Polarization and hydrodynamics with spin-1/2 particle

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Francesco Becattini, Bengt Friman, Amaresh Jaiswal, Avdhesh Kumar, Radosław Ryblewski, Rajeev Singh, Enrico Speranza

PRC97 (2018) 041901, Phys.Rev. D97 (2018) 116017, Phys.Rev. C98 (2018) 044906
Phys.Rev. C99 (2019) 011901, Phys.Lett. B789 (2019) 419,
Prog.Part.Nucl.Phys. 108 (2019) 103709 – review

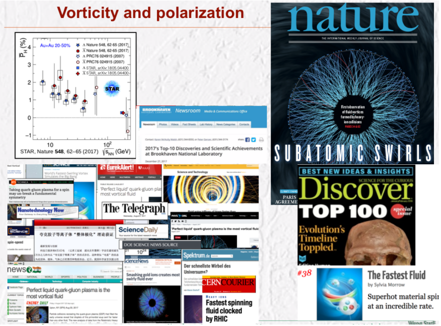
40th Max Born Symposium, Three Days on Strong Correlations in Dense Matter
University of Wroclaw, Poland, October 9th – 12th, 2019

PART 1: PHYSICS MOTIVATION

... the STAR experiment at Brookhaven National Laboratory (USA, Long Island) made the first positive measurements of spin polarization of Λ hyperons produced in relativistic heavy-ion collisions

www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever

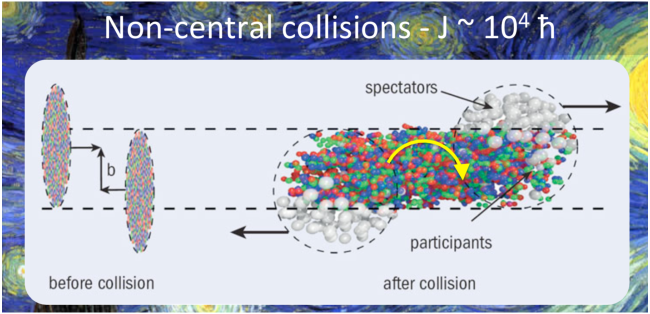
L. Adamczyk et al. (**STAR**), (2017), **Nature 548 (2017) 62-65**



(from Sergiei Voloshin's talk at „Hirschegg 2019 Workshop")

Non-central heavy-ion collisions create fireballs with large global angular momenta, some part of the angular momentum can be transferred from the orbital to the spin part

$$\boldsymbol{J}_{\rm init} = \boldsymbol{L}_{\rm init} = \boldsymbol{L}_{\rm final} + \boldsymbol{S}_{\rm final}$$

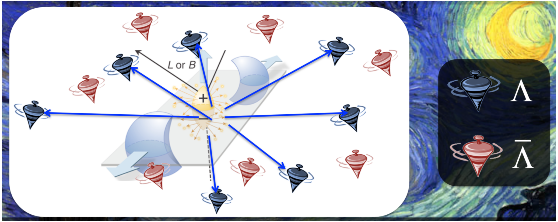


e. g. $\pi^+ + \pi^- \to \rho^0$

(Michael Lisa, talk „Strangeness in Quark Matter 2016")

The measurement of the weak decay of Λ brings the information about its spin polarization

Production of a polarized Lambda

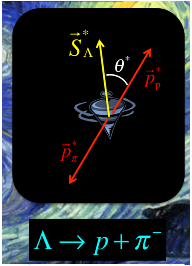


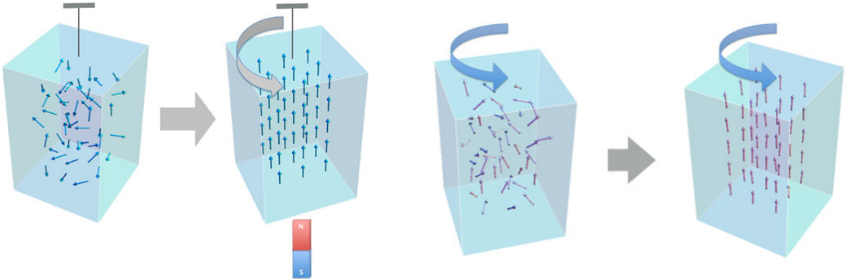
Polarization is measured through the analysis of the weak decay

$$\Lambda \rightarrow p + \pi^-$$

Proton prefers the emission direction that agrees with the spin orientation of Λ (in the rest frame of Λ)

(figures from Michael Lisa, talk at "Strangeness in Quark Matter 2016")



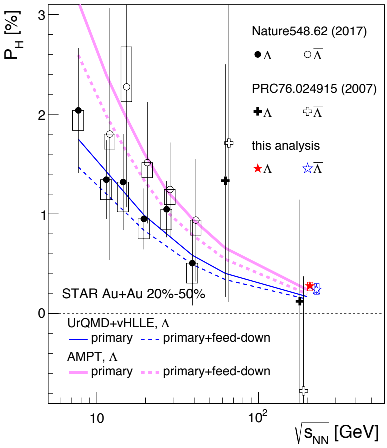


Einstein-de-Haas effect, 1915 (Richardson, 1908)
magnetization induces rotation

Barnett effect, 1915: rotation induces magnetization

Warning: the magnetic field aligns magnetic moments, those are opposite with respect to spin projection for particles and antiparticles, for systems with zero baryon number the magnetic field cannot induce the spin polarization

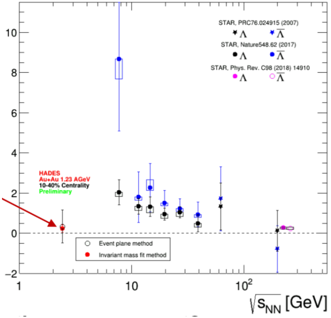
- polarization grows with decreasing beam energy, non-zero even for the highest RHIC energies
- within the exp. errors, the spin polarization is the same for particles and antiparticles — most likely, the observed effect has no connection to magnetic fields



(Takafumi Niida, arXiv:1808.10482, talk at "Quark Matter 2016")

New results from HADES

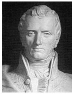
- Au+Au collisions at s_{NN} = 2.4 GeV
- no effect observed
- what really happens at low energies???

