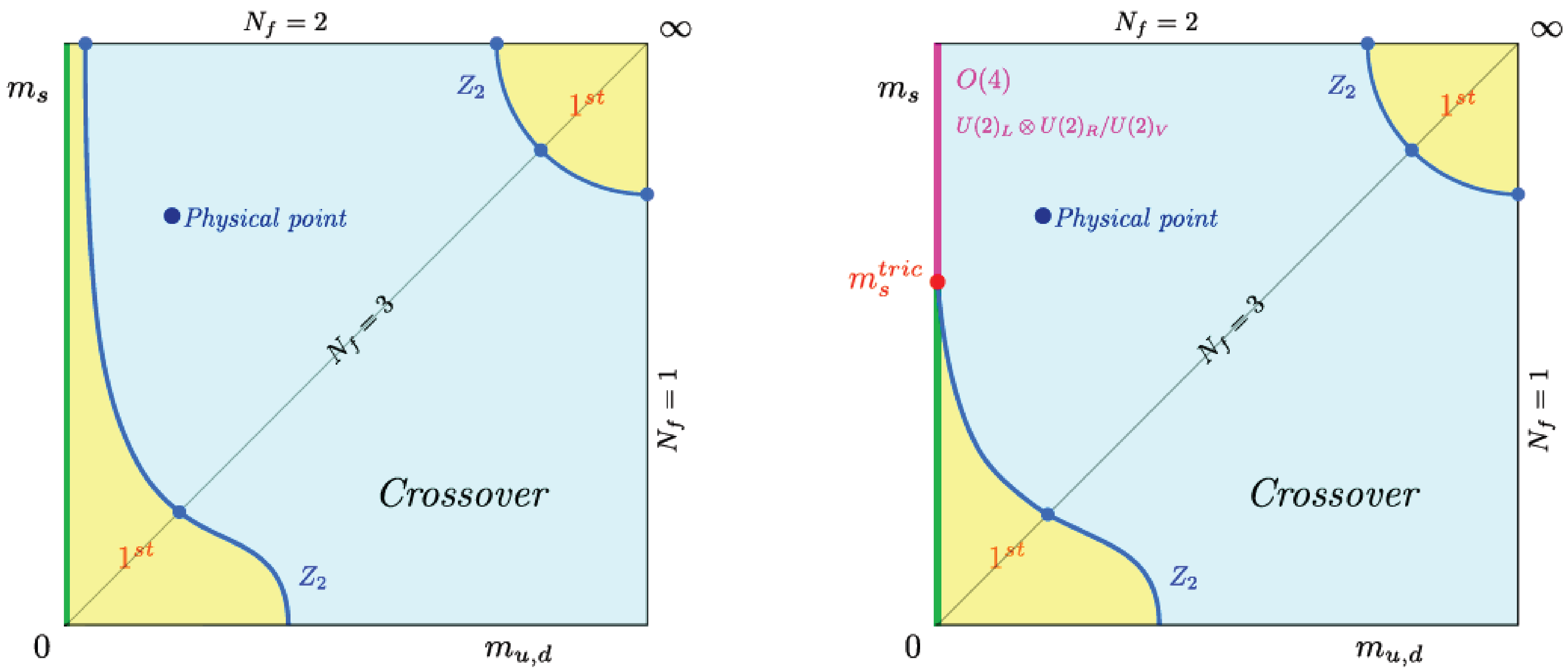


spectral function



analog excited states

Columbia Plot



(a) First order scenario in the $m_s - m_{u,d}$ plane

(b) Second order scenario in the $m_s - m_{u,d}$ plane.

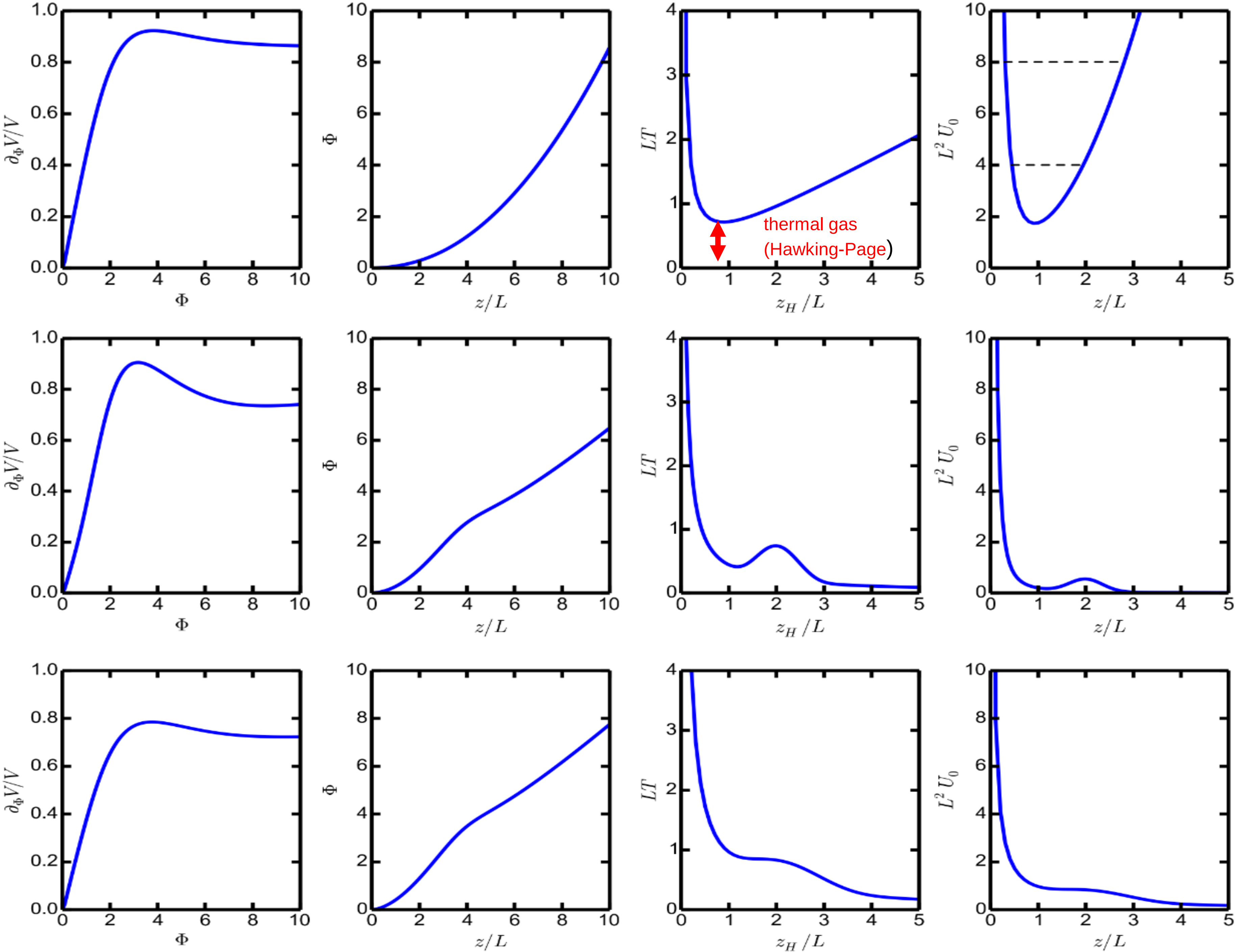
EPJ Web Conf. 175 (2018) 07032, Philipsen et al.

$$S = \frac{1}{k}\int \sqrt{g}(R - \frac{1}{2}(\partial\Phi)^2 - V(\Phi))\,\mathrm{d}^5 x$$

