Mesons in hot and dense matter in the framework of the NJL like models

D. Blaschke, A. V. Friesen, Yu. L. Kalinovsky, A. E. Radzhabov, V. D. Toneev

Joint Institute for Nuclear Research, Dubna, Russia

Wroclaw, October 10, 2019

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Motivation

- The most intriguing region of the QCD phase diagram is a subject of the nonperturbative study.
- We need the model is capable to describe the matter properties at finite T and μ_{B} in nonpeturbative region.
- The NJL model is successful effective model, which describes the spontaneous chiral symmetry breaking, formation of the quark condensate and the chiral phase transition.
- Polyakov loop extention solves the problem of a lack of deconfinement.
- To describe the fluctuation one need to go beyond mean field approximation.

SU(3) PNJL model

The Lagrangian (P. Costa et al. PRD79, 116003 (2009); E. Blanquier J. Phys. G: NPP 38, 105003 (2011), A. Friesen et al. Phys.-Usp 59, 367 (2017)):

$$\begin{aligned}\mathcal{L} &= \bar{q}\,(\,i\,\gamma^{\mu}\,D_{\mu}\,-\,\hat{m}-\gamma_0\mu)\,q+\frac{1}{2}\,G_s\,\sum_{a=0}^{8}[\,(\,\bar{q}\,\lambda^a\,q\,)^2\,+\,(\,\bar{q}\,i\,\gamma_5\,\lambda^a\,q\,)^2\,] \\ &+ K\,\{\det[\bar{q}\,(\,1\,+\,\gamma_5\,)\,q\,]+\det[\bar{q}\,(\,1\,-\,\gamma_5\,)\,q\,]\,\}-\mathcal{U}(\Phi,\bar{\Phi};T)\end{aligned}$$

$D_{\mu} = \partial^{\mu} - iA^{\mu}$, where A^{μ} is the gauge field with $A^0 = -iA_4$ and $A^{\mu}(x) = G_s A^{\mu}_a \frac{\lambda_a}{2}$
The effective potential has to reproduce the Lattice calculation in the pure gauge sector:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right)+\frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2,$$

$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

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$$+ K\,\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\} - \mathcal{U}(\Phi,\bar{\Phi};T)$$

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$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3.$$

We can:

- explain and describe spontaneous chiral symmetry breaking as $m_q = m_0 + <\bar{q}q>$;

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- simulate the confinement/deconfinement transition

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We can:

- explain and describe spontaneous chiral symmetry breaking as $m_q = m_0 + <\bar{q}q>$;
- simulate the confinement/deconfinement transition
- build the phase diagram with crossover at low chemical potential and 1st order transition at high chemical potential ($m_0 \neq 0$),

The mean-field approximation

The grand potential density:

$$\Omega = \mathcal{U}(\Phi,\bar{\Phi};T) + G_s \sum_{i=u,d,s} \langle \bar{q}_i q_i \rangle^2 + 4K \langle \bar{q}_u q_u \rangle \langle \bar{q}_d q_d \rangle \langle \bar{q}_s q_s \rangle - 2N_c \sum_{i=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_i - $$
$$- 2T \sum_{i=u,d,s} \int \frac{d^3p}{(2\pi)^3} (N_\Phi^+(E_i) + N_\Phi^-(E_i))$$

with the functions

$$N_\Phi^+(E_i) = \mathrm{Tr}_c \left[\ln(1 + L^\dagger e^{-\beta(E_i-\mu_i)}) \right] = \left[1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_i^+} \right) e^{-\beta E_i^+} + e^{-3\beta E_i^+} \right],$$

$$N_\Phi^-(E_i) = \mathrm{Tr}_c \left[\ln(1 + L e^{-\beta(E_p+\mu_i)}) \right] = \left[1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_p^-} \right) e^{-\beta E_p^-} + e^{-3\beta E_p^-} \right] ,$$