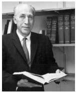


(Frederic Kornas, talk at "Hirschegg 2019 Workshop")

PART 2: HOW CAN WE INCLUDE SPIN POLARIZATION IN A HYDRODYNAMIC FRAMEWORK

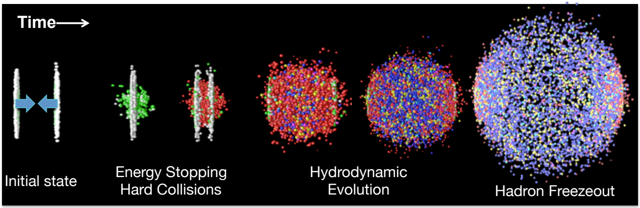
Claude-Louis Navier, 1785–1836, French engineer and physicist
Sir George Gabriel Stokes, 1819–1903, Irish physicist and mathematician





C. Eckart, Phys. Rev. 58 (1940) 919
L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York, 1959
10 out of 671 pages about relativistic hydrodynamics, mostly outdated
Eckart-Landau theory leads to acausal behavior


RELATIVISTIC HYDRODYNAMICS FORMS THE BASIC INGREDIENT OF THE STANDARD MODEL OF HEAVY-ION COLLISIONS



T. K. Nayak, Lepton-Photon 2011 Conference

significant progress done in Poland:
phenomenology: Piotr Bożek, Wojciech Broniowski, Radosław Ryblewski, ...
theory: Romuald Janik, Michał P. Heller, Michal Spaliński, Przemek Witaszczyk, ...

PERFECT-FLUID HYDRODYNAMICS = local equilibrium + conservation laws

one usually includes energy, linear momentum, baryon number, ...

T (temperature), u^μ (three independent components of flow), $\mu = T\xi$ (chemical potential)

$$\boxed{T^{\mu\nu} = [\varepsilon(T,\mu) + P(T,\mu)]\, u^\mu u^\nu - P(T,\mu) g^{\mu\nu}}$$

$$\boxed{\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu j^\mu = 0}$$

five equations for five unknown functions

dissipation does not appear

$$\boxed{\partial_\mu (s u^\mu) = 0}$$

entropy conservation follows from the energy-momentum conservation and the form of the energy-momentum tensor

FOR PARTICLES WITH SPIN, THE CONSERVATION OF ANGULAR MOMENTUM IS NOT TRIVIAL

new hydrodynamic variables should be introduced

$\Omega_{\mu\nu} = T\omega_{\mu\nu}$ (spin chemical potential = temperature $\times$ spin polarization tensor)

HYDRO WITH SPIN SHOULD EXPLAIN **SPACE-TIME CHANGES OF POLARIZATION**
MAY BE NOT POSSIBLE TO EXPLAIN THE ORIGINS OF SPIN POLARIZATION

Canonical energy-momentum $\widehat{T}^{\mu\nu}_{\rm can}$ and angular-momentum $\widehat{J}^{\mu,\lambda\nu}_{\rm can}$ tensors from the Noether Theorem:

$$\partial_\mu \widehat{T}^{\mu\nu}_{\rm can} \quad = \quad 0, \quad \partial_\mu \widehat{J}^{\mu,\lambda\nu}_{\rm can} = 0.$$

In general, the energy-momentum tensor is not symmetric

$$T^{\mu\nu}_{\rm can} \neq T^{\nu\mu}_{\rm can}$$

although classical $T^{\mu\nu}$ is always symmetric

$$T^{\mu\nu}_{\rm class} = \frac{\Delta p^\mu}{\Delta \Sigma_\nu} = \frac{\Delta p^\nu}{\Delta \Sigma_\mu} = T^{\nu\mu}_{\rm class} \quad \text{if} \quad \boldsymbol{v} = \frac{\boldsymbol{p}}{E} \quad \text{(1906 Planck)}$$

here $p^\mu = (E, \boldsymbol{p})$ is the four-momentum, while $\Delta\Sigma_\nu$ is a space-time volume element, so, for exmple

$$T^{10} = \frac{\Delta p^x}{\Delta x\, \Delta y\, \Delta z} = \frac{\Delta E\, \Delta x}{\Delta t\, \Delta x\, \Delta y\, \Delta z} = \frac{\Delta E}{\Delta t\, \Delta y\, \Delta z} = T^{01}$$

