# Hamiltonian approach to QCD at finite T

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Dear David


Dear David

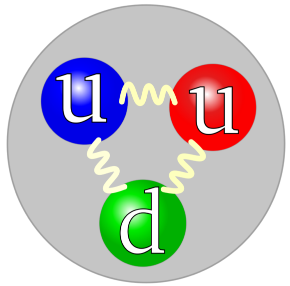
Mit der Reife wird man immer jünger

Hermann Hesse

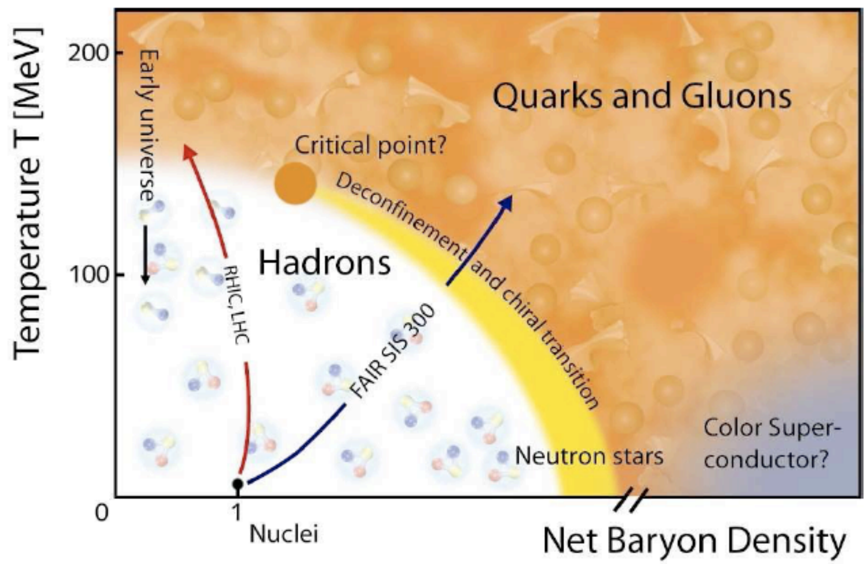
(With maturity one gets constantly younger)

# QCD

- *vacuum*
  - confinement
  - SB chiral symmetry




- *phase diagram*
  - deconfinement
  - rest. chiral symm.




- latticeMC-fail at large chemical potential
  continuum approaches required

# non-perturbative continuum approaches

- Dyson-Schwinger equations
  - Landau(+Coulomb)gauge
- FRG flow equations
  - Landau gauge
- Variational approaches
  - Covariant : Landau gauge
  - Hamiltonian: Coulomb gauge

# non-perturbative continuum approaches

- Dyson-Schwinger equations
    - Landau(+Coulomb)gauge
- FRG flow equations
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- Variational approaches
    - Covariant : Landau gauge
    - Hamiltonian: Coulomb gauge

# Outline

- introduction
- basics of the Hamiltonian approach to QCD in Coulomb gauge (T=0)
  - Yang-Mills theory
  - quark sector
- Hamiltonian approach at finite temperature by compactification of a spatial dimension
- QCD at finite T
  - quark condensate
  - Polyakov loop
- conclusions & outlook

# Canonical Quantization of Yang-Mills theory

cartesian coordinates $A_\mu^a(x)$

momenta $\Pi_i^a(x) = \delta S / \delta \dot{A}_i^a(x) = E_i^a(x)$

$\Pi_0^a(x) = 0$ Weyl gauge : $A_0^a(x) = 0$

$$H = \tfrac{1}{2} \int d^3x (\Pi^2(x) + B^2(x))$$

quantization: $\Pi_k^a(x) = \delta / i\delta A_k^a(x)$

Gauß law: $D\Pi\Psi = \rho_m \Psi$

residual gauge invariance $\mathrm{U}(\vec{\mathrm{x}})$ : $\Psi(A^U) = \Psi(A)$

# Coulomb gauge

$$\partial A = 0, \qquad A = A^{\perp}$$

curved space

$$\langle \Psi | \Phi \rangle = \int DA^{\perp} J(A^{\perp}) \Psi^{*}(A^{\perp}) \Phi(A^{\perp})$$

Faddeev-Popov

$$J(A^{\perp}) = Det(-D\partial)$$

$$\Pi = \Pi^{\perp} + \Pi^{\parallel}, \qquad \Pi^{\perp} = \delta / i\delta A^{\perp}$$

Gauß law:

$$D\Pi\Psi = \rho_m \Psi$$

resolution of Gauß´ law

$$\Pi^{\parallel} = -\partial(-D\partial)^{-1}\rho, \qquad \rho = (-\hat{A}^{\perp}\Pi^{\perp} + \rho_m)$$

# Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J\Pi + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density $\rho^a = -f^{abc}A^b\Pi^c + \rho^a_m \qquad \rho^a_m = q^{\dagger}t^a q$

$$\langle\phi|\ldots|\psi\rangle = \int DA\, J(A)\phi^*(A)\ldots\psi(A)$$

$$H\psi[A] = E\psi[A]$$

# Variational approach to YMT

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\,\omega(x,y)\,A(y)\right]$$

gluon propagator $\langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

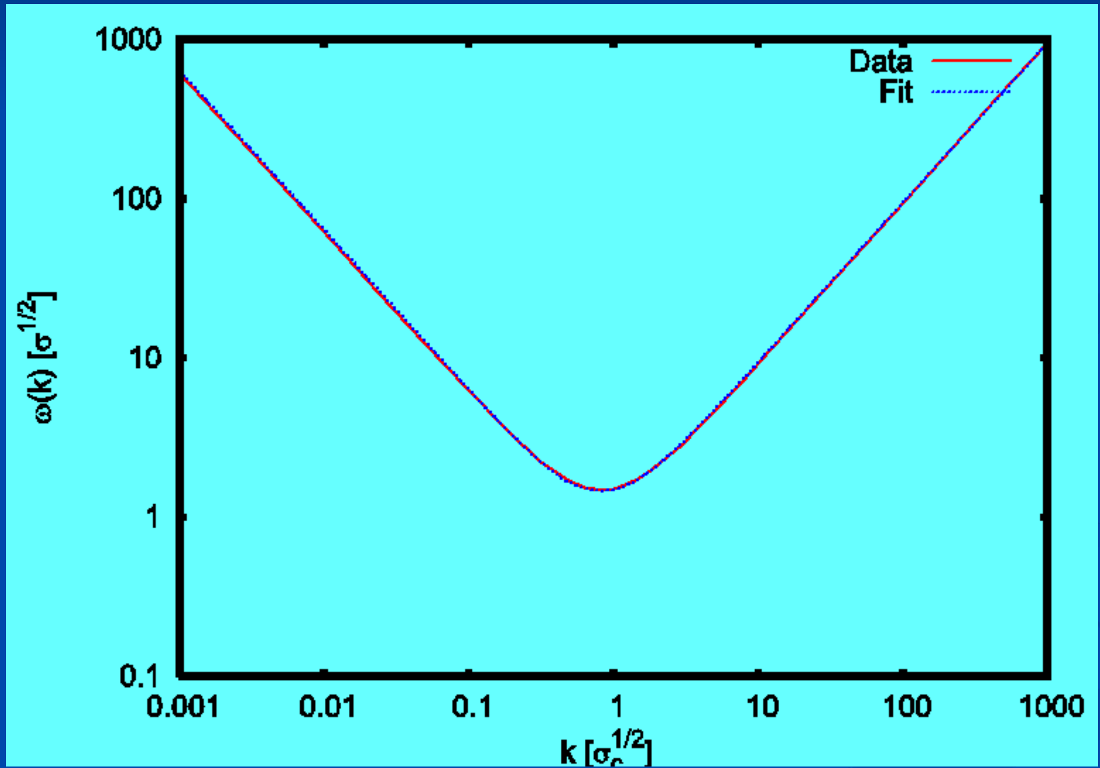
variational kernel $\omega(x,x')$ determined from