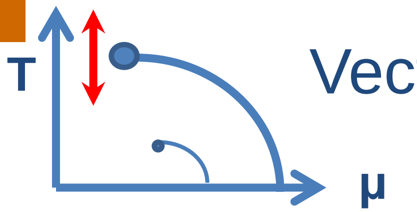
$$\mathrm{d}s^2 = e^A(f\mathrm{d}t^2 - \mathrm{d}\vec{x}^2 - \mathrm{d}z^2/f)$$

$$\begin{aligned} f'' + \frac{3}{2}A'f' &= 0 \\ A'' - \frac{1}{2}A'^2 + \frac{1}{3}\Phi'^2 &= 0 \\ (A'^2 - \frac{1}{6}\Phi'^2)f + \frac{1}{2}A'f' - \frac{1}{3}e^A V &= 0 \end{aligned}$$

$$\Phi'' + \left(\frac{3}{2}A' + \frac{f'}{f}\right)\Phi' + \frac{e^A}{f}\dot{V} = 0$$



thermal gas (Hawking-Page)

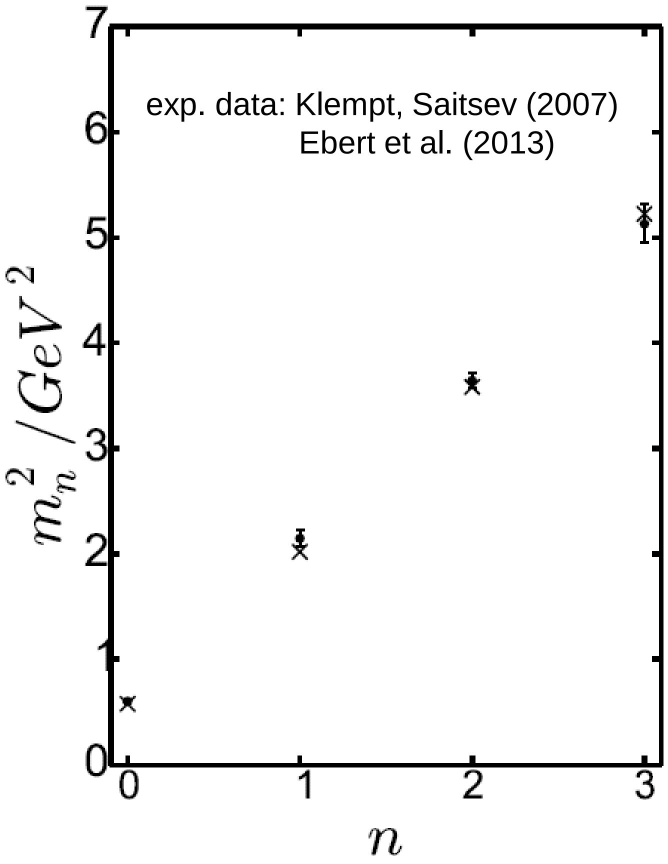


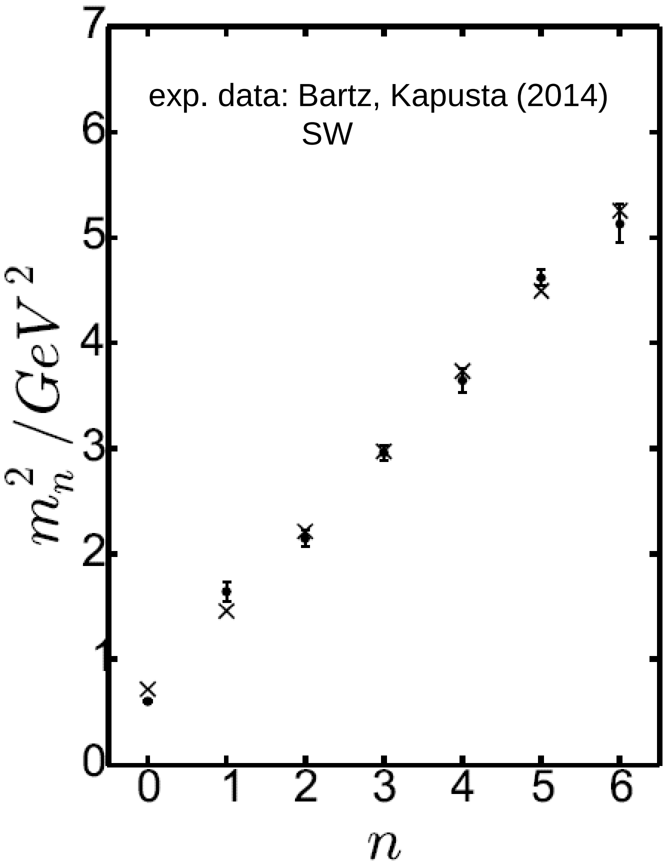
Vector mesons in AdS/CFT – extended soft wall model

$$S_V = F(\overbrace{\text{warp factor, blackening function}}^{\text{5D gravity}}, \overbrace{\text{dilaton}}^{\text{conf. symmetry breaker}}, \overbrace{\text{V wave function}}^{\text{sourced by } \bar{q}\gamma^{\mu}q})$$

soft wall (probe limit): A(z) = ln $(L/z)^2$ f(z) = 1 - $(\frac{z}{H})^4$ Φ(z) = $(cz)^2$