Total angular momentum $\widehat{J}^{\mu,\lambda\nu}_{\rm can}$ has orbital $\widehat{L}^{\mu,\lambda\nu}_{\rm can}$ and spin $\widehat{S}^{\mu,\lambda\nu}_{\rm can}$ parts:

$$\widehat{J}^{\mu,\lambda\nu}_{\rm can} = x^\lambda \widehat{T}^{\mu\nu}_{\rm can} - x^\nu \widehat{T}^{\mu\lambda}_{\rm can} + \widehat{S}^{\mu,\lambda\nu}_{\rm can} \equiv \widehat{L}^{\mu,\lambda\nu}_{\rm can} + \widehat{S}^{\mu,\lambda\nu}_{\rm can}$$

$$\partial_\mu \widehat{J}^{\mu,\lambda\nu}_{\rm can} = \widehat{T}^{\lambda\nu}_{\rm can} - \widehat{T}^{\nu\lambda}_{\rm can} + \partial_\mu \widehat{S}^{\mu,\lambda\nu}_{\rm can} = 0, \qquad \partial_\mu \widehat{S}^{\mu,\lambda\nu}_{\rm can} = \widehat{T}^{\nu\lambda}_{\rm can} - \widehat{T}^{\lambda\nu}_{\rm can}$$

Antisymmetry in the last two indices:

$$\widehat{J}^{\mu,\lambda\nu}_{\rm can} = -\widehat{J}^{\mu,\nu\lambda}_{\rm can}, \quad \widehat{L}^{\mu,\lambda\nu}_{\rm can} = -\widehat{L}^{\mu,\nu\lambda}_{\rm can}, \quad \widehat{S}^{\mu,\lambda\nu}_{\rm can} = -\widehat{S}^{\mu,\nu\lambda}_{\rm can}$$

Pseudo-gauge transformation

(different localization of energy density and angular momentum, global charges not changed)

$$\begin{aligned} \widehat{T}'^{\mu\nu} &= \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu} \right), \\ \widehat{S}'^{\lambda,\mu\nu} &= \widehat{S}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu} \end{aligned}$$

Belinfante's construction (Rosenfeld): superpotential defined as $\widehat{\Phi} = \widehat{S}^{\lambda,\mu\nu}_{\rm can}$

$$\widehat{T}^{\mu\nu}_{\rm Bel} = \widehat{T}^{\mu\nu}_{\rm can} + \frac{1}{2}\partial_\lambda \left(\widehat{S}^{\lambda,\mu\nu}_{\rm can} - \widehat{S}^{\mu,\lambda\nu}_{\rm can} - \widehat{S}^{\nu,\lambda\mu}_{\rm can} \right), \quad \widehat{S}^{\lambda,\mu\nu}_{\rm Bel} = 0$$

in this talk the canonical tensors are considered

physical system under consideration: hadronic gas (Λ hyperons + ...)

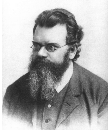
Dirac's equation treated as an effective description of baryons with spin 1/2

no EM fields included

SPIN TENSOR IS KEPT AND USED
ALSO BY THE COMMUNITY THAT STUDIES PROTON'S SPIN

Connection of spin polarization to vorticity

Local-equilibrium density operator (Boltzmann → Zubarev, Becattini, ...)



Canonical operators

$$e^{-(E-\mu)/T} \longrightarrow e^{-p\cdot\beta(x)+\xi(x)} \longrightarrow \widehat{\rho}_{\rm LEQ} = \frac{1}{Z}\exp\left[-\int_\Sigma d\Sigma_\mu \left(\widehat{T}^{\mu\nu}_{\rm can}\beta_\nu - \frac{1}{2}\omega_{\lambda\nu}\widehat{J}^{\mu,\lambda\nu}_{\rm can} - \xi\widehat{j}^{\mu}\right)\right]$$

Σ is a space-like hypersurface, for example, corresponding to a constant LAB time t, in this case $\widehat{\rho}_{\rm LEQ} = \widehat{\rho}_{\rm LEQ}(t)$

β_ν is the ratio between the local four-velocity u^μ and temperature T (a four-temperature vector)

an antisymmetric tensor field $\omega_{\lambda\nu}$ is the **spin polarization tensor**

Connection of spin polarization to vorticity

Global equilibrium

in global equilibrium we require that $\widehat{\rho}_{\rm LEQ}$ = const.

$\widehat{T}^{\mu\nu}_{\rm can} \neq \widehat{T}^{\nu\mu}_{\rm can} \quad \longrightarrow \beta_\mu$ **satisfies the Killing equation**
at the same time, spin polarization is given by thermal vorticity $\varpi_{\lambda\nu}$

$$\partial_\lambda \beta_\nu + \partial_\nu \beta_\lambda = 0, \qquad \varpi_{\lambda\nu} = -\frac{1}{2}\left(\partial_\lambda \beta_\nu - \partial_\nu \beta_\lambda\right), \qquad \varpi_{\lambda\nu} = \omega_{\lambda\nu} = \mathrm{const.}, \quad \xi = \mathrm{const.}$$

solution of the Killing equation:

$$\beta_\mu = \beta^0_\mu + \varpi_{\mu\nu} x^\nu, \quad \beta^0_\mu = \mathrm{const}, \quad \varpi_{\mu\nu} = -\varpi_{\nu\mu} = \mathrm{const}$$

uniform motion, rigid rotation (special boundary conditions)
constant acceleration along the stream lines
special case of the Tolman-Klein conditions for thermodynamic equilibrium of fluids in gravitational fields

Local equilibrium – first way

based on the works by F. Becattini and collaborators
also in local equilibrium $\omega_{\lambda\nu}(x) = \varpi_{\lambda\nu}(x)$

1) Run any type of hydro, perfect or viscous, or transport, or whatsoever, without spin

