

Effective Coupling Constant of Plasmons

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The lady's man

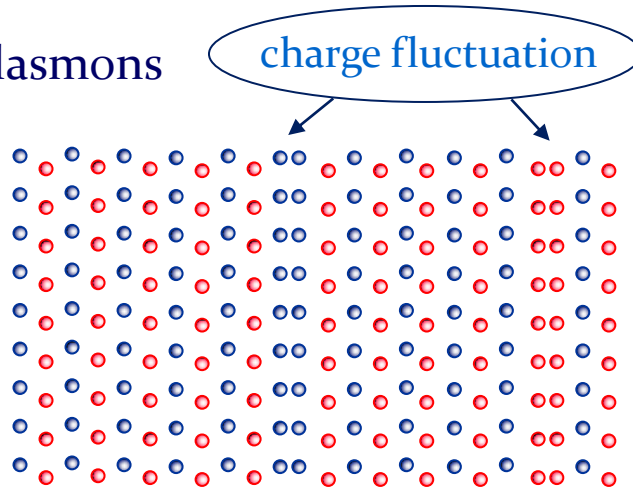
Happy Birthday!



Background: plasmons

Longitudinal plasmons

$\mathbf{E} \parallel \mathbf{k}$



$$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}_0 \cos(\omega(\mathbf{k})t - \mathbf{k} \cdot \mathbf{r} + \varphi)$$

plasma or Langmuir frequency

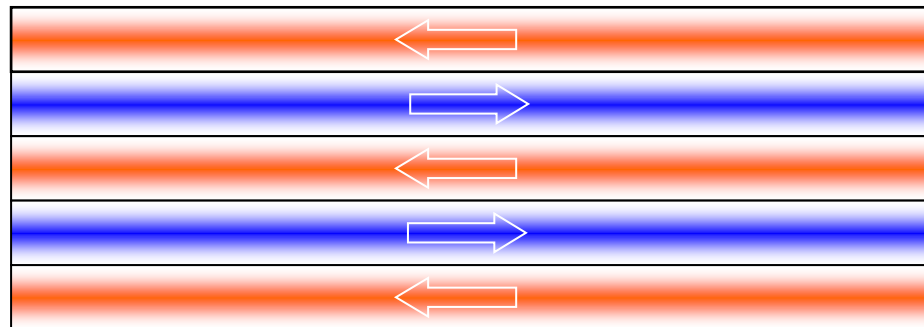
$$\omega(\mathbf{k}) \approx \omega_p \sim \frac{eT}{\hbar} \quad \mathbf{k} \rightarrow 0$$