EoM of V→Schrödinger eq. in tortoise coordinate, T = 0 → Regge type spectrum

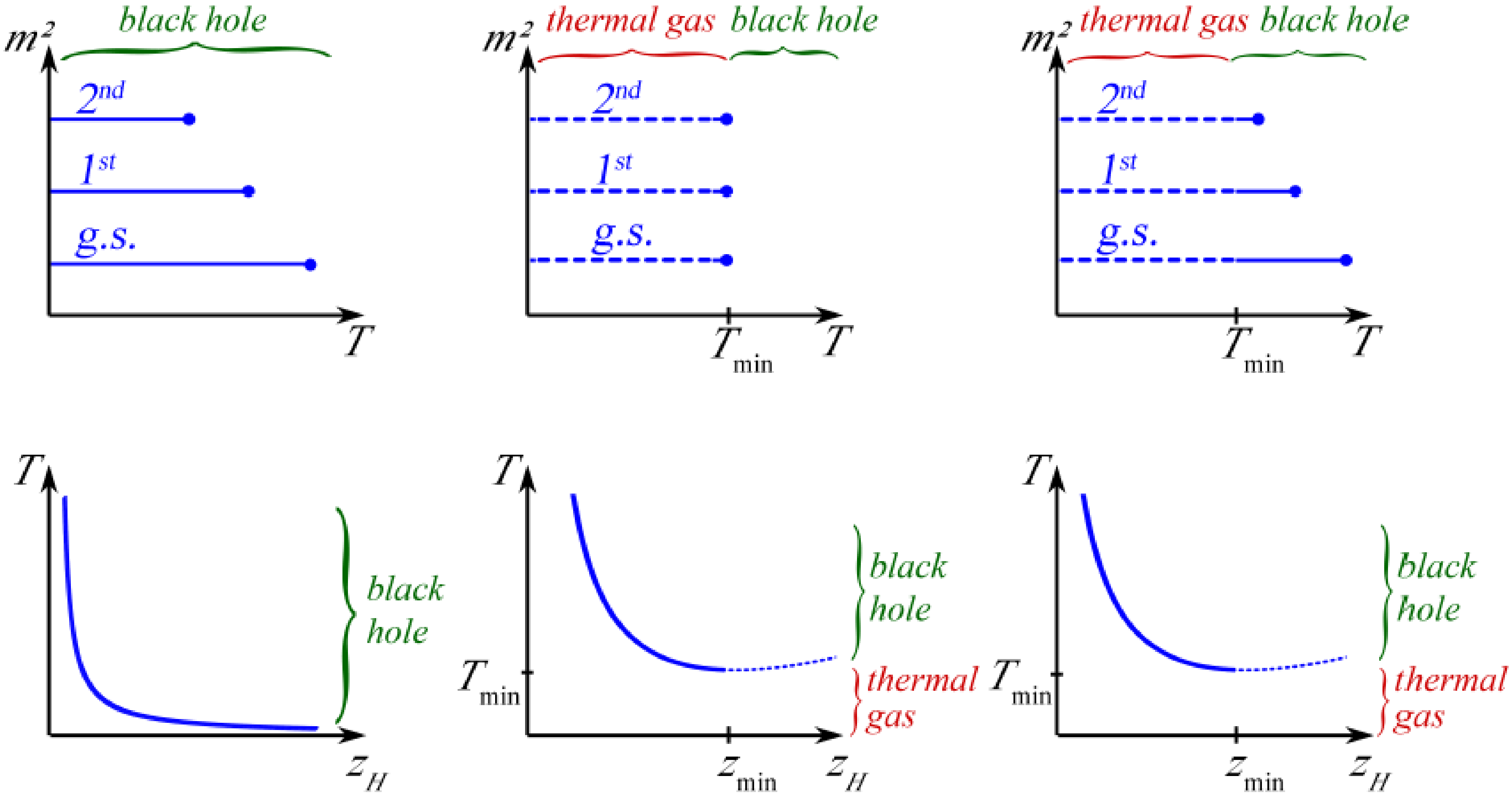




rho trajectory from mod. SW:

$\tilde{A}, \tilde{f}, \tilde{\phi}$

SW & theor. reasoning:
Karch, Katz, Son, Stephanov
PRD (2006)

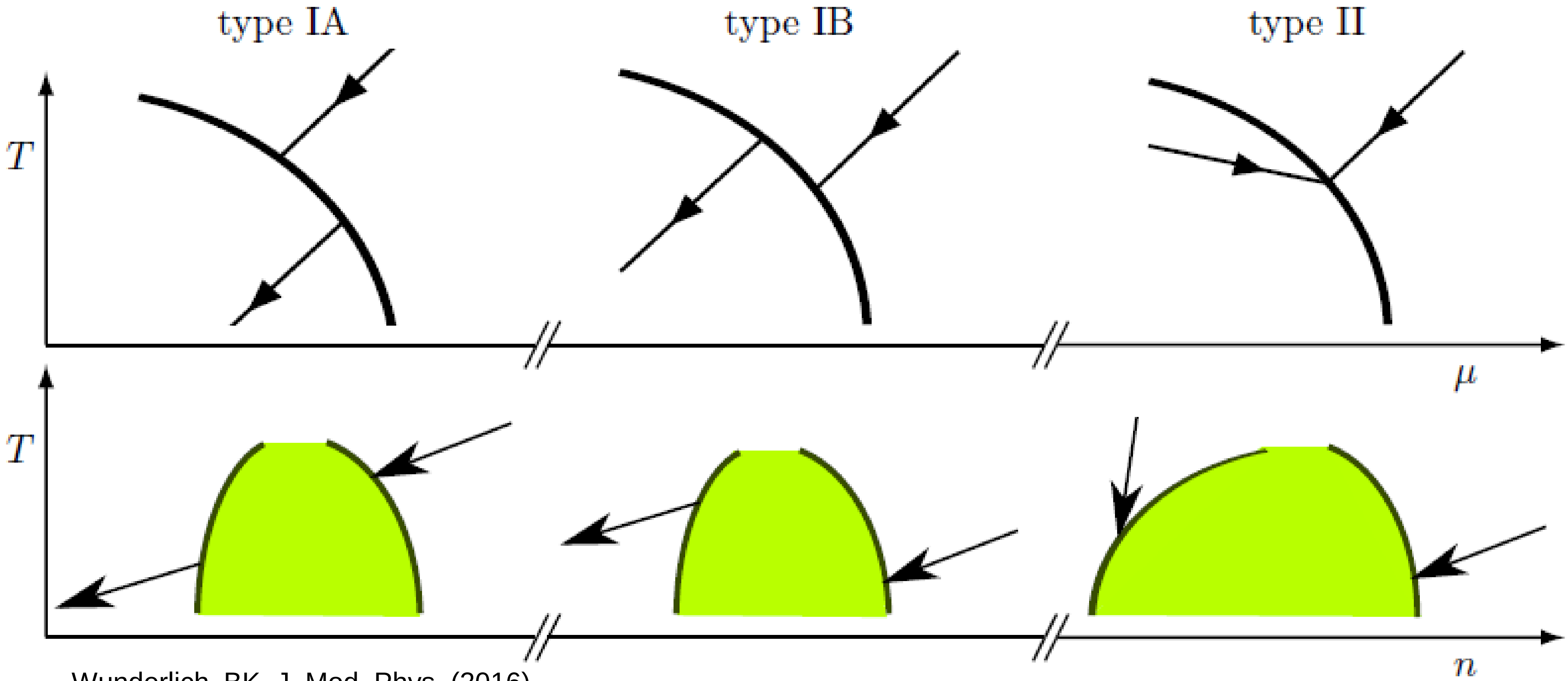


sequential vs. instantan. vs. mixed sequential disappearance

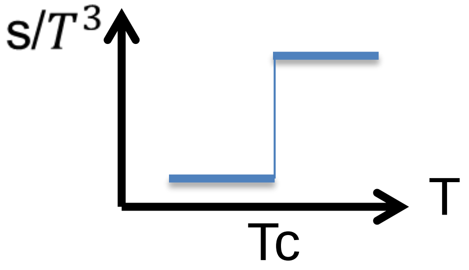
thermodyn. options:

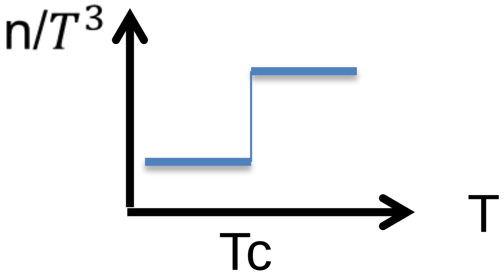
continuous – cross over – 2nd order – 1st order transitions

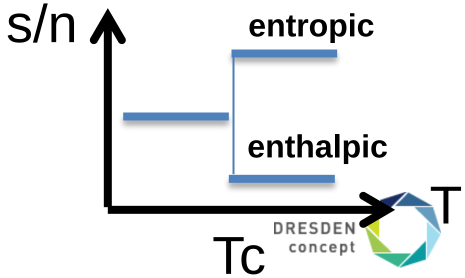
Crossing the phase border line



Wunderlich, BK, J. Mod. Phys. (2016)