2) Find $\beta_\mu(x) = u_\mu(x)/T(x)$ on the freeze-out hypersurface
(defined often by the condition T=const)

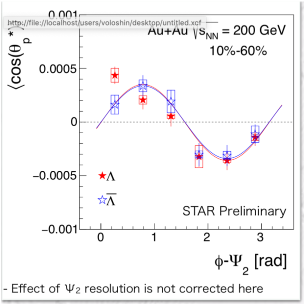
3) Calculate thermal vorticity $\varpi_{\alpha\beta}(x) \neq$ const

4) Identify thermal vorticity with the spin polarization tensor $\omega_{\mu\nu}$

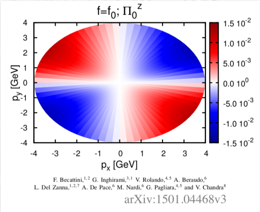
5) Make predictions about spin polarization

**SUCH A METHOD WORKS WELL,
DESCRIBES MOST OF THE DATA, BUT...**

...CAN WE TAKE IT FOR GRANTED?
THERE ARE PROBLEMS WITH MORE DETAILED DESCRIPTION



Hydro:



Exp: opposite sign to hydro predictions

Local equilibrium – second way

in local equilibrium $\widehat{\rho}_{\rm LEQ}$ is approximately constant (with dissipation effects neglected)

$$T^{\mu\nu} = \mathrm{tr}\left(\widehat{\rho}_{\rm LEQ}\,\widehat{T}^{\mu\nu}\right), \quad S^{\mu,\lambda\nu} = \mathrm{tr}\left(\widehat{\rho}_{\rm LEQ}\,\widehat{S}^{\mu,\lambda\nu}\right), \quad j^{\mu} = \mathrm{tr}\left(\widehat{\rho}_{\rm LEQ}\,\widehat{j}^{\mu}\right)$$

these tensors are all functions of the hydrodynamic variables β_μ, $\omega_{\mu\nu}$, and ξ

$$T^{\mu\nu} = T^{\mu\nu}[\beta,\omega,\xi], \quad S^{\mu,\lambda\nu} = S^{\mu,\lambda\nu}[\beta,\omega,\xi], \quad j^{\mu} = j^{\mu}[\beta,\omega,\xi]$$

and satisfy the conservation laws

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}, \quad \partial_\mu j^{\mu} = 0$$

Spin dependent phase-space distribution functions

standard scalar functions $f(x,p)$ are generalized to 2×2 Hermitean matrices in spin space for each value of the space-time position x and four-momentum p

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Annals Phys. 338 (2013) 32

$$
\begin{aligned}
f^{+}_{rs}(x,p) &= \bar{u}_r(p) X^{+} u_s(p) \\
f^{-}_{rs}(x,p) &= -\bar{v}_s(p) X^{-} v_r(p)
\end{aligned}
$$

$$
X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x) p^\mu\right] M^{\pm}, \quad M^{\pm} = \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]
$$

here $\Sigma^{\mu\nu}$ is the Dirac spin operator, electric- and magnetic-like three-vectors

$$
\omega_{\mu\nu} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix}
$$

special case in this talk

$$
M^{\pm} = 1 \pm \frac{1}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}
$$

Spin dependent phase-space distribution functions

The spin observables are represented by the Pauli matrices $\boldsymbol{\sigma}$ and the expectation values of $\boldsymbol{\sigma}$ provide information on the polarization of spin-1/2 particles in their rest frame

$$f^{\pm}(x,p) = e^{\pm\xi - p\cdot\beta}\left[1 - \frac{1}{2}\,\boldsymbol{P}\cdot\boldsymbol{\sigma}\right]$$

average polarization per particle

$$\boldsymbol{P} \quad = \quad \frac{1}{m}\left[E_p\,\boldsymbol{b} - \boldsymbol{p}\times\boldsymbol{e} - \frac{\boldsymbol{p}\cdot\boldsymbol{b}}{E_p+m}\boldsymbol{p}\right] = \boldsymbol{b}_* \quad (\boldsymbol{b}\text{ field in the particle rest frame})$$

$$\langle \boldsymbol{P}(x,p)\rangle = \frac{1}{2}\,\frac{\mathrm{tr}_2\left[(f^+ + f^-)\boldsymbol{\sigma}\right]}{\mathrm{tr}_2\left[f^+ + f^-\right]} = -\frac{1}{4}\boldsymbol{P}$$

relativistic spin is a very subtle concept, people use helicity = spin projection along momentum
here we deal with a canonical spin of massive particles, defined in the particle rest frame
„Spin in relativistic quantum theory" W.N. Polyzou, W. Glöckle, H. Witala, Few Body Syst. 54 (2013) 1667

x - space-time position, k - off-shell momentum, p on-shell momentum
De Groot, van Leeuwen, van Weert: *Relativistic Kinetic Theory. Principles and Applications*

GLW framework

$$\mathcal{W}^{+}_{\rm eq}(x,k) = \frac{1}{2}\sum_{r,s=1}^{2}\int dP\,\delta^{(4)}(k-p)u^{r}(p)\bar{u}^{s}(p)f^{+}_{rs}(x,p)$$

$$\mathcal{W}^{-}_{\rm eq}(x,k) = -\frac{1}{2}\sum_{r,s=1}^{2}\int dP\,\delta^{(4)}(k+p)v^{s}(p)\bar{v}^{r}(p)f^{-}_{rs}(x,p)$$