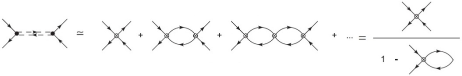
where $E_i^\pm = E_i \mp \mu_i$, $\beta = 1/T$, $E_i = \sqrt{{p_i}^2 + m_i^2}$ is the energy of quarks and $\langle \bar{q}_i q_i \rangle$ is the quark condensate.
The equations of motion

$$\frac{\partial \Omega}{\partial \sigma_f} = 0, \quad \frac{\partial \Omega}{\partial \Phi} = 0, \frac{\partial \Omega}{\partial \bar{\Phi}} = 0.$$

and gap equations:

$$m_i = m_{0i} + 4G < \bar{q}_i q_i > +2K < \bar{q}_j q_j >< \bar{q}_k q_k >$$

The mesons mass $\mu_B = 0$



The meson masses are defined by the Bethe-Salpeter equation at $P = 0$

$$1 - P_{ij}\Pi^{P}_{ij}(P_0 = M, P = 0) = 0 \ ,$$

with

$$P_\pi = G_s + K \langle \bar{q}_s q_s \rangle \ , \quad P_K = G_s + K \langle \bar{q}_u q_u \rangle$$

and the polarization operator:

$$\Pi^{P}_{ij}(P_0) = 4 \left((I^i_1 + I^j_1) - [P_0^2 - (m_i - m_j)^2] \ I^{ij}_2(P_0) \right) ,$$

where

$$I^i_1 = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_i^2}, \quad I^{ij}_2(P_0) = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m_i^2)((p + P_0)^2 - m_j^2)}$$

When $T > T_{Mott}$ $(P_0 > m_i + m_j)$ the meson $\rightarrow$ the resonance state $\rightarrow$ $P_0 = M_M - 1/2i\Gamma_M$.

The mesons mass $\mu_B = 0$: result

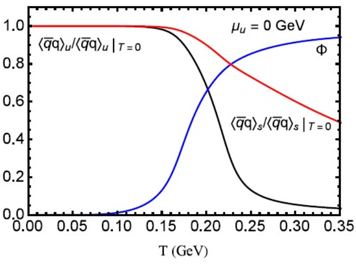
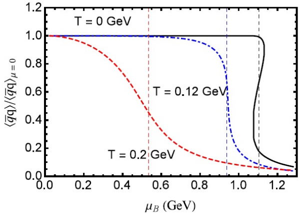
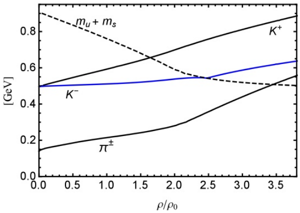


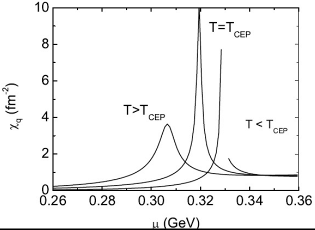
Figure 1: The mass spectra at zero mu$_B$

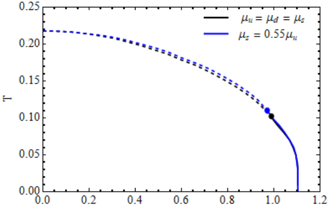
The model with finite μ_B and density

$$a)\ \mu_u = \mu_d = \mu_s, \quad b)\ \mu_u = \mu_d; \mu_s = 0.55\mu_u$$
$$\mu_B = 3\mu_u, \quad \rho_B = \frac{\rho_u + \rho_d + \rho_s}{3}$$

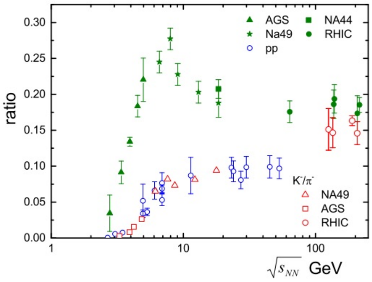








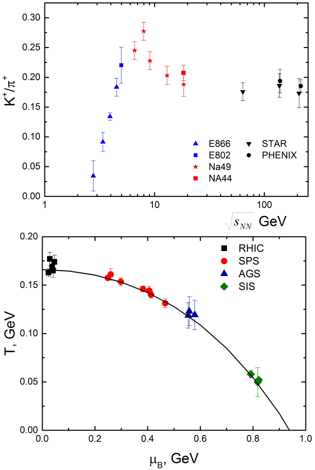
Do we can apply the model to the 'horn' description?