$$\langle\Psi|H|\Psi\rangle \to \min$$

# Numerical results

gluon energy

D. Epple, H. R. & W.Schleifenbaum, PRD 75 (2007)




$$IR: \quad \omega(k) \sim 1/k \qquad UV: \quad \omega(k) \sim k$$

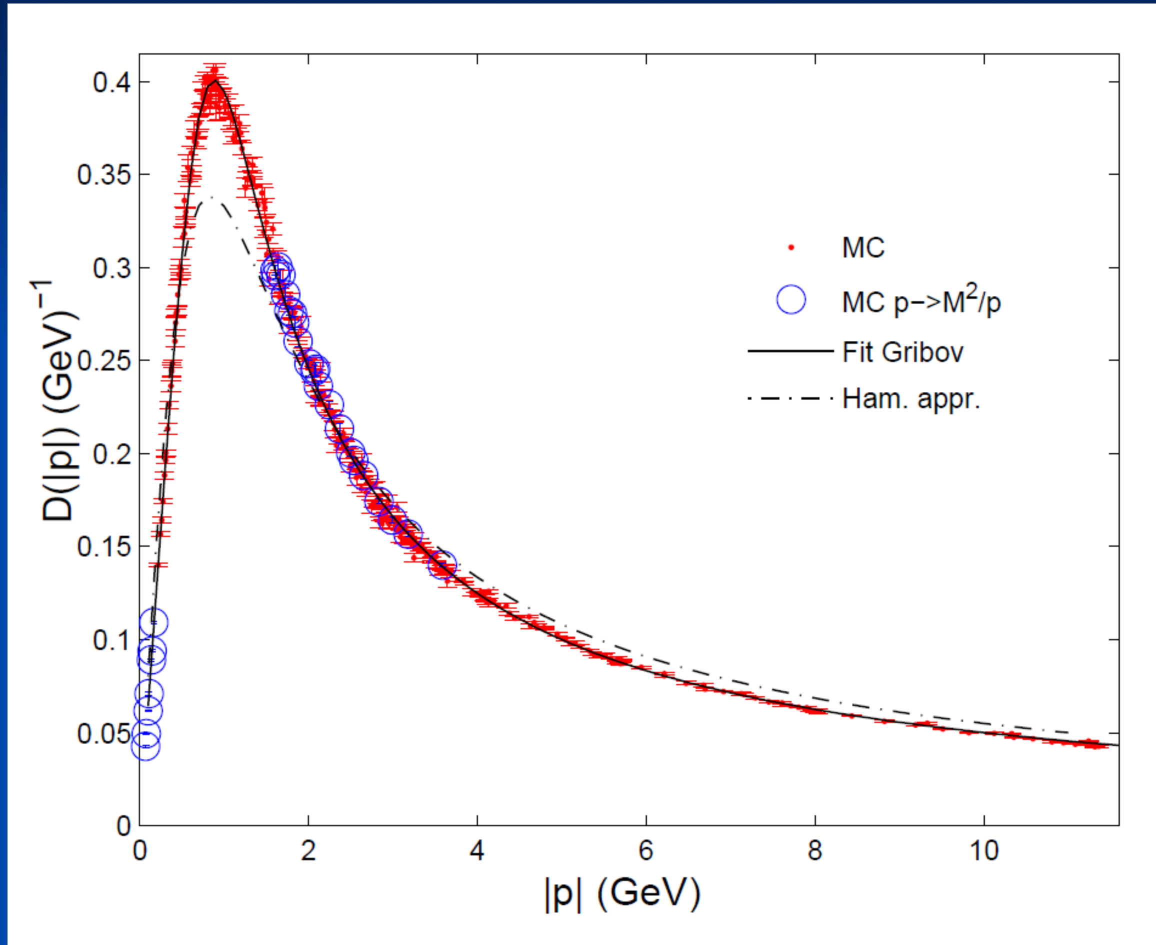
# Static gluon propagator in D=3+1

$$D(k) = (2\omega(k))^{-1}$$

*Gribov's formula*

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88\,GeV$$




missing strength in
mid momentum regime

lattice: G. Burgio, M.Quandt , H.R., **PRL102(2009)**

continuum: D. Epple, H. R., W.Schleifenbaum, PRD 75 (2007)

# Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,
Phys.Rev.D82(2010)
Phys.Rev.D92(2015)

*wave functional*

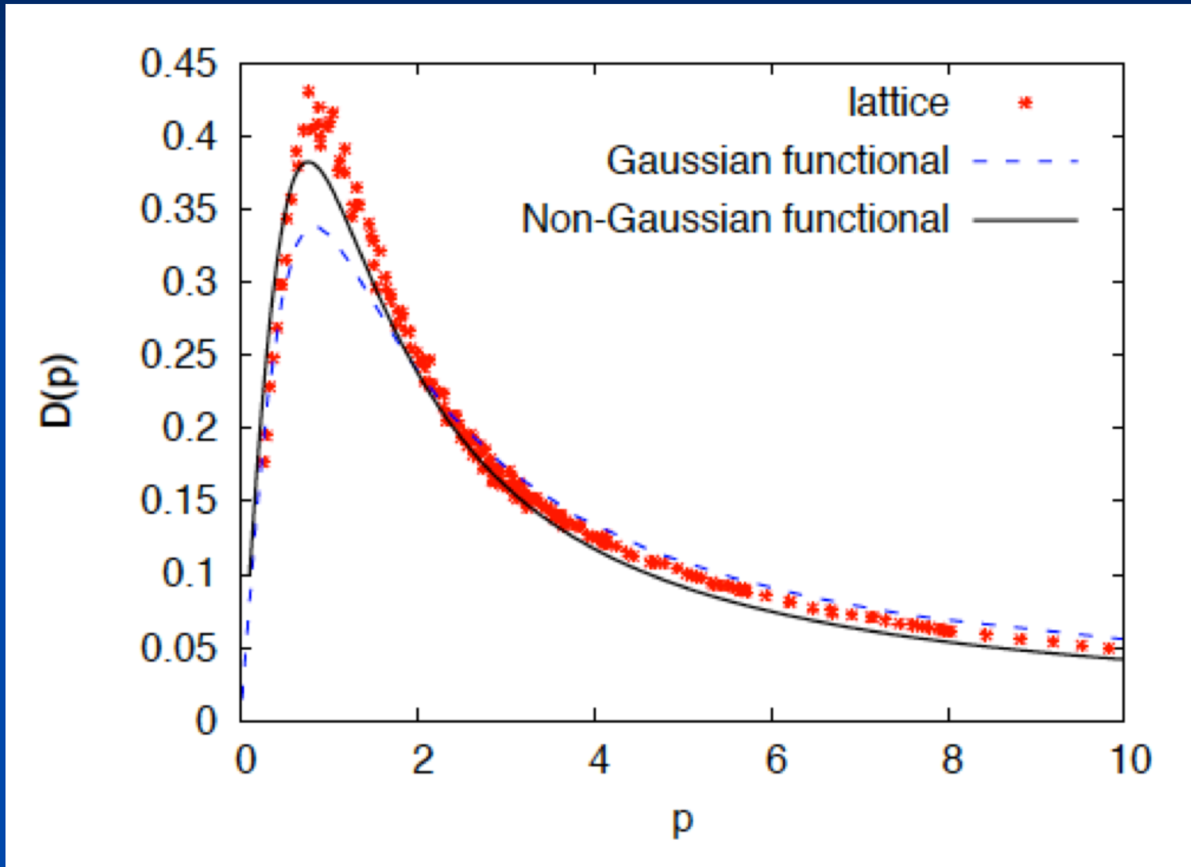
$$|\psi[A]|^2 = \exp(-S[A])$$

*ansatz*

$$S[A] = \int \omega A^2 + \tfrac{1}{3!}\int \gamma^{(3)} A^3 + \tfrac{1}{4!}\int \gamma^{(4)} A^4$$