Clifford-algebra expansion

(used in many early works on QED and QGP plasma, e.g., H.T. Elze, M. Gyulassy, D. Vasak, Phys.Lett. B177 (1986) 402)

$$\mathcal{W}^{\pm}(x,k) \quad = \quad \frac{1}{4}\left[\mathcal{F}^{\pm}(x,k) + i\gamma_5\mathcal{P}^{\pm}(x,k) + \gamma^{\mu}\mathcal{V}^{\pm}_{\mu}(x,k) + \gamma_5\gamma^{\mu}\mathcal{A}^{\pm}_{\mu}(x,k) + \Sigma^{\mu\nu}\mathcal{S}^{\pm}_{\mu\nu}(x,k)\right]$$

Global-equilibrium Wigner function

WF, A. Kumar, R. Ryblewski, Phys. Rev. C98 (2018) 044906

$$\left(\gamma_\mu K^\mu - m\right)\mathcal{W}(x,k) = C[\mathcal{W}(x,k)]$$

Here K^μ is the operator defined by the expression

$$K^\mu = k^\mu + \frac{i\hbar}{2}\,\partial^\mu$$

In the case of global equilibrium, with the vanishing collision term, the Wigner function $\mathcal{W}(x,k)$ exactly satisfies the equation

$$\left(\gamma_\mu K^\mu - m\right)\mathcal{W}(x,k) = 0$$

the leading order terms in $\hbar$ can be taken from Becattini's $\mathcal{W}^{\pm}_{\rm eq}(x,k)$

in the NLO in $\hbar$ we get the kinetic equation (well known in the literature)

$$k^\mu \partial_\mu \mathcal{F}_{\rm eq}(x,k) = 0$$

$$k^\mu \partial_\mu \mathcal{A}^\nu_{\rm eq}(x,k) = 0, \quad k_\nu \mathcal{A}^\nu_{\rm eq}(x,k) = 0$$

Global equilibrium — these equations are exactly fulfilled, what about local equilibrium

Local equilibrium — only moments of the kinetic equations are satisfied, standard method for going from the kinetic-theory description to hydrodynamics

$$\partial_\alpha N^\alpha_{\rm eq}(x) = 0, \quad \partial_\alpha T^{\alpha\beta}_{\rm GLW}(x) = 0, \quad \partial_\lambda S^{\lambda,\mu\nu}_{\rm GLW}(x) = 0$$

GLW — forms proposed by de Groot, van Leeuwen, van Weert

EQUATIONS DEFINED ABOVE ARE EXACTLY THE HYDRODYNAMIC EQUATIONS WE HAVE BEEN LOOKING FOR
11 equations for 11 unknown functions of space and time

LO generates corrections in the NLO

$$\mathcal{P}^{(1)} = -\frac{1}{2m}\,\partial^\mu \mathcal{A}_{\mathrm{eq},\mu}$$

$$\mathcal{V}^{(1)}_\mu = -\frac{1}{2m}\partial^\nu \mathcal{S}_{\mathrm{eq},\nu\mu}$$

$$\mathcal{S}^{(1)}_{\mu\nu} = \frac{1}{2m}\left(\partial_\mu \mathcal{V}_{\mathrm{eq},\nu} - \partial_\nu \mathcal{V}_{\mathrm{eq},\mu}\right)$$

IMPORTANT IF the canonical formalism is used

$$T^{\mu\nu}_{\mathrm{GLW}}(x) = \frac{1}{m}\mathrm{tr}_4 \int d^4k\, k^\mu\, k^\nu \mathcal{W}(x,k) = \frac{1}{m}\int d^4k\, k^\mu\, k^\nu \mathcal{F}(x,k)$$

$$T^{\mu\nu}_{\mathrm{can}}(x) = \int d^4k\, k^\nu \mathcal{V}^\mu(x,k)$$

quantum corrections induce asymmetry $T^{\mu\nu}_{\mathrm{can}}(x) \neq T^{\nu\mu}_{\mathrm{can}}(x)$

Including the components of $\mathcal{V}^{\mu}(x,k)$ up to the first order in the equilibrium case we obtain

$$T^{\mu\nu}_{\rm can}(x) = T^{\mu\nu}_{\rm GLW}(x) + \delta T^{\mu\nu}_{\rm can}(x)$$

where

$$\delta T^{\mu\nu}_{\rm can}(x) = -\frac{\hbar}{2m}\int d^4k k^{\nu}\partial_{\lambda}\mathcal{S}^{\lambda\mu}_{\rm eq}(x,k) = -\partial_{\lambda}S^{\nu,\lambda\mu}_{\rm GLW}(x)$$

The canonical energy-momentum tensor is conserved

$$\partial_{\alpha}T^{\alpha\beta}_{\rm can}(x) = 0$$

$$\begin{aligned} S^{\lambda,\mu\nu}_{\rm can} &= \hbar\cosh(\xi)\int dP\, e^{-\beta\cdot p}\left(\omega^{\mu\nu}p^{\lambda} + \omega^{\nu\lambda}p^{\mu} + \omega^{\lambda\mu}p^{\nu}\right) \\ &= S^{\lambda,\mu\nu}_{\rm GLW} + S^{\mu,\nu\lambda}_{\rm GLW} + S^{\nu,\lambda\mu}_{\rm GLW} \end{aligned}$$

The canonical spin tensor is not conserved!