Explanation of the "horn"

- a jump is a signal of deconfinement (SMES M. Gazdzicki, M.I. Gorenstein, Acta Phys. Pol. B 30, 2705 (1999)).
- the quick increase is a result of the partial chiral symmetry restoration (A. Palmese, et al. PRC 94, 044912 (2016)- PHSD; K. Bugaev - statistical model; J. Nayak - microscopic model).

The experimental data



The model approach:

- all mesons were created during hadronization and we skip the rescattering, decays and so on..
- freeze-out line is coincide with the chiral phase transition line (it is not absolutely legitimate :)
- Experiment: for each energy of collision we can find T^* and μ_B^* of the freeze-out
- Experiment: we can rescale the data as function of T^*/μ_B^*
- Theory: now we can calculate the kaon to pion ratio as a function T/μ_B where T and μ_b are chosen along the phase transition line.

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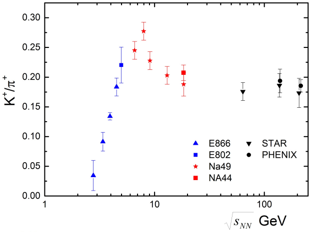

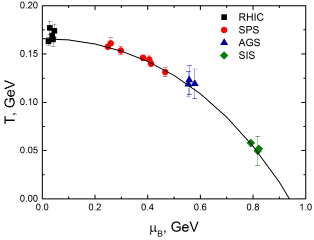

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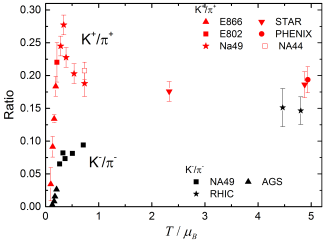


Kaon to pion ratio in PNJL model

$$n_{K^\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{K^\pm}} \mp \mu_{K^\pm})/T} - 1},$$
$$n_{\pi^\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{\pi^\pm}} \mp \mu_{\pi^\pm})/T} - 1}.$$

with parameter $\mu_\pi = 0.135$ (M. Kataja, P.V. Ruuskanen PLB 243, 181 (1990)) and $\mu_K = \mu_u - \mu_s$ (see for example A. Lavagno and D. Pigato, EPJ Web of Conferences 37, 09022 (2012)).

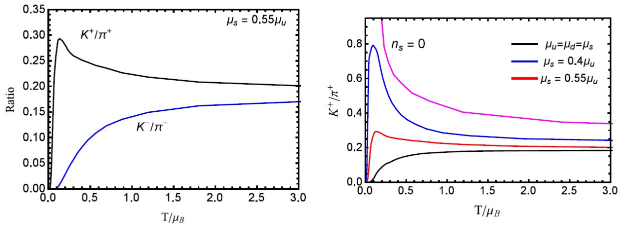


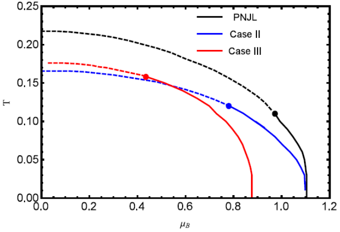
Figure 2: A. V. Friesen, Yu. L. Kalinovsky, V. D. Toneev PRC 99, 045201 (2019)

Phase diagram and kaon to pion ratio

- introduce a phenomenological dependence of $G_s(\Phi)$ (Y. Sakai et al PRD 82, 076003 (2010), P. de Forcrand, O. Philipsen NPB 642, 290(2002), A. Friesen et al. IJMPA30, 1550089 (2015).)

$$\tilde{G}_s(\Phi) = G_s[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)]$$

  with $\alpha_1 = \alpha_2 = 0.2$.
- the effect of axial symmetry and the coupling $K = K_0 \exp(-(\rho/\rho_0)^2)$ on the dense states (K. Fukushima PR77, 114028 (2008); P. Costa, Yu. Kalinovsky et al AIP Conf.Proc. 775 (2005) 173; arXiv::0503258) + $G_s(\Phi)$ with $\alpha_1 = \alpha_2 = 0.2$.