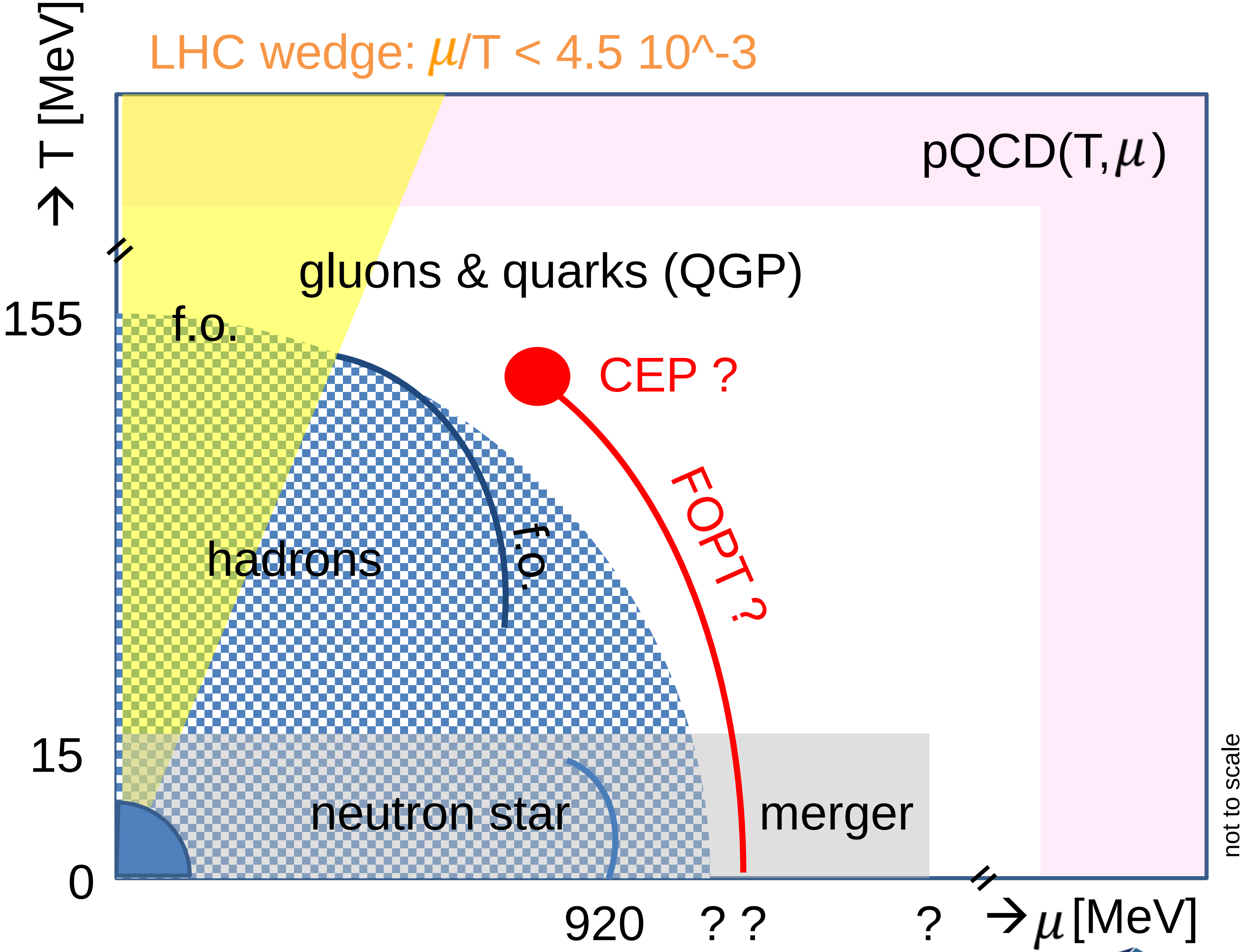






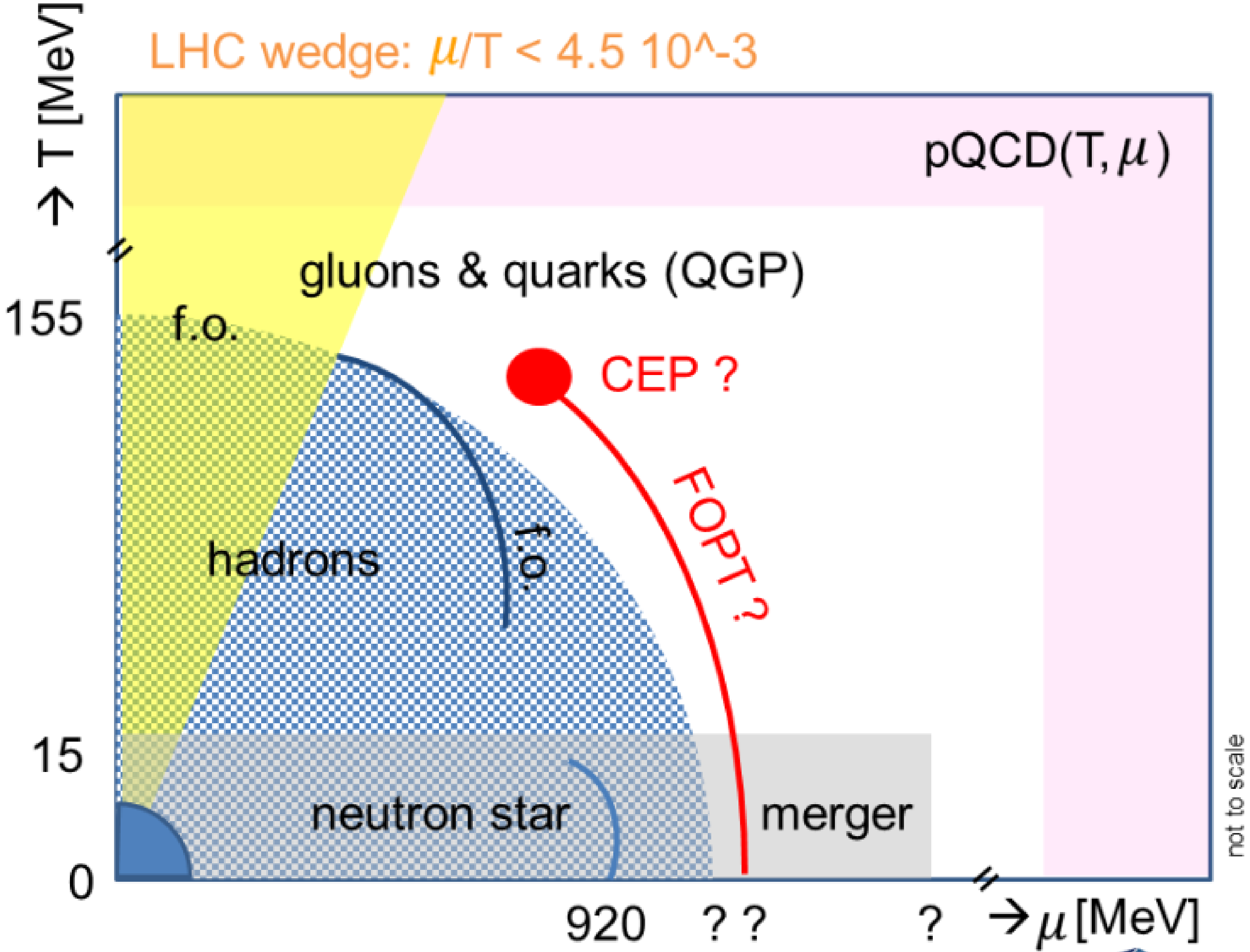
mu = const

LHC wedge: $\mu/T < 4.5\ 10\text{^}-3$



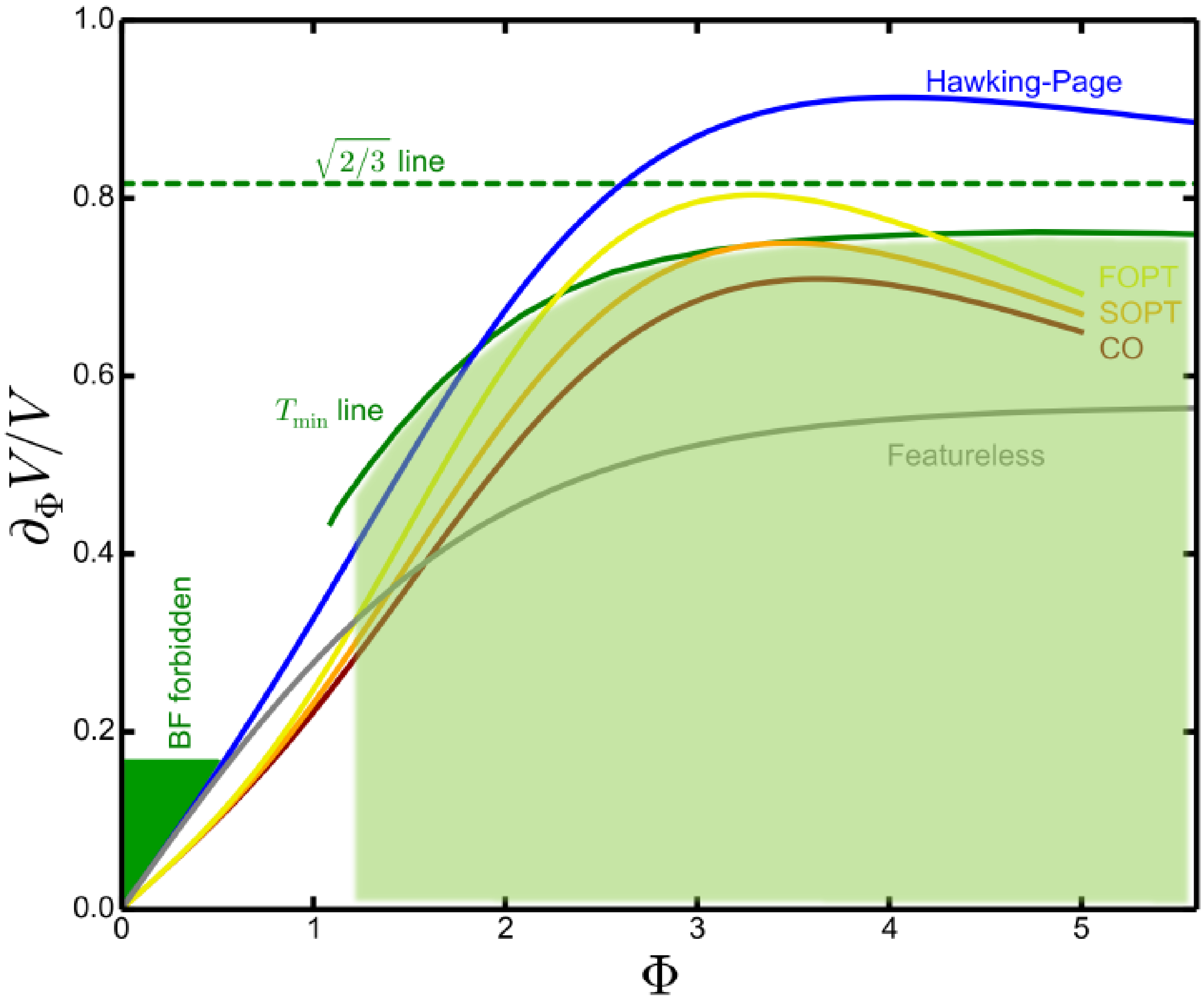
Gregor XV (1622): sacra congregatio de propaganda fide

LHC wedge: μ/T < 4.5 10^-3