$$\partial_{\lambda}S^{\lambda,\mu\nu}_{\rm can}(x) = T^{\nu\mu}_{\rm can} - T^{\mu\nu}_{\rm can} = -\partial_{\lambda}S^{\mu,\lambda\nu}_{\rm GLW}(x) + \partial_{\lambda}S^{\nu,\lambda\mu}_{\rm GLW}(x)$$

if we introduce the tensor $\Phi_{\rm can}^{\lambda,\mu\nu}$ defined by the relation

$$\Phi_{\rm can}^{\lambda,\mu\nu} \equiv S_{\rm GLW}^{\mu,\lambda\nu} - S_{\rm GLW}^{\nu,\lambda\mu}$$

we can write

$$S_{\rm can}^{\lambda,\mu\nu} = S_{\rm GLW}^{\lambda,\mu\nu} - \Phi_{\rm can}^{\lambda,\mu\nu}$$

and

$$T_{\rm can}^{\mu\nu} = T_{\rm GLW}^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi_{\rm can}^{\lambda,\mu\nu} + \Phi_{\rm can}^{\mu,\nu\lambda} + \Phi_{\rm can}^{\nu,\mu\lambda}\right)$$

The canonical and GLW frameworks are connected by a pseudo-gauge transformation. Similarly to Belinfante, it leads to a symmetric energy-momentum tensor, however, does not eliminate the spin degrees of freedom.

we introduce the phase-space density $\Delta\Pi_\mu$ of the **Pauli-Lubański vector**

$$E_p \frac{d\Delta\Pi_\mu(x,p)}{d^3p} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\,\Delta\Sigma_\lambda(x)\,E_p\frac{dJ^{\lambda,\nu\alpha}(x,p)}{d^3p}\,\frac{p^\beta}{m}$$

only the spin-part contributes here, the results are the same for the canonical and GLW versions dividing by the total density of particles and antiparticles, we find

$$\pi_\mu(x,p) \equiv \frac{\Delta\Pi_\mu(x,p)}{\Delta\mathcal{N}(x,p)} \quad = \quad -\frac{\hbar}{4m}\,\tilde{\omega}_{\mu\beta}\,p^\beta$$

in PRF

$$\pi^0_* = 0, \qquad \boldsymbol{\pi}_* = -\frac{\hbar}{4}\boldsymbol{P}$$

This is an important result showing that the space part of the PL vector in PRF agrees with the mean polarization three-vector

Simulations done by R. Ryblewski

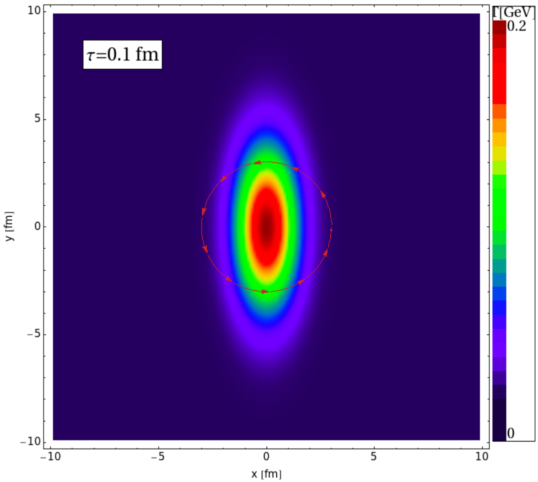
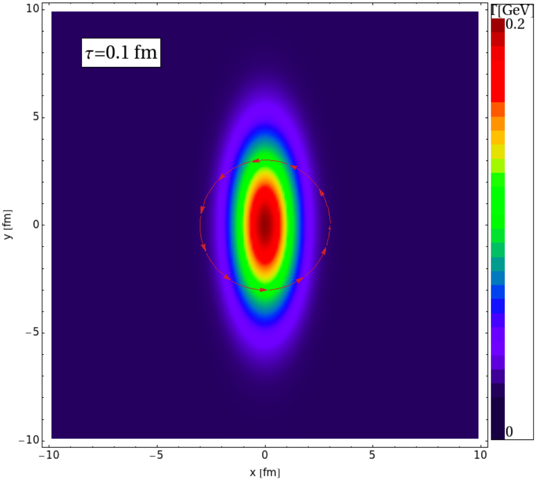


Figure: Initial conditions for a quasi-realistic model

Quasi-realistic model for low-energy collisions



1. Foundations for the theoretical framework of perfect-fluid hydrodynamics with spin have been laid.
2. Relations between different definitions of the energy-momentum tensors have been clarified in this context.
3. Intensive work is going on now aiming at the numerical implementation of our ideas and making more detailed comparisons with the experimental data.
4. More conceptual work is needed to include dissipation.

more in Prog.Part.Nucl.Phys. 108 (2019) 103709

DEAR DAVID, ALL THE BEST ON THE OCCASION OF YOUR 60TH BIRTHDAY!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2021
29TH INTERNATIONAL
CONFERENCE ON ULTRARELATIVISTIC
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CONGRESS CENTRE ICE KRAKÓW, POLAND

PART 3: BACK-UP SLIDES

Back-up slides 1: Jan Weyssenhoff's circle

cc

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CRACOW INVESTIGATIONS IN THE THEORY OF RELATIVISTIC SPIN PARTICLES

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Abstract

The first papers of M. Mathisson on the theory of relativistic spin particles, written in early thirties in Warsaw are presented. Then the beginnings of the work on this theory in Cracow, performed by J. Weyssenhoff, M. Mathisson and their collaborators in the last years before the outbreak of the II World War is described. Further the development of this theory by Weyssenhoff and A. Raabe during the years of the war is discussed. Their work demonstrated the similarity of the properties of relativistic spin particles with those of Dirac's electron. After the end of the war Weyssenhoff and his collaborators, B. Średniawa, Z. Borelowski and A. Białas studied the properties of spin particles, working especially on the canonical formalism of this theory and on the radiation and motion of magnetic dipoles. The research in these problems lasted in Cracow until the late sixties.