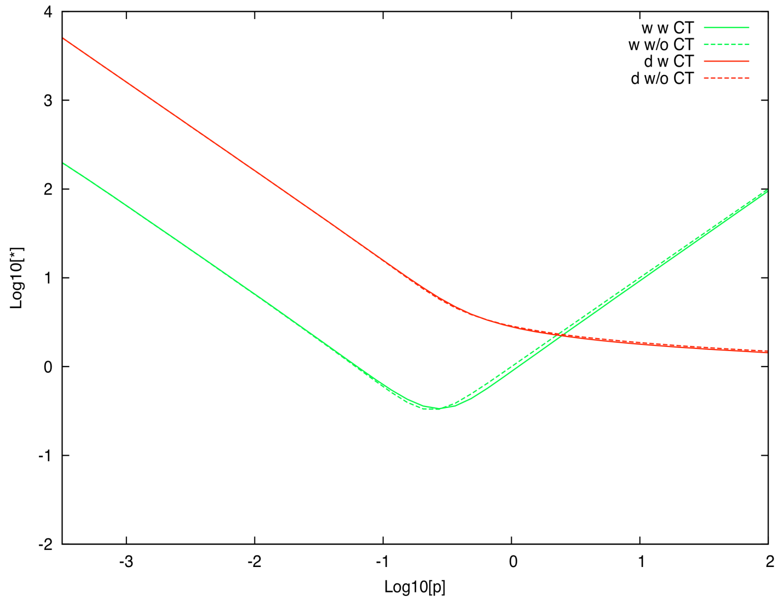
exploit DSE

# Static gluon propagator in D=3+1



# The Ghost Propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d/(-\Delta)$$



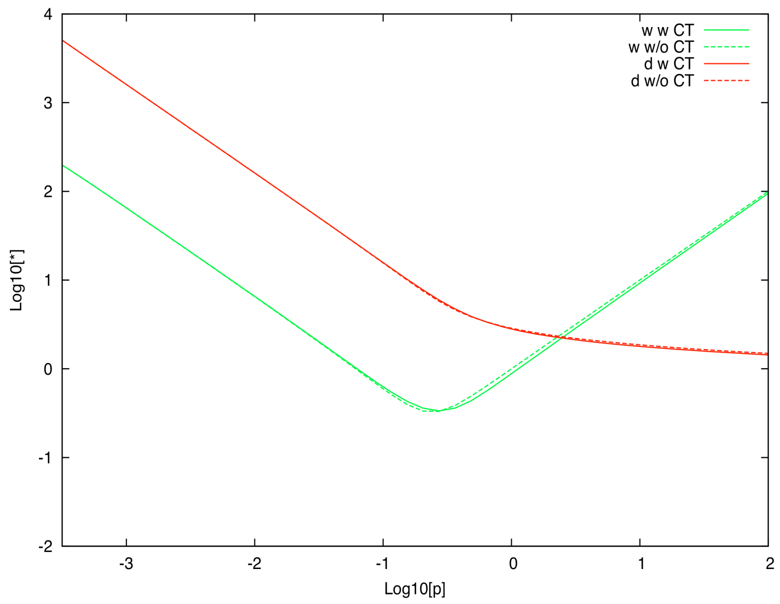

**horizon condition**

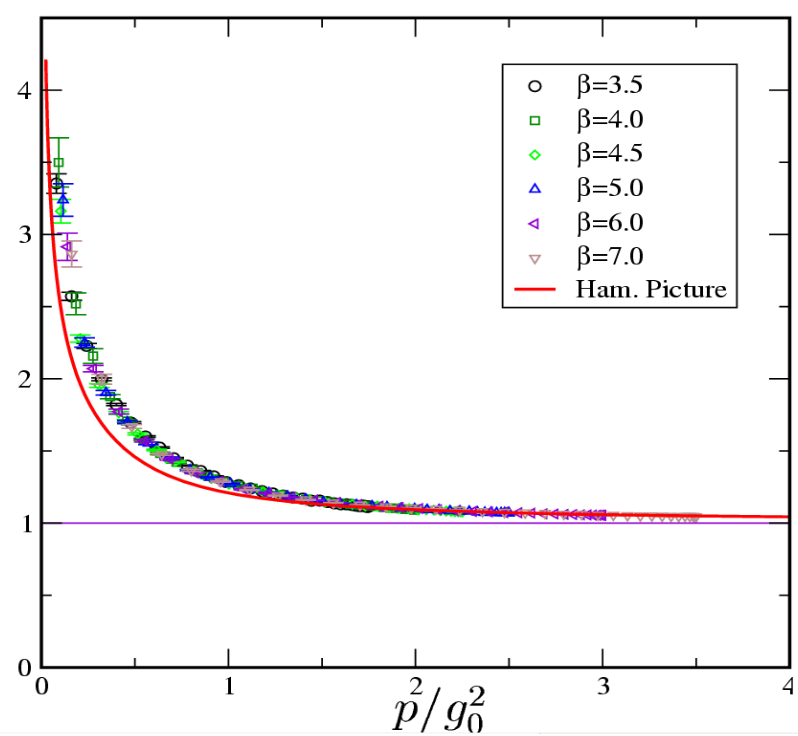
$$d^{-1}(0) = 0$$

necessary for confinement

# The Ghost Propagator

$$\langle (-D\partial)^{-1} \rangle = d/(-\Delta)$$




**horizon condition**

$$d^{-1}(0) = 0$$

necessary for confinement

Lattice-data: L.Moyaerts, Diss, Tübingen, 2005

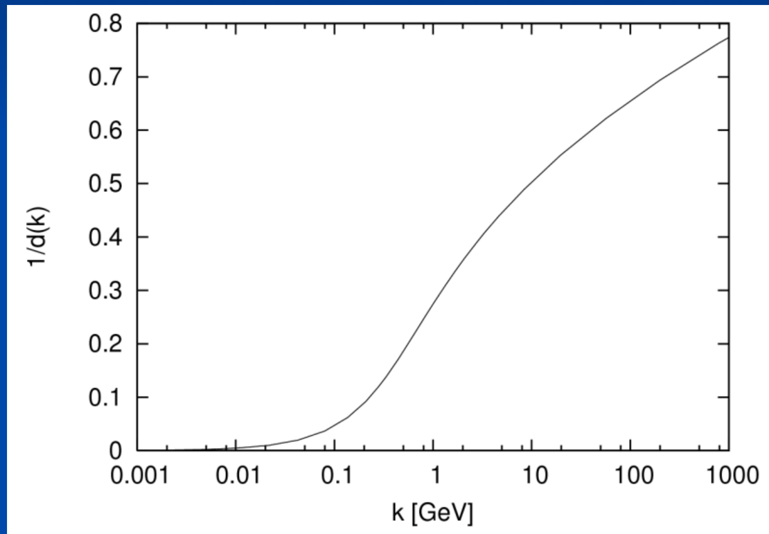
continuum: C.Feuchter, H. Reinhardt, Phys.Rev.D77(2008)