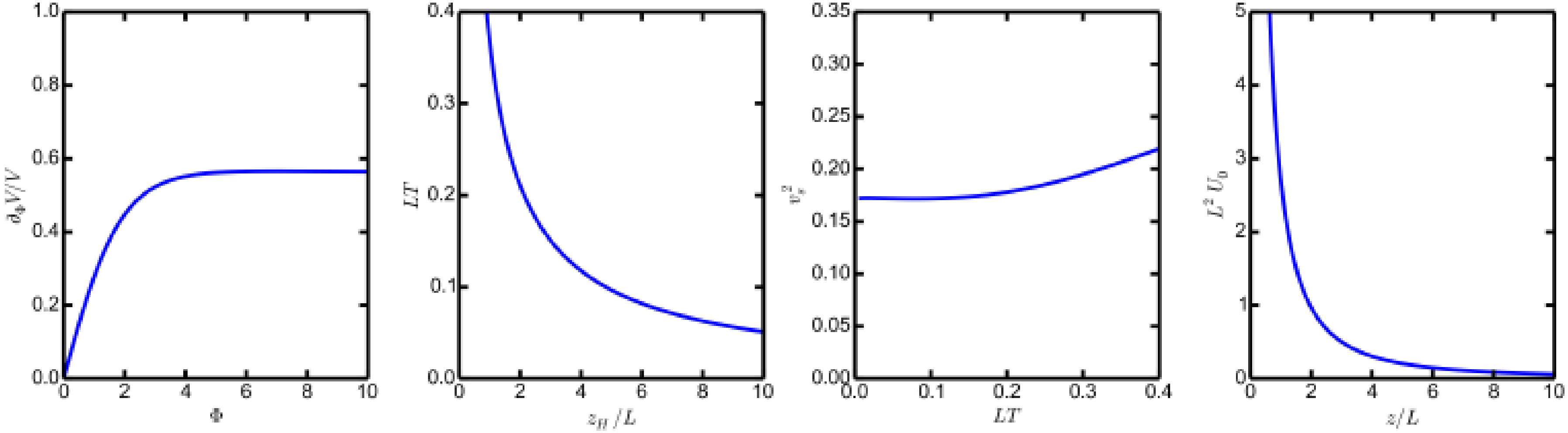


Gregor XV (1622): sacra congregatio de propaganda fide

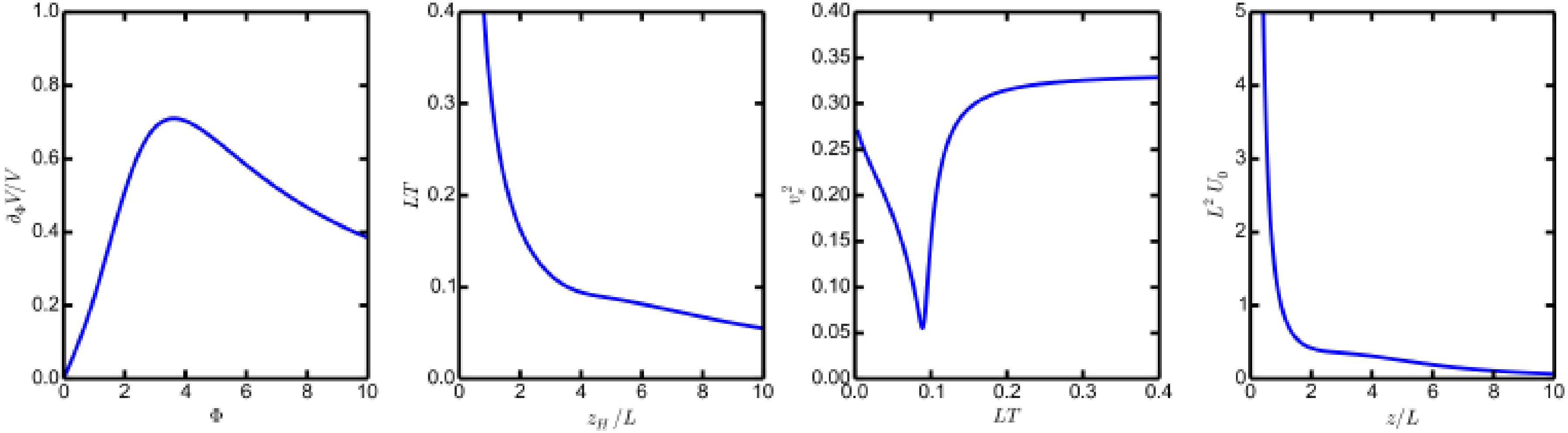
$$-L^2V_1(\Phi) = 12\exp(a\Phi^2 + b\Phi^4)$$
$$-L^2V_2(\Phi) = 12\cosh(\gamma\Phi) + a\Phi^2 + b\Phi^4$$