ultrarelativistic EM plasma

Transverse plasmons

current fluctuation

$\mathbf{E} \perp \mathbf{k}$



Background: plasma instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

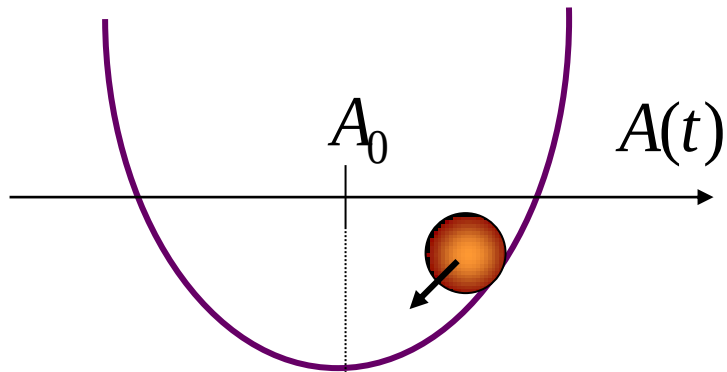
fluctuation

Instability

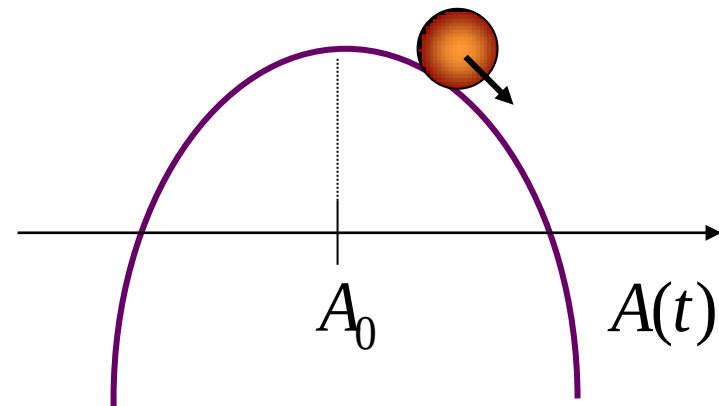
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Background: plasma instabilities

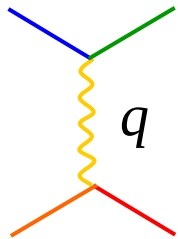
Anisotropic QGP is unstable

$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma \sim gT$$

T - hard momentum scale

Damping due to parton-parton scattering



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Frequency of collisions

$$\nu_{\text{hard}} \sim g^4 \ln(1/g) T$$

$$\nu_{\text{soft}} \sim g^2 \ln(1/g) T$$

$$\text{If } g^2 \ll 1 \Rightarrow \nu_{\text{hard}} \ll \nu_{\text{soft}} \ll \gamma$$

In weakly coupled plasma instabilities play an important role!

Background: running coupling constant

QED - Landau's pole

$$\alpha(q^2) \equiv \frac{e^2(q^2)}{4\pi} = \frac{3\pi}{\ln(\Lambda_{\text{QED}}^2 / q^2)}$$

$$\Lambda_{\text{QED}} \approx 10^{287} \text{ eV}$$

QCD - asymptotic freedom

$$\alpha(q^2) \equiv \frac{g^2(q^2)}{4\pi} = \frac{12\pi}{(33 - 2N_f) \ln(q^2 / \Lambda_{\text{QCD}}^2)}$$

$$\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$$

$$N_c = 3$$

What is $\alpha(q^2)$ for plasma collective modes?

What is q^2 ?

Plasmons in equilibrium QED plasma

Formulation of the problem

Plasmons – poles of propagator

Example of scalar fields

Dyson-Schwinger equation $\Delta(k) = \Delta_0(k) + \Delta(k) \Pi(k) \Delta_0(k)$

Free propagator
$$\Delta_0(p) = \frac{1}{p^2 - m^2}$$

Resumed propagator
$$\Delta(p) = \frac{1}{p^2 - m^2 - \Pi(p)}$$

Dispersion equation
$$\Delta^{-1}(p) = p^2 - m^2 - \Pi(p) = 0$$

Photon propagator

Dyson-Schwinger equation

$$D(k) = D_0(k) + D(k) \Pi(k) D_0(k)$$

Free retarded photon propagator

► General covariant gauge (GCG)

$$D_0^{\mu\nu}(k) = \frac{1}{k^2 + ik_0 0^+} \left(g^{\mu\nu} - (1 - \zeta) \frac{k^\mu k^\nu}{k^2} \right)$$

► Temporal axial gauge (TAG)

$$D_0^{\mu\nu}(k) = \frac{1}{k^2 + ik_0 0^+} \left(g^{\mu\nu} + (1 + \zeta) \frac{k^\mu k^\nu}{(k \cdot n)^2} - \frac{k^\mu n^\nu + n^\mu k^\nu}{(k \cdot n)} \right)$$

$n^\mu = (1, 0, 0, 0)$ - rest frame of the heat bath

$$\text{Strict TAG } \zeta = 0 \quad \left\{ \begin{array}{l} D_0^{00}(k) = D_0^{0i}(k) = D_0^{i0}(k) = 0 \\ D_0^{ij}(k) = -\frac{1}{k^2 + ik_0 0^+} \left(\delta^{ij} - \frac{k^i k^j}{k_0^2} \right) \end{array} \right.$$