# The color dielectric function of the QCD vacuum

- ghost propagator
- dielectric „constant“

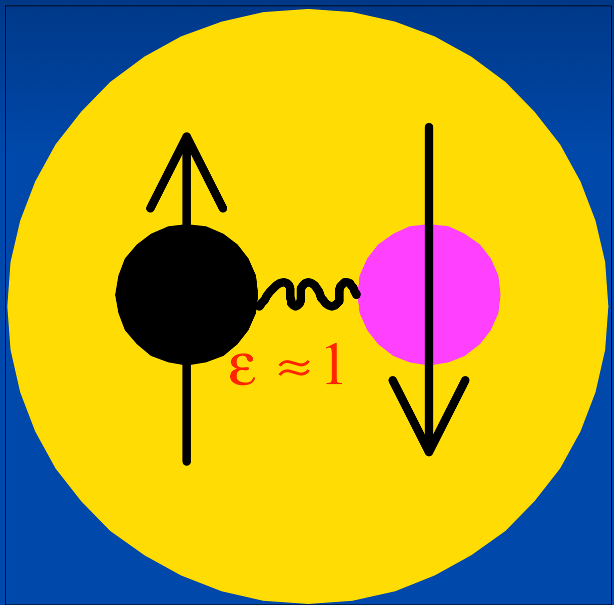
$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)

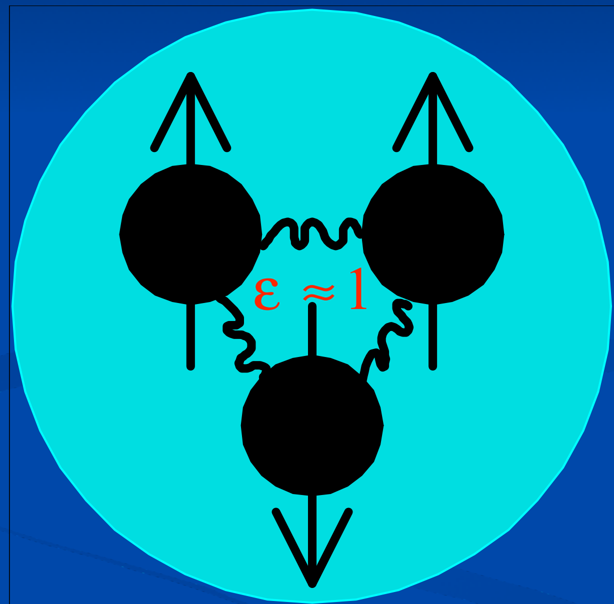



- horizon condition:
  - : $d^{-1}(k=0)=0 \qquad \varepsilon(k=0)=0$
  -

$$D = \varepsilon E \qquad \partial D = \rho_{free}$$




$$\varepsilon = 0$$



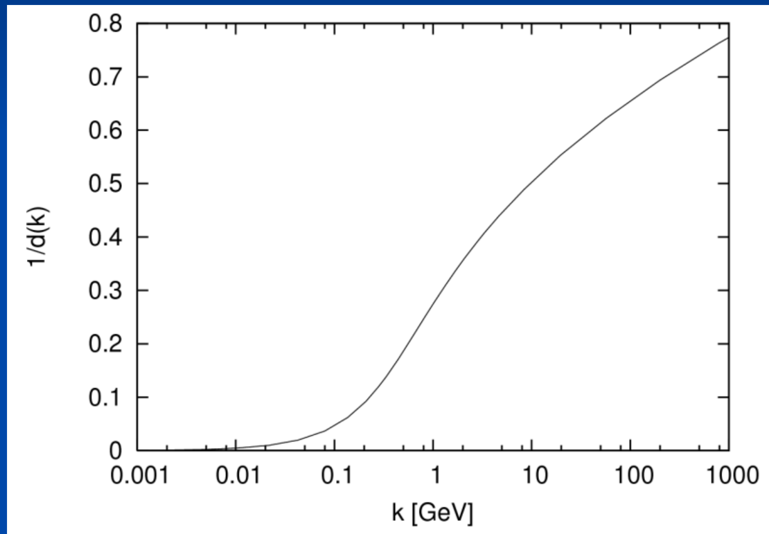

no free color charges in the vacuum: confinement

# The color dielectric function of the QCD vacuum

- ghost propagator
- dielectric „constant“

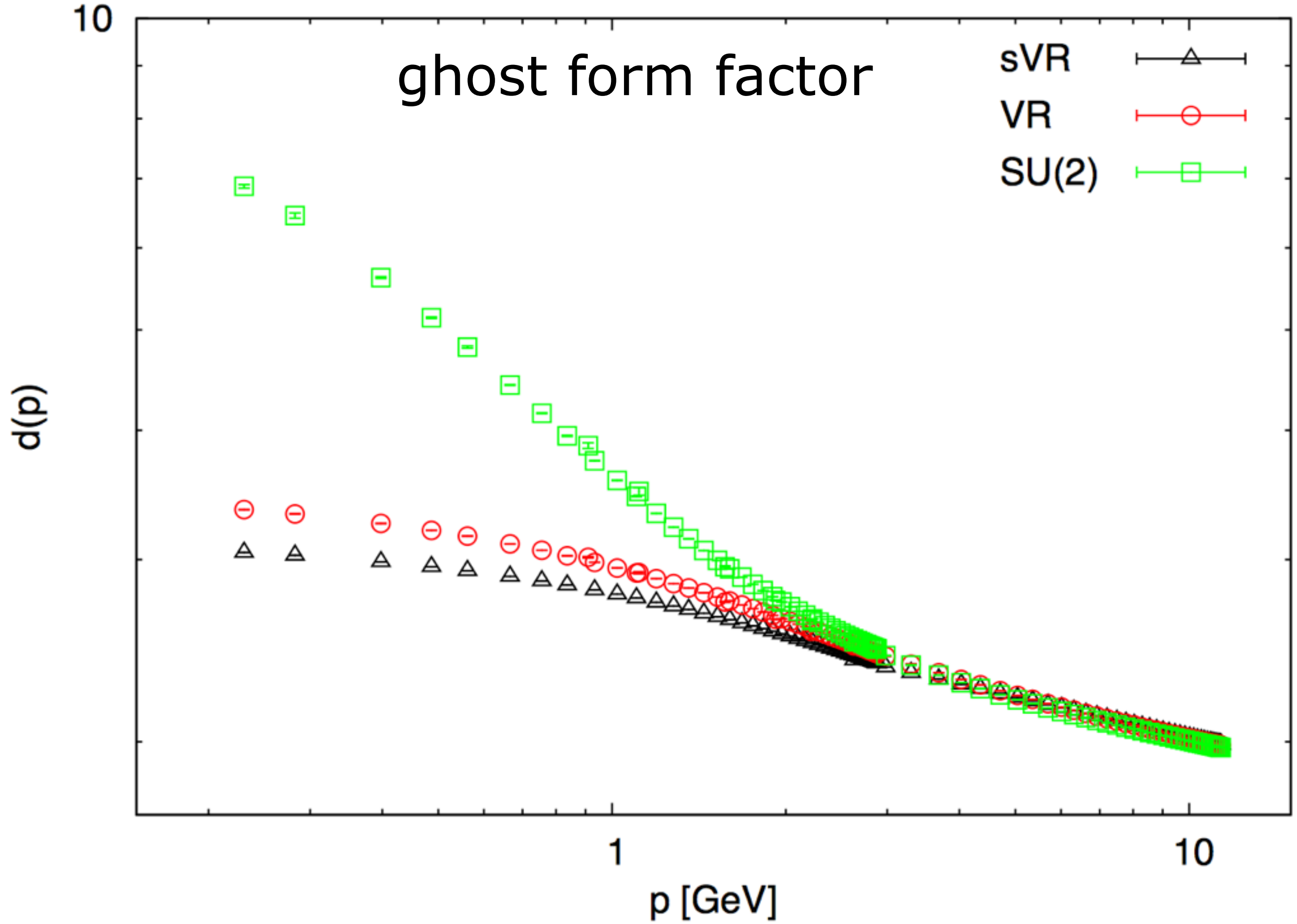
$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)