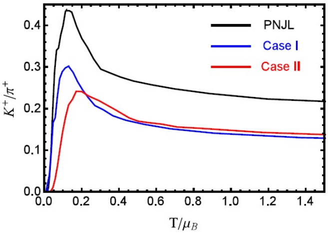


Figure 3: $\mu_s = 0.5\mu_u$

Meson fluctuation in PNJL 1+2 model)

The meson spectra beyond the mean field approximation can be obtained from pole condition for meson propagator

$$[\mathcal{S}_{\mathrm{M}}(\mathrm{M},\bar{0})]^{-1} = 2\mathrm{G_s} - \Pi(\mathrm{M} - \mathrm{i}\eta, \bar{0}) = 0$$

then using "polar" representation for propagator of meson $\mathcal{S}_{\mathrm{M}}(\omega,\bar{\mathrm{q}}) = |\mathcal{S}_{\mathrm{M}}(\omega,\bar{\mathrm{q}})| \exp^{\delta_{\mathrm{M}}(\omega,\bar{\mathrm{q}})}$,
where mesonic phase shift has the form $\delta_{\mathrm{M}}(\omega,\bar{\mathrm{q}}) = -\arctan\{\frac{\mathrm{Im}[\mathcal{S}_{\mathrm{M}}(\omega - \mathrm{i}\eta,\bar{\mathrm{q}})]^{-1}}{\mathrm{Re}[\mathcal{S}_{\mathrm{M}}(\omega + \mathrm{i}\eta,\bar{\mathrm{q}})]^{-1}}\}$

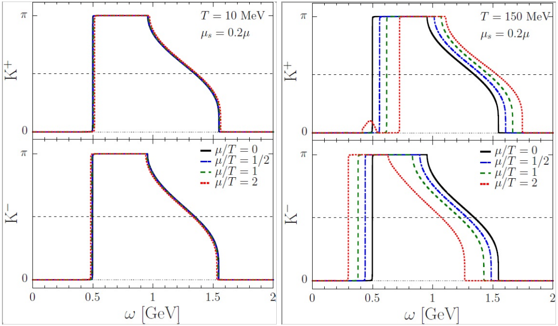


Figure 4: $\mu_{\mathrm{s}} = 0.2\mu_{\mathrm{u}}$

(see for discussion A. Dubinin, D. Blaschke, A. Radzhabov Phys. Rev. D 96, 094008 (2017))

Meson fluctuation and K/π ratio (preliminary results)

The partial number densities is

$$n_M = d_M \int \frac{dM}{\pi} \delta_M(M) \int \frac{d^3q}{(2\pi)^3} \frac{M}{E} g(E \pm \mu_M)$$

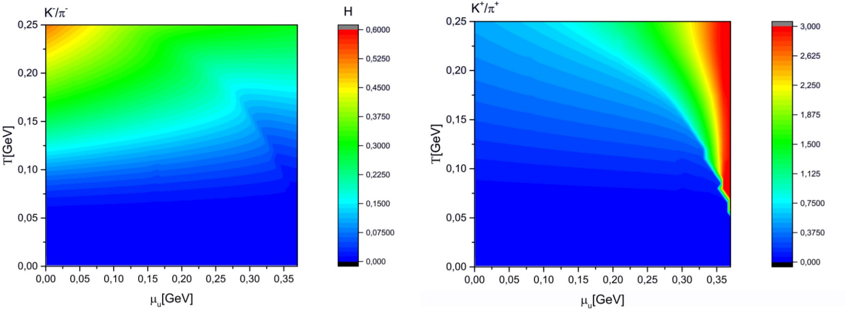


Figure 5: $\mu_s = 0.2\mu_u$

Results and outlooks

- splitting of kaons masses at high densities ⇒ the difference in the behaviour of the K/π at low energies.
- the hight of the peak in the model depends on the properties of the matter (strange chemical potential, T and μ_{B}).
- the position of the peak pretends to be depend on curvature of phase diagram/CEP position.
- it is interesting to consider baryon-to-pion ratio in the PNJL model


Спасибо
Gracias
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Obrigado
Спасибо
Dank U
Grazie
Ευχαριστώ
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Merci
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