Tensor decomposition for GCG

$n^\mu = (1, 0, 0, 0)$ - rest frame of the heat bath

$$n_T^\mu \equiv \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) n_\nu$$

Tensor
basis

$$A^{\mu\nu}(k) \equiv g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} - \frac{n_T^\mu n_T^\nu}{n_T^2}, \quad B^{\mu\nu}(k) \equiv \frac{n_T^\mu n_T^\nu}{n_T^2}$$

$$C^{\mu\nu}(k) \equiv k^\mu n_T^\nu + n_T^\mu k^\nu, \quad E^{\mu\nu}(k) \equiv \frac{k^\mu k^\nu}{k^2}$$

$$D_0^{\mu\nu}(k) = \frac{1}{k^2 + ik_0 0^+} (A^{\mu\nu} + B^{\mu\nu}) + \frac{\zeta}{k^2 + ik_0 0^+} E^{\mu\nu}$$

$$k_\mu \Pi^{\mu\nu}(k) = 0 \quad \Rightarrow \quad \Pi^{\mu\nu}(k) = \Pi^T(k) A^{\mu\nu} + \Pi^L(k) B^{\mu\nu}$$

Dyson-Schwinger equation provides

$$D^{\mu\nu}(k) = D^T(k) A^{\mu\nu}(k) + D^L(k) B^{\mu\nu}(k) + \frac{\zeta}{k^2 + ik_0 0^+} E^{\mu\nu}(k)$$

$$D^{T,L}(k) \equiv \frac{1}{k^2 - \Pi^{T,L}(k)}$$

Tensor decomposition for TAG

$$T^{ij}(k) \equiv \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad L^{ij}(k) \equiv \frac{k^i k^j}{\mathbf{k}^2}$$

$$k_\mu \Pi^{\mu\nu}(k) = 0 \quad \Rightarrow \quad \Pi^{ij}(k) = \Pi^T(k) T^{ij} + \frac{k_0^2}{k^2} \Pi^L(k) L^{ij}$$

$$-D^{ij}(k) = D^T(k) T^{ij}(k) + \frac{k^2}{k_0^2} D^L(k) L^{ij}(k)$$

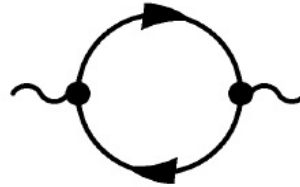
$$D^{T,L}(k) \equiv \frac{1}{k^2 - \Pi^{T,L}(k)}$$

Dispersion equation in GCG & TAG

$$k^2 - \Pi^{T,L}(k) = 0$$

Polarization tensor

Keldysh-Schwinger formalism



$$\Pi^{\mu\nu}(k) = 2e^2 \sum_{n=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \frac{1 - n_f(|\mathbf{p}|)}{|\mathbf{p}|} \frac{2p^\mu p^\nu + p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k \cdot p)}{(p+k)^2 + i(p_0 + k_0)0^+} \Big|_{p_0=n|\mathbf{p}|}$$

$$n_f(E) \equiv \frac{1}{e^{\beta E} + 1}$$

► $k_\mu \Pi^{\mu\nu}(k) = 0 \Rightarrow \Pi^{\mu\nu}(k) = \Pi^T(k) A^{\mu\nu}(k) + \Pi^L(k) B^{\mu\nu}(k)$

► $\Pi^{\mu\nu}(k) = \Pi_{\text{vac}}^{\mu\nu}(k) + \Pi_{\text{med}}^{\mu\nu}(k)$

UV divergent

UV & IR finite

Vacuum polarization tensor

One usually ignores the vacuum contribution as subleading when collective modes are studied but the vacuum makes the coupling run.

$$\Pi_{\text{vac}}^{\mu\nu}(k) = \left(k^2 g^{\mu\nu} - k^\mu k^\nu \right) P(k^2)$$