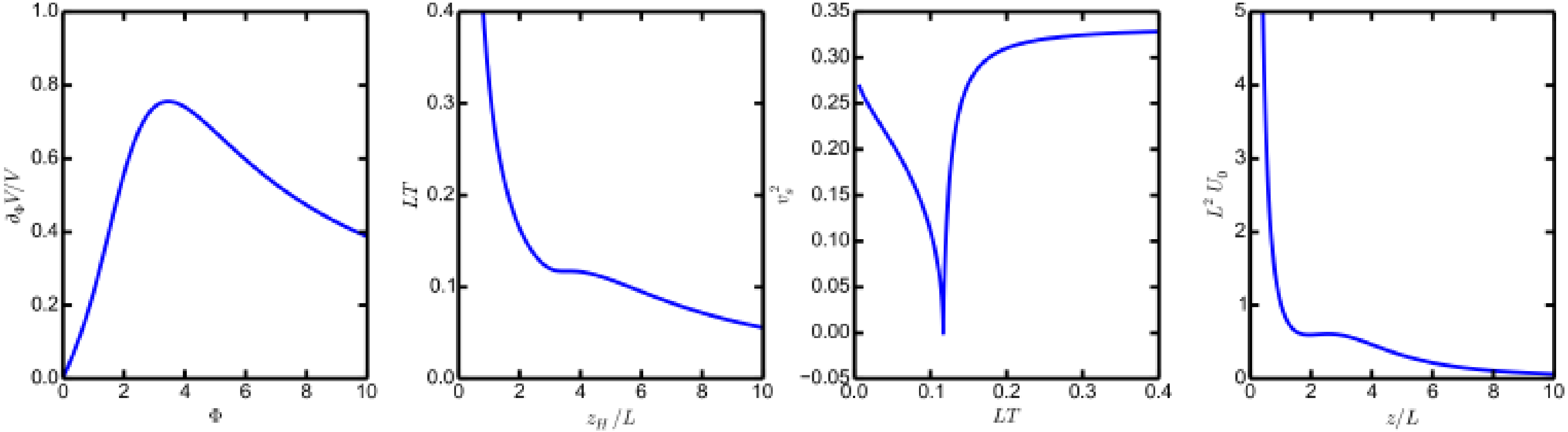
Featureless



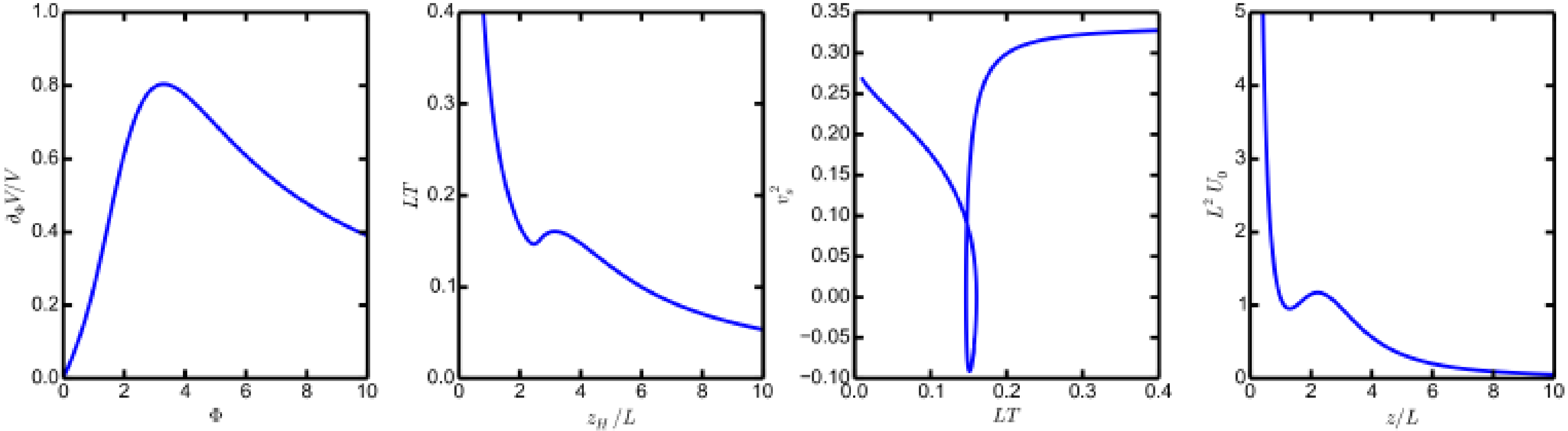
Cross-over



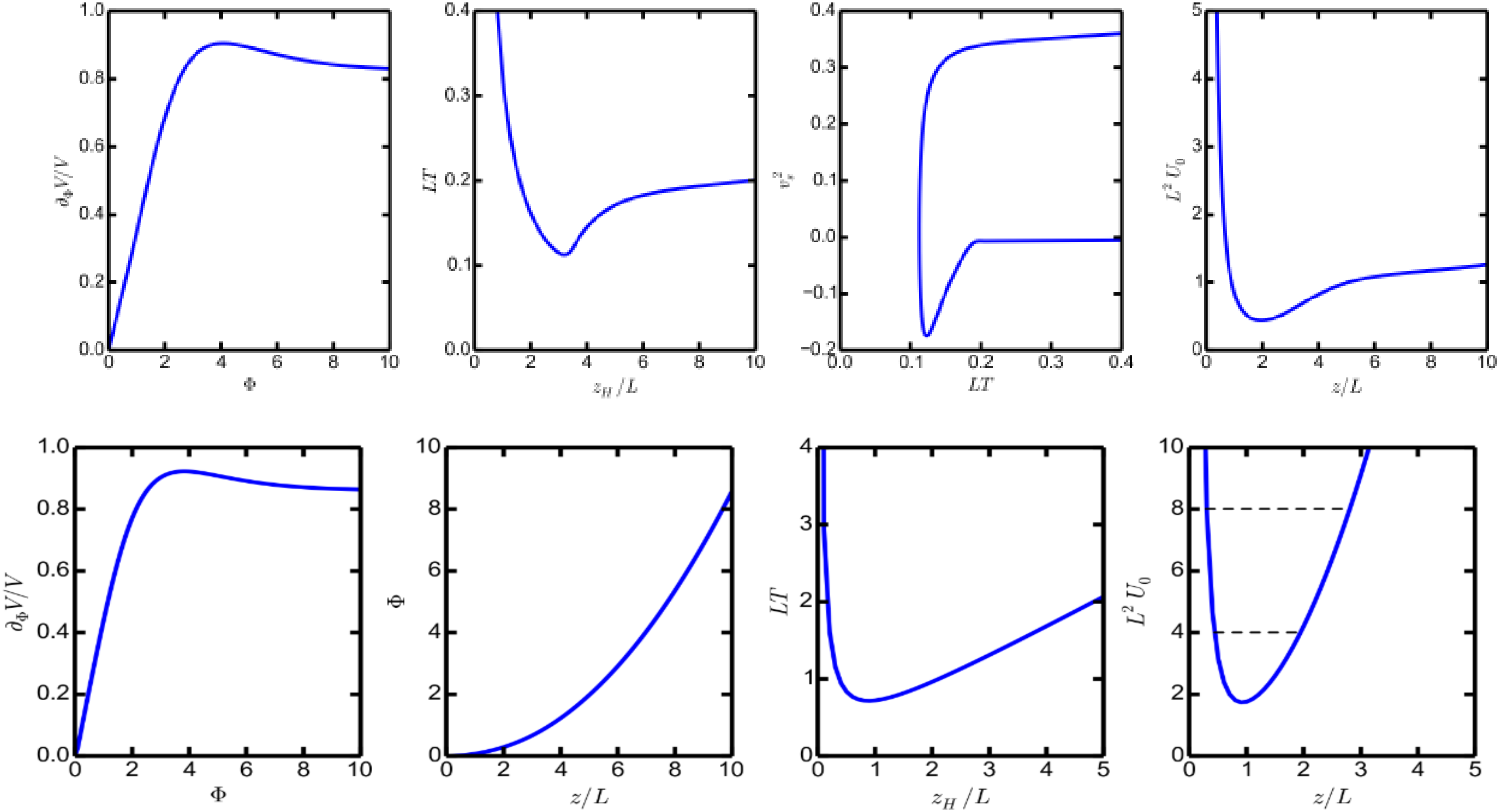
Second-order phase transition

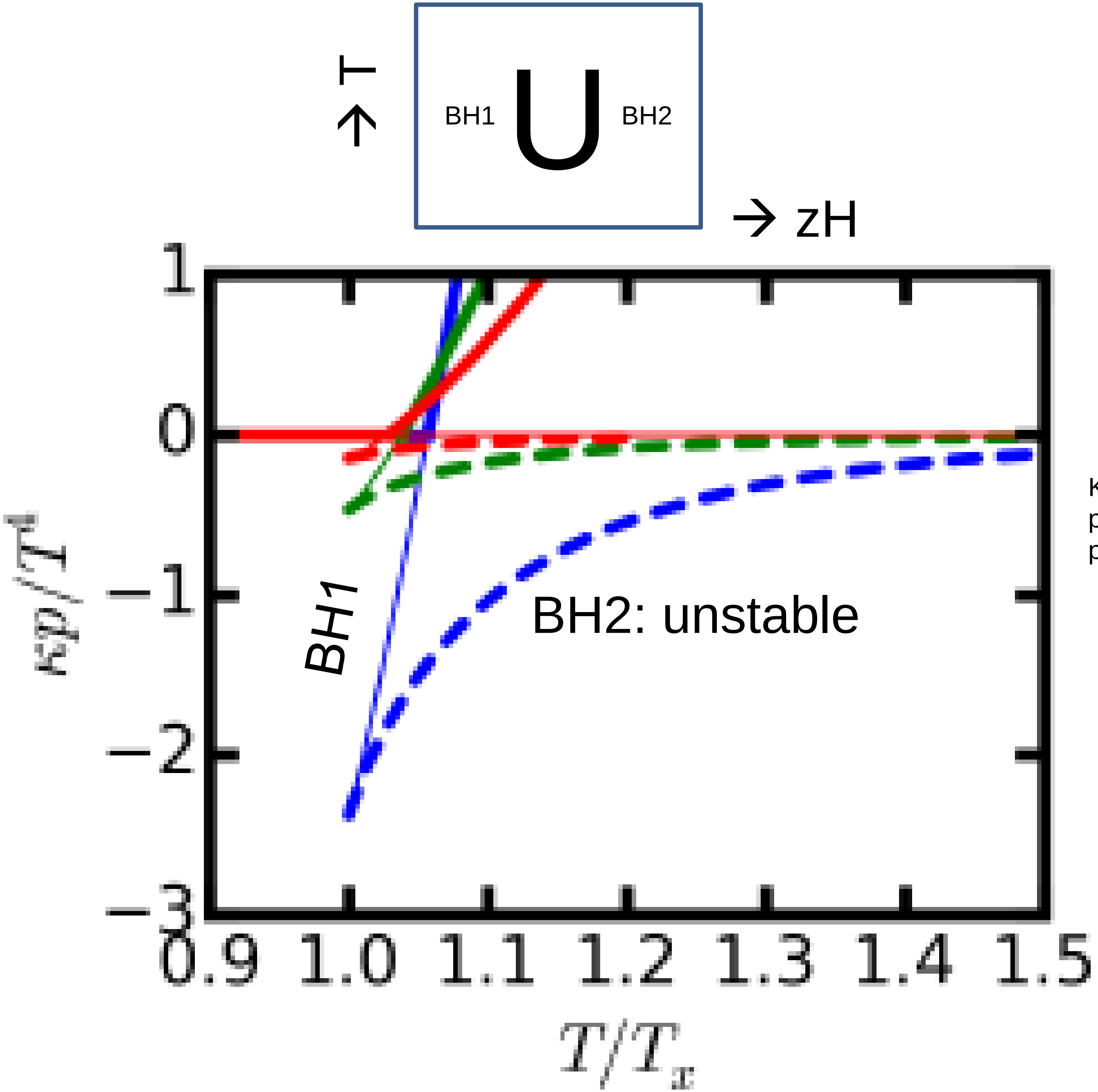