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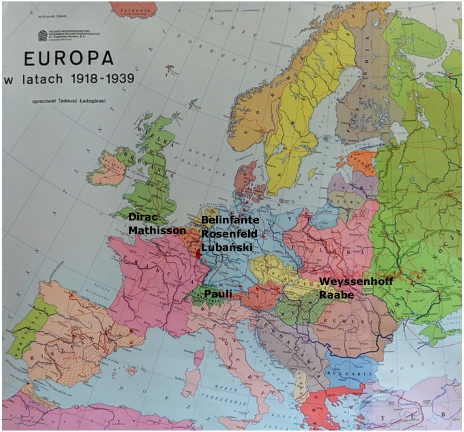

Jan Weyssenhoff
1889-1972

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J. K. LUBAŃSKI

Obituary notice by L. ROSENFELD, Manchester.

The circle of Lubański's friends was not large. During the wartime which was so difficult for him he lived very retired and he was rather shy and taciturn. But those who were in closer contact with him have been able to discover his sensitive and refined personality. After the liberation a striking change came over him, that made us only then understand how much he had suffered during the war. He surprised us by an unknown alacrity and optimism. His new task in Delft gave him great satisfaction and he fulfilled it with an enthusiasm and energy that authorised the greatest expectations for his scientific and personal future. This made the shock the more violent for his friends when the news reached them of his unexpected death on the 8-th December 1946, after only a very short illness.

Jozeph Kazimir Lubański was born in 1914 in Rumania from Polish parents. He spent his youth in Russia; only in 1926 did he come to Poland where he studied Physics at the Universities of Wilno and Kraków. In Kraków he worked under the direction of the very original theoretician Mathisson (who died in England during the war); his first paper in 1937 is based on Mathisson's theories. Until the autumn of 1938 Lubański was assistant at the Institute of Theoretical Physics at Wilno. In December 1938 he came to Leiden with a stipend from the Polish Government to work under the direction of Professor Kramers. Since that time he lived in Holland, where he was greatly helped during the war by the Lorentz Fund. As a Polish citizen he was forced already in 1940 to leave the coastal region and after a short stay in the country-side he settled in Utrecht, where he stayed until his appointment at Delft in October 1945.

In Leiden he worked with Kramers and Belinfante on the Theory of Particles with arbitrary spin. His investigations on this subject are set-out in three papers, published in the Dutch journal Physica. These papers witness his profound knowledge of the abstract theories of modern algebra and his mastery in applying them to fundamental problems of theoretical physics. The same qualities characterize his further publications which contained the results of the work he did in Utrecht.

Back-up slides 2: classical treatment of spin

internal angular momentum tensor $s^{\alpha\beta}$

M. Matthison, Neue mechanik materieller systemes, Acta Phys. Polon. 6 (1937) 163.

$$s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta} p_\gamma s_\delta. \tag{1}$$

$$s \cdot p = 0, \quad s^\alpha = \frac{1}{2m}\epsilon^{\alpha\beta\gamma\delta} p_\beta s_{\gamma\delta} \tag{2}$$

A straightforward generalization of the phase-space distribution function $f(x, \boldsymbol{p})$ is a spin dependent distribution $f(x, \boldsymbol{p}, s)$

$$\int dS \ldots = \frac{m}{\pi \mathfrak{s}} \int d^4 s \, \delta(s \cdot s + \mathfrak{s}^2) \, \delta(p \cdot s) \ldots \tag{3}$$

$$\mathfrak{s}^2 = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4} \tag{4}$$

$$\int dS = \frac{m}{\pi \mathfrak{s}} \int d^4 s \, \delta(s \cdot s + \mathfrak{s}^2) \, \delta(p \cdot s) = 2 \tag{5}$$

equilibrium distribution functions for particles and antiparticles in the form

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x)s^{\alpha\beta}\right). \tag{6}$$

conserved "currents"

$$N^{\mu}_{\rm eq} = \int dP \int dS \; p^{\mu} \left[f^{+}_{\rm eq}(x,p,s) - f^{-}_{\rm eq}(x,p,s)\right], \tag{7}$$

$$T^{\mu\nu}_{\rm eq} \quad = \quad \int dP \int dS \; p^{\mu} p^{\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right] \tag{8}$$

$$S^{\lambda,\mu\nu}_{\rm eq} \quad = \quad \int dP \int dS \; p^{\lambda} \, s^{\mu\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right] \tag{9}$$

For $|\omega_{\mu\nu}| < 1$ we obtain the formalism that agrees with that based on the quantum description of spin (in the GLW version)