- horizon condition:
  - : $d^{-1}(k=0)=0 \qquad \varepsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor
  - $\varepsilon(k)<1$ anti-screening

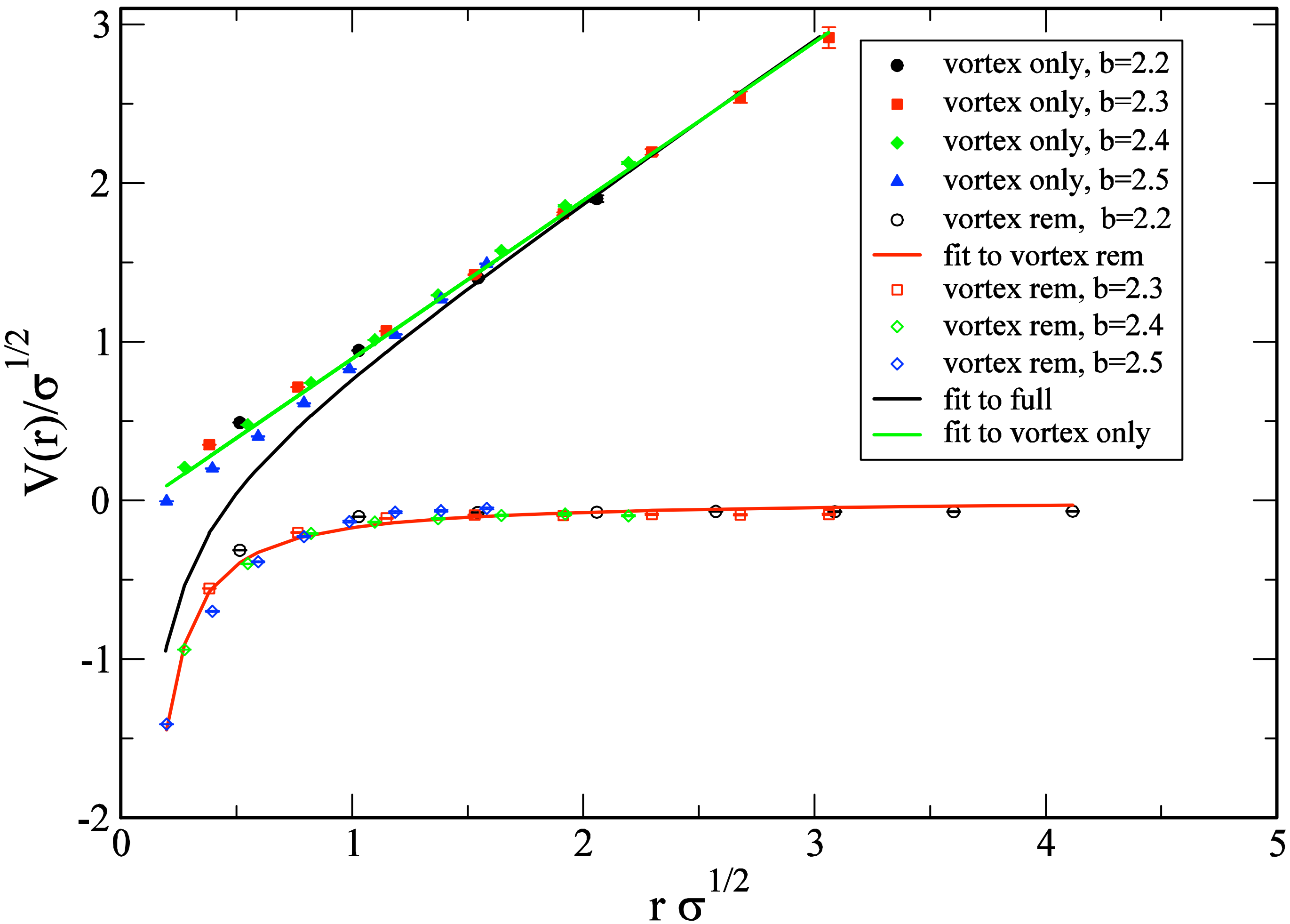
# connection to the center vortex picture of confinement




*G. Burgio, M. Quandt, H.R. & H.Vogt, Phys. Rev.D92(2015)*

- elimination of center vortices: IR enhancement disappears
- horizon condition $d^{-1}(0) = 0$ is lost

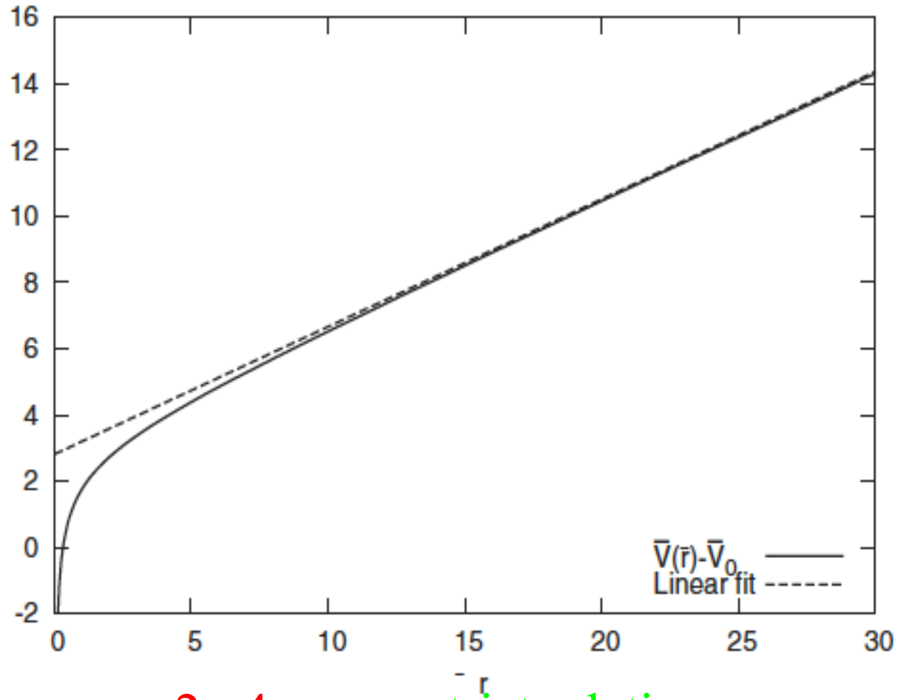
# Q-Q-potential: SU(2)




$$\sigma_{vortices} \approx \sigma$$

# Non-Abelian Coulomb potential

$$V_C(\vec{x},\vec{y}) = \langle \vec{x} | (-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} | \vec{y} \rangle$$
$$= \int d^3p \exp[i\vec{p}(\vec{x}-\vec{y})](d(\vec{p}))^2 / \vec{p}^2$$