First-order phase transition

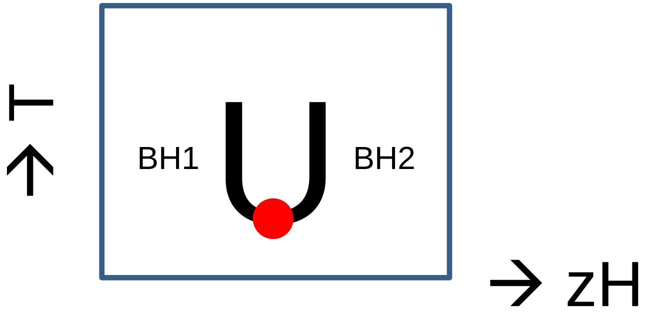


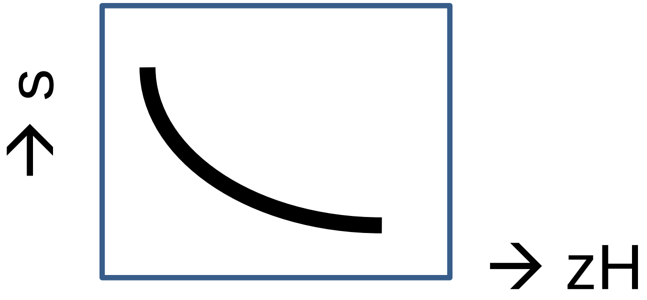
Hawking-Page



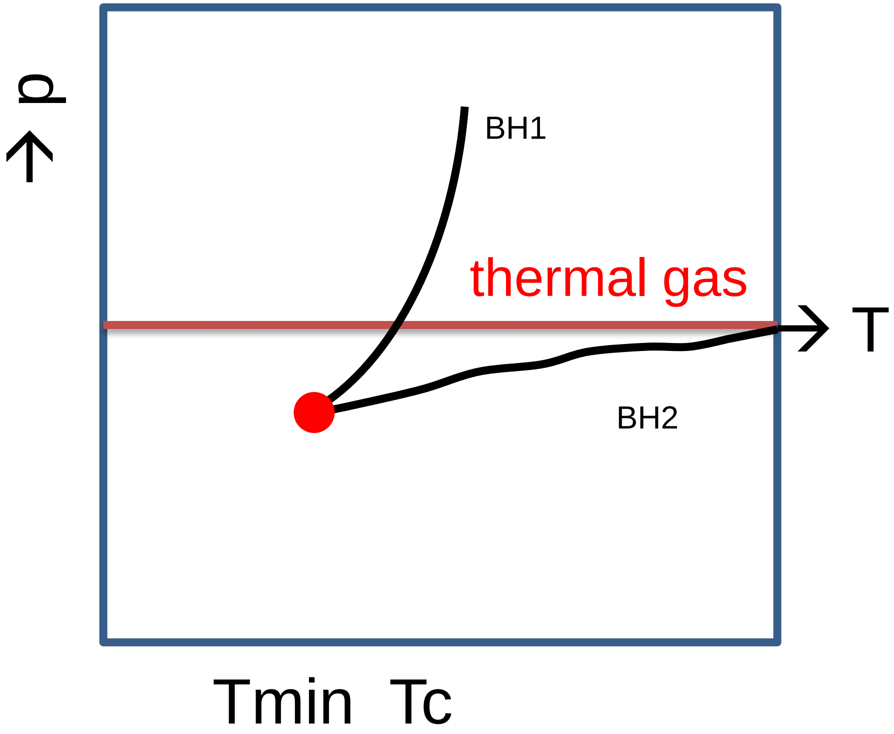


Kiritsis et al. (2008):
p(BH) ~ Nc^2
p(therm.gas) ~ O(Nc^0)