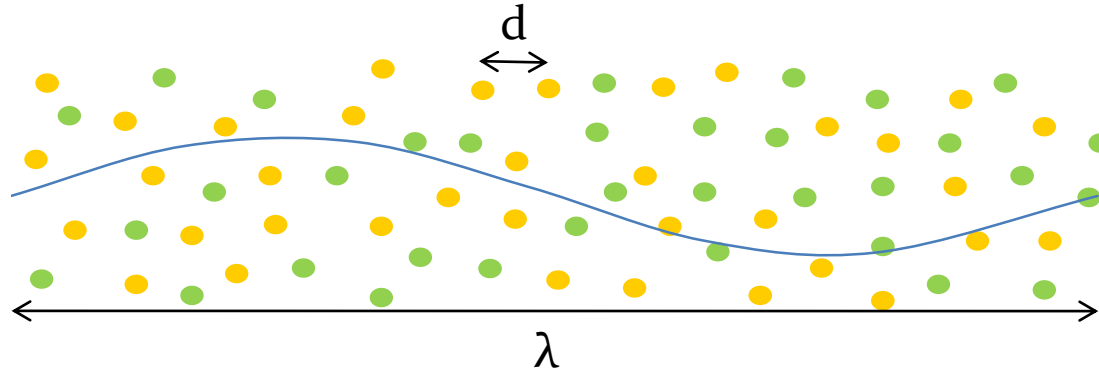
$$\Pi_{\text{vac}}^T(k) = \Pi_{\text{vac}}^L(k) = k^2 P(k^2)$$

$$P(k^2) = -\frac{e^2}{2\pi^2} \left[\frac{1}{6} \left(\frac{1}{\delta} - \gamma_E \right) - \int_0^1 dx (1-x) \ln \left(-\frac{x(1-x)k^2}{4\pi M^2} \right) \right]$$

$$m_e = 0$$

Dimensionally regularized, divergent as $\delta \rightarrow 0$

Medium contribution



Expansion in $\left(\frac{k_0}{T}, \frac{|\mathbf{k}|}{T} \right)$ $d \ll \lambda \iff (k_0, |\mathbf{k}|) \ll T$

Leading order, HTL approximation

$$\left\{ \begin{array}{l} \Pi_{\text{HTL}}^L(k) = -\frac{k^2}{\mathbf{k}^2} m_D^2 \left[1 - \frac{k_0}{2|\mathbf{k}|} \left(\ln \left| \frac{|\mathbf{k}| + k_0}{|\mathbf{k}| - k_0} \right| - i\pi \Theta(-k^2) \right) \right] \\ \Pi_{\text{HTL}}^T(k) = \frac{k_0^2}{2\mathbf{k}^2} m_D^2 \left[1 - \left(\frac{k_0}{2|\mathbf{k}|} - \frac{|\mathbf{k}|}{2k_0} \right) \left(\ln \left| \frac{|\mathbf{k}| + k_0}{|\mathbf{k}| - k_0} \right| - i\pi \Theta(-k^2) \right) \right] \end{array} \right. \quad m_D^2 \equiv \frac{e^2 T^2}{3}$$

Medium contribution cont.

Expansion in $\left(\frac{k_0}{T}, \frac{|\mathbf{k}|}{T}\right)$

Next-to-leading order

$$\left\{ \begin{array}{l} \Pi_{\text{med}}^L(k) = \Pi_{\text{HTL}}^L(k) - \frac{k^2}{12\pi^2} \ln\left(\frac{k^2}{T^2}\right) + \dots \\ \Pi_{\text{med}}^T(k) = \Pi_{\text{HTL}}^T(k) - \frac{k^2}{12\pi^2} \ln\left(\frac{k^2}{T^2}\right) + \dots \end{array} \right.$$