D. Epple, H. Reinhardt
W.Schleifenbaum,
PRD 75 (2007)

$$V(r) = \xrightarrow[r\to 0]{} \sim 1/r$$

$$V(r) = \xrightarrow[r\to\infty]{} \sigma_C r,$$

lattice: $\sigma_C = 2...4\sigma_W$ strict relation $\sigma_W < \sigma_C$ *D. Zwanziger*

$\sigma_C$ is linked to $\sigma_{Wspatial}$ and not to $\sigma_{Wtemporal}$

*G. Burgio, M. Quandt, H. R. & H. Vogt, Phys.Rev.D92(2015)*

# The QCD Hamiltonian in Coulomb gauge

$$H_{QCD} = H_{YM} + H_q + H_C$$

*gluon part*

$$H_{YM} = \tfrac{1}{2}\int (J^{-1}\Pi J\Pi + B^2) \qquad \Pi = -i\delta/\delta A \qquad J(A^{\perp}) = Det(-D\partial)$$

*quark part*

$$H_q = \int \Psi^{\dagger}(x)[\vec{\alpha}(\vec{p} + g\vec{A}) + \beta m_0]\Psi(x) \qquad \vec{\alpha},\beta - Dirac\ matrices$$

*Coulomb term*

$$H_C = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

*color charge density*

$$\rho^a = -f^{abc}A^b\Pi^c + \Psi^{\dagger}(x)t^a\Psi(x)$$

# quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

$s, v, w$ – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

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$s, v, w$ – variational kernels  $\vec{\alpha}, \beta$ – *Dirac matrices*

$\mathrm{v} = \mathrm{w} = 0$ : *BCS – wave function*

Finger & Mandula
Adler & Davis,
Alkofer & Amundsen

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$\mathrm{v}=\mathrm{w}=0$ : *BCS – wave function*

Finger & Mandula
Adler & Davis,
Alkofer & Amundsen

$\mathrm{v}\neq 0, \mathrm{w}=0$ : *quark - gluon - coupling*

Pak & Reinhardt,

# quark wave functional

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

$s, v, w$ – variational kernels $\quad$ $\vec{\alpha}, \beta$ – *Dirac matrices*

> calculate ‹$H_{QCD}$› up to 2 loops

> variation w.r.t. **s, v, w**

$$v(p,q) = f_v[s,\omega] \qquad w(p,q) = f_w[s,\omega]$$

$$s(p) = f_s[s,v,w;p]$$

*gap equation*

*cancelation of all UV-divergencies*

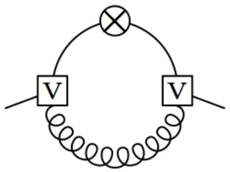
*D. Campagnari, Ebadati, H.R & Vastag, PRD94(2016)*

# cancellation of UV-divergencies

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^{\dagger}(s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle \quad \cdots$$

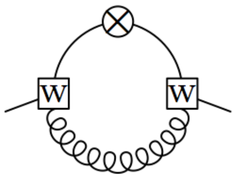
divergent loop contributions to the gap equation

> kernel **V**




$$\frac{C_F}{16\pi^2} g^2 S(k)\left[-2\Lambda + k\ln\frac{\Lambda}{\mu}\left(-\frac{2}{3} + 4P(k)\right)\right]$$

> kernel **W**




$$\frac{C_F}{16\pi^2} g^2 S(k)\left[2\Lambda + k\ln\frac{\Lambda}{\mu}\left(\frac{10}{3} - 4P(k)\right)\right]$$

> Coulomb term $V_C$

$$-\frac{C_F}{6\pi^2} g^2 k S(k)\ln\frac{\Lambda}{\mu}$$

# cancellation of UV-divergencies

> occurs not only in 3+1 but also in

2+1 dimension

# quark wave functional

P. Vastag, H. R.
D. Campagnari
Phys.Rev.D93(2016)

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

$s, v, w$ – variational kernels $\quad \vec{\alpha}, \beta$ – *Dirac matrices*

# numerical calculation

D. Campagnari , E. Ebadati, H.R. and P: Vastag,
arXiv:1608.06820,PRD94(2016)074027

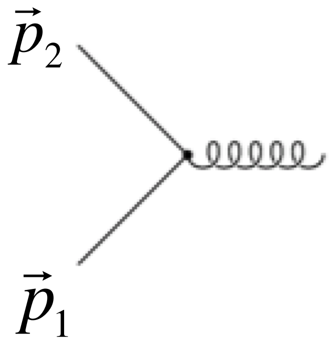
**input:** $\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}\ M = 0.88 GeV$

$lattice: \quad \sigma_C = 2.5\sigma$

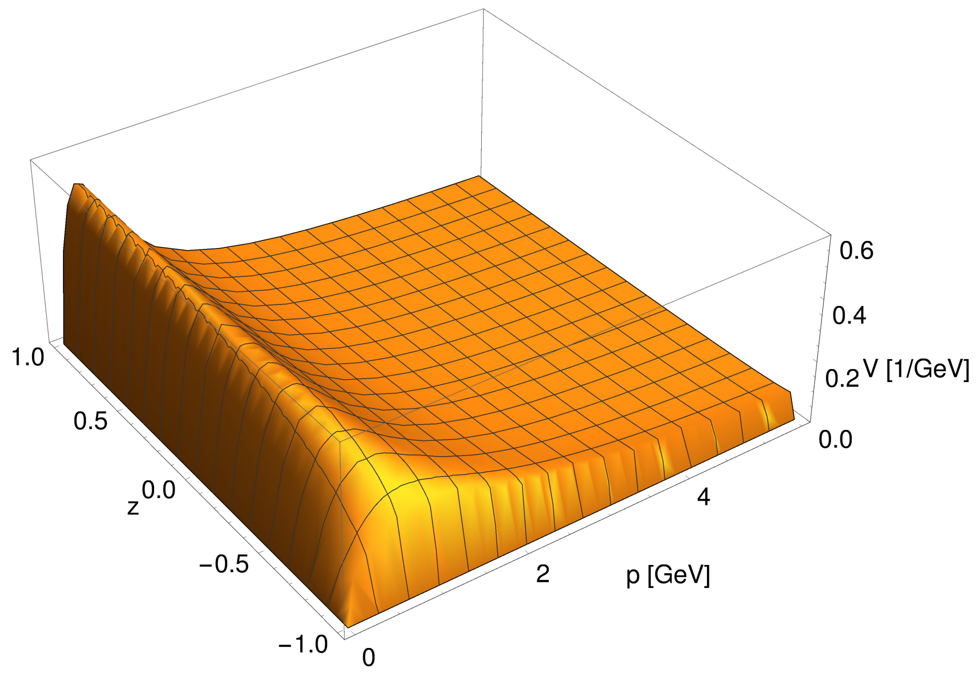
G. Burgio, M.Quandt , H.R.,
PRL102(2009)