PL vector can be expressed by the simple expression

$$\pi_\mu = -\mathfrak{s}\,\frac{\tilde{\omega}_{\mu\beta}}{P}\,\frac{p^\beta}{m}\,L(P\,\mathfrak{s}), \tag{10}$$

where $L(x)$ is the Langevin function defined by the formula

$$L(x) = \coth(x) - \frac{1}{x}. \tag{11}$$

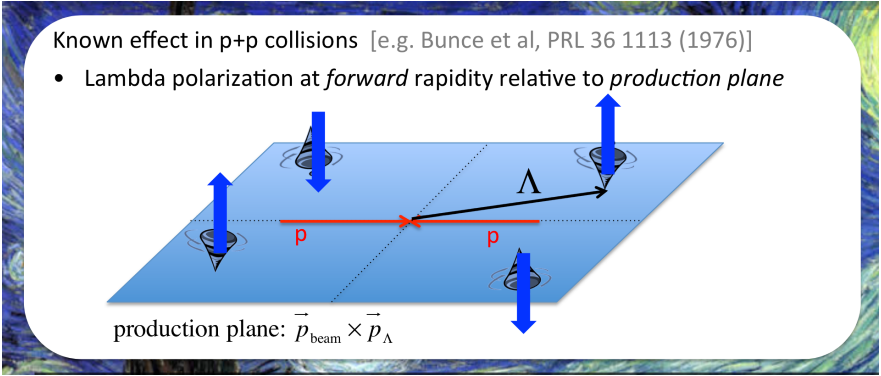
in PRF the direction of the PL vector agrees with that of the polarization vector $\boldsymbol{P}$. For small and large P we obtain two important results:

$$\boldsymbol{\pi}_* = -\mathfrak{s}\,\frac{\boldsymbol{P}}{P}, \quad |\boldsymbol{\pi}_*| = \mathfrak{s} = \sqrt{\frac{3}{4}}, \quad \text{if} \quad P \gg 1 \tag{12}$$

and

$$\boldsymbol{\pi}_* = -\mathfrak{s}^2\,\frac{\boldsymbol{P}}{3}, \quad |\boldsymbol{\pi}_*| = \mathfrak{s}^2\,\frac{P}{3} = \frac{P}{4}, \quad \text{if} \quad P \ll 1. \tag{13}$$

Back-up slides 3: *pp* collisions



no integrated effect at midrapidity

Back-up slides 4: GLW expressions

charge current

$$N^{\alpha}_{\rm GLW} = n u^{\alpha}, \qquad n = 4\,\sinh(\xi)\,n_{(0)}(T)$$

energy-momentum tensor (with a perfect-fluid form)

$$T^{\mu\nu}_{\rm GLW}(x) \quad = \quad (\varepsilon + P) u^{\mu} u^{\nu} - P g^{\mu\nu}$$

$$\varepsilon = 4\,\cosh(\xi)\,\varepsilon_{(0)}(T)\,, \quad P = 4\,\cosh(\xi)\,P_{(0)}(T)$$

$n_{(0)}(T)$, $\varepsilon_{(0)}(T)$, $P_{(0)}(T)$ — particle density, energy density, and pressure of classical particles at the temperature T

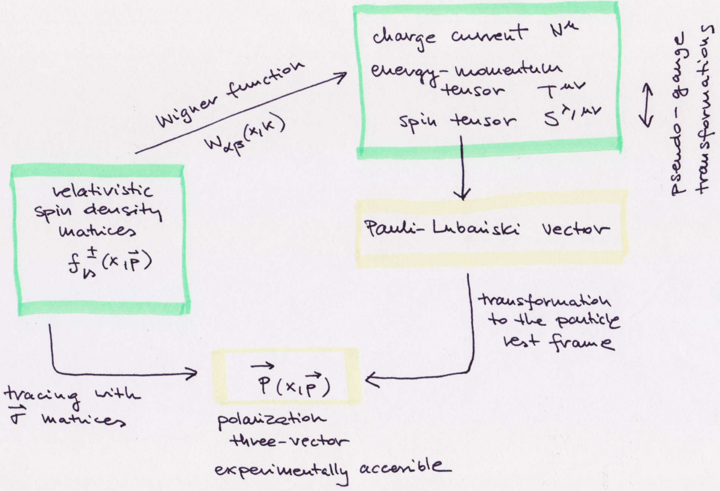
spin tensor

$$S^{\lambda,\mu\nu}_{\rm GLW} \quad = \quad \frac{\hbar \cosh(\xi)}{m^2} \int dP\, e^{-\beta\cdot p} p^{\lambda} \left(m^2 \omega^{\mu\nu} + 2 p^{\alpha} p^{[\mu} \omega^{\nu]}{}_{\alpha} \right) = S^{\lambda,\mu\nu}_{\rm ph} + S^{\lambda,\mu\nu}_{\Delta}.$$

only $S^{\lambda,\mu\nu}_{\rm ph}$ was used in WF, B. Friman, A. Jaiswal, E.Speranza, Phys.Rev. C97 (2018) 041901

Back-up slides 5: consistency checks

consistency checks 1



relativistic spin density matrices $f^{\pm}_{rs}(x,\vec{p})$

kinetic equations

for the Wigner function coefficients

$$k^{\mu}\partial_{\mu}\mathcal{F}_{eq}(x,k)=0$$
$$k^{\mu}\partial_{\mu}\mathcal{A}^{\nu}_{eq}(x,k)=0$$
$$k_{\nu}\mathcal{A}^{\nu}_{eq}(x,k)=0$$

moments (integrals over k)

$$\partial_{\mu}N^{\mu}=0$$
$$\partial_{\mu}T^{\mu\nu}_{GLW}=0$$
$$\partial_{\lambda}S^{\lambda,\mu\nu}_{GLW}=0$$

hydrodynamic equations

classical spin $f(x,\vec{p},s)$

$s^2=\frac{1}{2}(1+\frac{1}{2})$

$|\omega_{\mu\nu}|<1$

arbitrary $\omega_{\mu\nu}$

anisotropic hydrodynamics with spin