Renormalization

$$\hat{D}^{T,L}(k, \mu) \equiv \frac{1}{Z_3(\mu)} D^{T,L}(k)$$

Renormalization condition

$$k^2 \rightarrow \mu^2 \quad \& \quad T = 0 \quad \Rightarrow \quad \hat{D}^{T,L}(k, \mu) = \frac{1}{k^2}$$

The scale μ is arbitrary.

$$Z_3(\mu) = 1 + P(-\mu^2)$$

$$\hat{\Pi}_{\text{vac}}^T(k, \mu) = \hat{\Pi}_{\text{vac}}^T(k, \mu) = k^2 \hat{P}(k^2, \mu)$$

$$\hat{P}(k^2, \mu) \equiv P(k^2) - P(-\mu^2) = \frac{1}{12\pi^2} \ln \left(-\frac{k^2}{\mu^2} \right)$$

Charge renormalization

$$\hat{\alpha}(\mu) = Z_3(\mu) \alpha$$

$$\alpha \equiv \frac{e^2}{4\pi}$$

$$\mu \frac{d\hat{\alpha}(\mu)}{d\mu} = \beta(\mu)$$

At one-loop level:

$$\beta(\mu) = \frac{2}{3\pi} \hat{\alpha}(\mu)$$

$$\hat{\alpha}(\mu) = \frac{\hat{\alpha}(\mu_0)}{1 - \frac{\hat{\alpha}(\mu_0)}{3\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)}$$

Renormalized propagator

$$\hat{D}^{T,L}(k, \mu) = \frac{1}{k^2 \left(1 - \hat{P}(k^2, \mu)\right) - \Pi_{\text{med}}^{T,L}(k)}$$

$$\hat{D}^{T,L}(k, \mu) = \frac{1}{k^2 \left(1 - \frac{\hat{\alpha}(\mu)}{3\pi} \ln\left(\frac{T^2}{\mu^2}\right)\right) - \hat{\alpha}(\mu) \pi^{T,L}(k)}$$

$$\frac{k^2}{12\pi^2} \ln\left(\frac{|k^2|}{\mu^2}\right) - \frac{k^2}{12\pi^2} \ln\left(\frac{|k^2|}{T^2}\right) = \frac{k^2}{12\pi^2} \ln\left(\frac{T^2}{\mu^2}\right)$$

from vacuum

from medium

$\hat{\alpha}(\mu) \pi^{T,L}(k)$ includes all contributions to $\Pi^{T,L}(k, \mu)$ except log terms

Scale dependent dispersion equation?

$$k^2 \left(1 - \frac{\hat{\alpha}(\mu)}{3\pi} \ln \left(\frac{T^2}{\mu^2} \right) \right) - \hat{\alpha}(\mu) \pi^{T,L}(k) = 0$$

No!

A physical quantity must be μ independent!

Plasmons – poles of

$$\hat{\alpha}(\mu) \hat{D}^{T,L}(k, \mu)$$

which is a renormalization-group invariant.

Coupling constant of plasmons

$$\begin{aligned}\hat{\alpha}(\mu)\hat{D}^{T,L}(k,\mu) &= \frac{\hat{\alpha}(\mu)}{k^2 \left(1 - \frac{\hat{\alpha}(\mu)}{3\pi} \ln\left(\frac{T^2}{\mu^2}\right)\right) - \hat{\alpha}(\mu)\pi^{T,L}(k)} \\ &= \frac{\hat{\alpha}(T)}{k^2 - \hat{\alpha}(T)\pi^{T,L}(k)} = \hat{\alpha}(T)\hat{D}^{T,L}(k,T)\end{aligned}$$

$$\hat{\alpha}(\mu) - \hat{\alpha}(\mu_0) = O(\hat{\alpha}^2(\mu))$$

T is the scale of the effective coupling constant.

Conclusions

- ▶ Collective modes need to be defined through a renormalization-group invariant.
- ▶ Temperature is the scale of the effective coupling constant in equilibrium plasmas.

Outlook

- ▶ Anisotropic plasma
- ▶ QCD