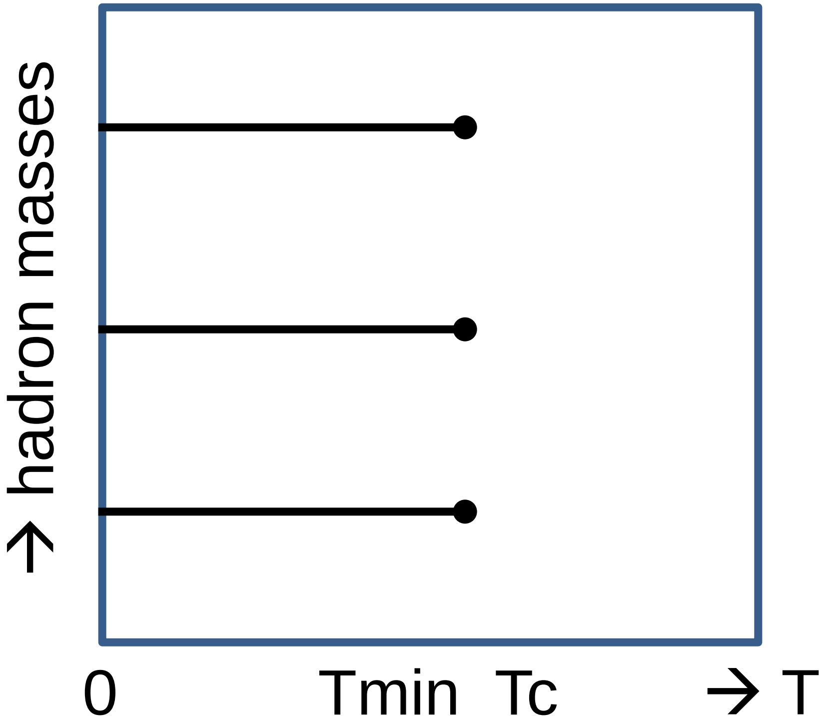




Hawking –Page as FOPT

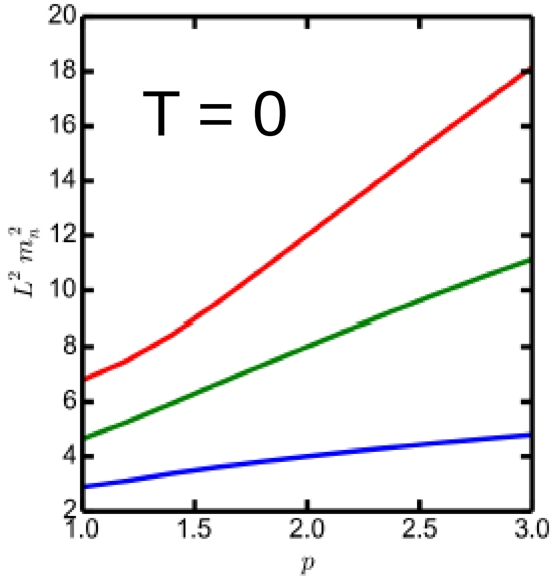


instant. melting



opposite way: start with

$$U_0 = \frac{3}{4z^2} + \left(\frac{z}{L}\right)^p$$



L. Die Dgl. $U_0 = \frac{1}{2}s'' + \frac{1}{4}s'^2$ besitzt die allgemeine Lösung

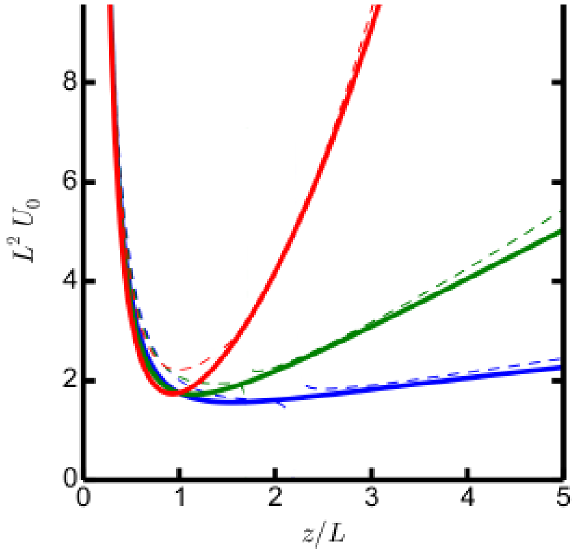
$$s = 2\ln\left(c_1 \hat{z}^{-\frac{1}{2}}\,{}_0F_1\left(\frac{p}{p+2}, \frac{\hat{z}^{p+2}}{(p+2)^2}\right) + c_2 \hat{z}^{\frac{3}{2}}\,{}_0F_1\left(\frac{p+4}{p+2}, \frac{\hat{z}^{p+2}}{(p+2)^2}\right)\right)$$

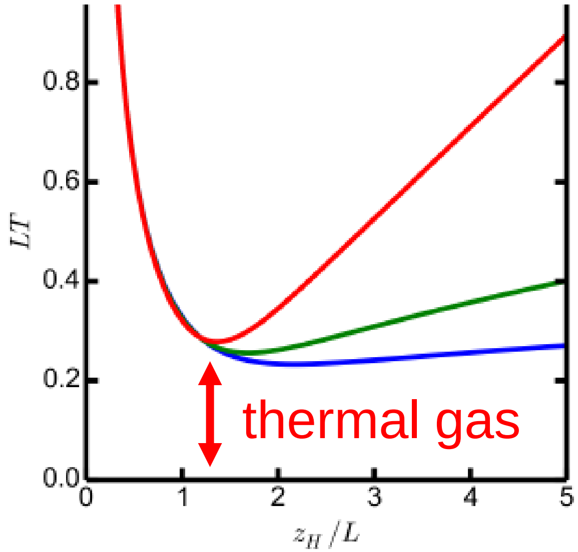
s = A/2 – 2 phi /3

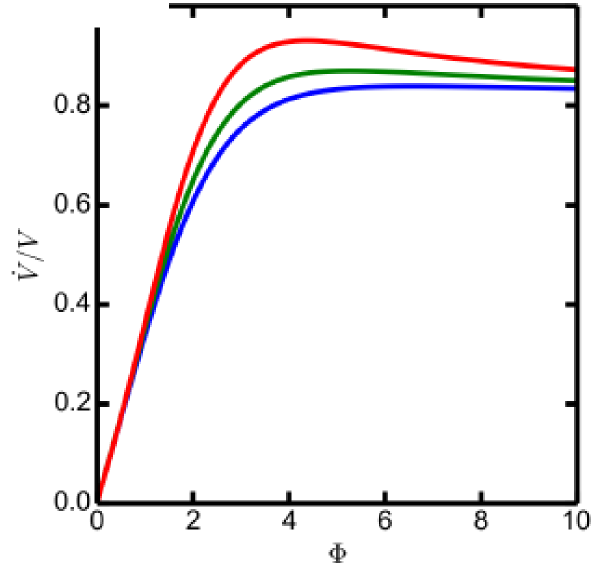
Wenn man U_0 global annimmt, folgt daraus eindeutig c_2 als

$$c_2 = -\frac{1}{2}\frac{\Gamma(\frac{p}{2+p})}{\Gamma(\frac{2}{p+2})}(p+2)^{\frac{p-2}{p+2}}$$

, c1 = 1







0.20

2 +1 QCD:
T_co = 155 MeV, m_rh0 = 777 MeV