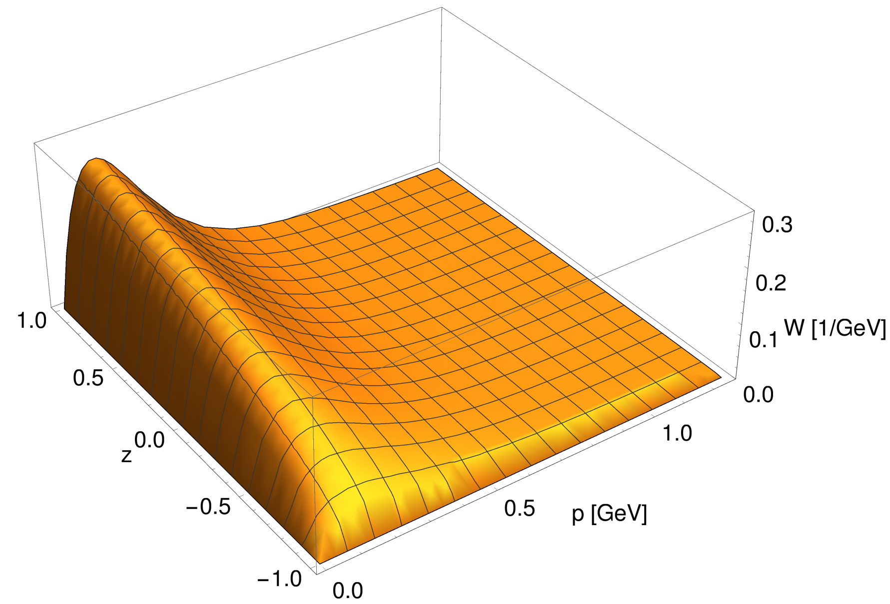
*choose* g *to reproduce* $\langle \bar{q}q\rangle = (-235 MeV)^3 \quad \Rightarrow g \simeq 2.1$

# vector form factors v , w

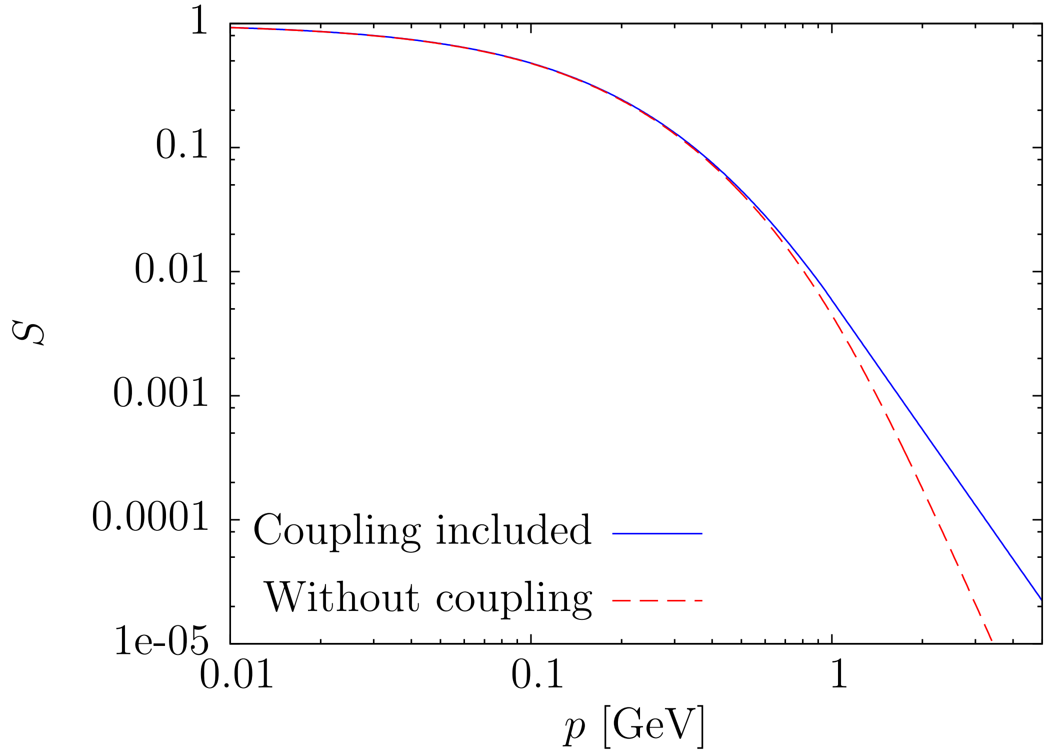



$$v, w(\vec{p}_1, \vec{p}_2): \quad p := |\vec{p}_1| = |\vec{p}_2|, \;\; z = \cos\sphericalangle(\vec{p}_1, \vec{p}_2)$$



# scalar form factor



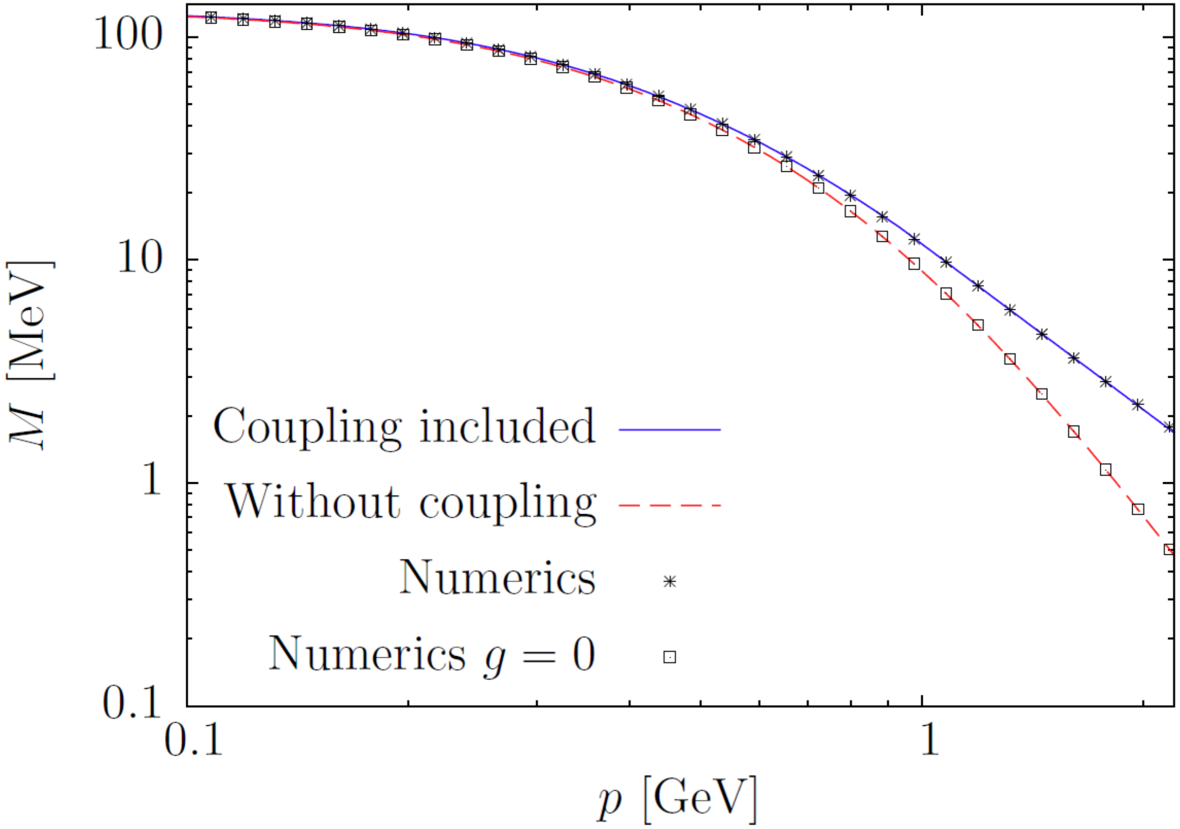

-quark-gluon coupling modifies only the mid- and high-momentum regime

-low-momentum regime is dominated by Coulomb term

# effective quark mass

$$M(\vec{p}) = \frac{2pS(\vec{p})}{1 - S^2(\vec{p})}$$

D.Campagniari, E.Ebadati, H. Reinhardt, P.Vastag, PRD **94** 074027 (2016)




Quark condensate

$$\langle \bar{q}q \rangle = (-236\,\text{Mev})^3$$

$$g = 2.1$$

Adler-Davis (g=0):

$$\langle \bar{q}q \rangle = (-185\,\text{Mev})^3$$

IR-mass:

M(0)=140MeV

> coupling to transversal gluons increases quark condensate

# Covariant vs Constituent Quark Mass

$$S_3(p) = \int \frac{dp^4}{2\pi} S(p)$$

- ***massive Dirac particle***

$$S^{-1}(p) = \not{p} - m \qquad S(p) = \frac{\not{p} + m}{p^2 - m^2} \qquad S_3(p) = \frac{\vec{\gamma}\vec{p} - m}{2E_{\vec{p}}} \qquad E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

- ***momentum dependent mass***

$$S^{-1}(p) = \not{p}A(p^2) - B(p^2) \qquad M(p^2) = B(p^2)/A(p^2)$$

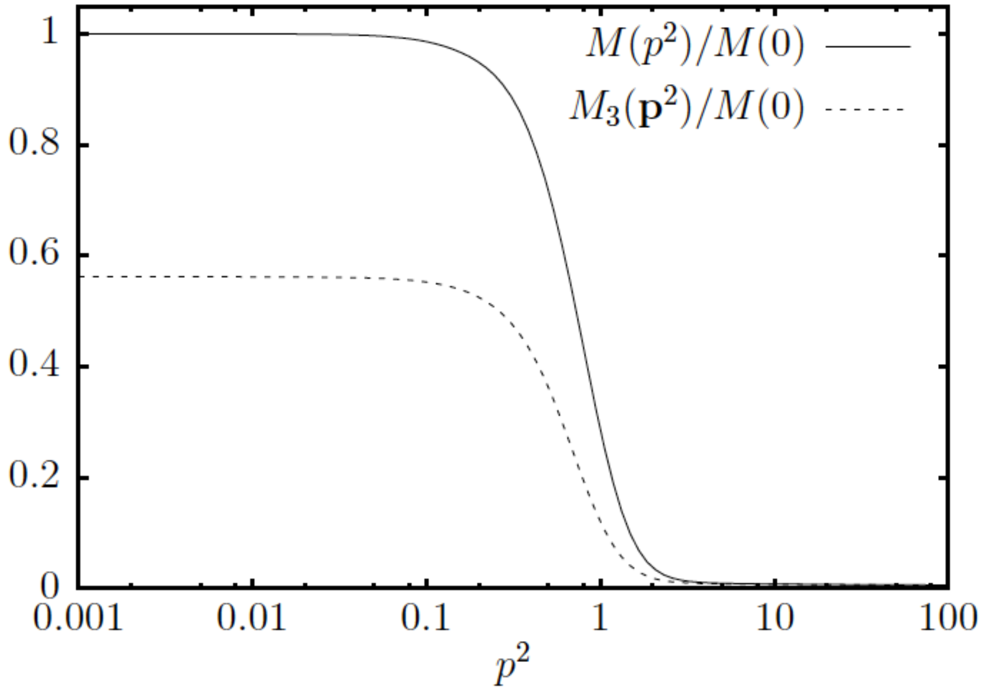
$$S_3(p) = \frac{1}{Z(p^2)} \frac{\vec{\gamma}\vec{p} - M_3(\vec{p}^2)}{2E_{\vec{p}}}$$

$$E_{\vec{p}} = \sqrt{p^2 + M_3{}^2(p^2)}$$

$$M_3(\mathbf{p}^2) = \frac{\int_0^\infty \mathrm{d}p_4 \frac{1}{A(p_4^2+\mathbf{p}^2)} \frac{M(p_4^2+\mathbf{p}^2)}{p_4^2+\mathbf{p}^2+M^2(p_4^2+\mathbf{p}^2)}}{\int_0^\infty \mathrm{d}p_4 \frac{1}{A(p_4^2+\mathbf{p}^2)} \frac{1}{p_4^2+\mathbf{p}^2+M^2(p_4^2+\mathbf{p}^2)}}$$

*D. Campagnari & H. R , PRD97(2018)*

# Covariant vs Constituent Quark Mass




*D. Campagnari & H. R ,*
*PRD97(2018)*

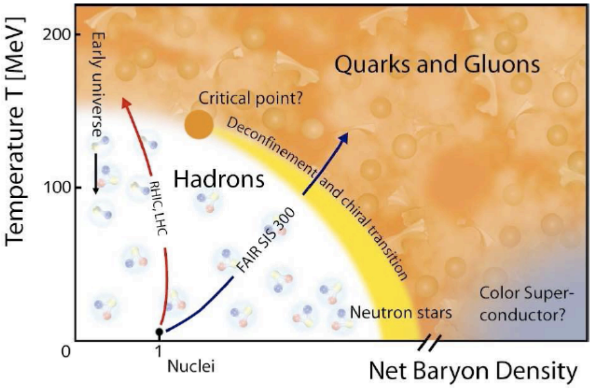
*M(p) from DSE,*
*M. Huber*

$$M_3(0) \approx \tfrac{1}{2} M(0)$$

# *summary of T=0 calculation*

- Hamiltonian approach to QCD in Coulomb gauge:
  - decent discription of the IR properties
    - confinement
    - SB of chiral symmetry
  - reasonable agreement with lattice data

# Hamiltonian approach to finite temperature QFT




- partition function

$$Z(L) = Tr\exp(-LH) \qquad T = L^{-1}$$

- necessitates approximation to density operator $\exp(-LH)$
- common: quasiparticle approximation(Wick´s theorem)

>alternative Hamiltonian approach to finite temperature QFT:

*compactification of a spatial dimension*

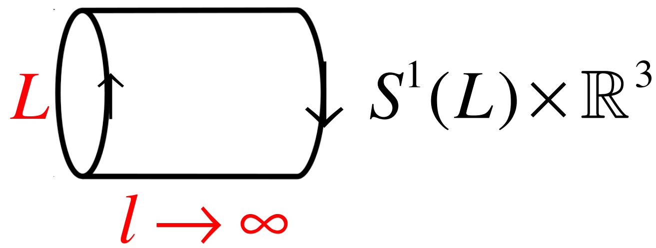
*H. R. Phys.Rev.D94(2016)045016*

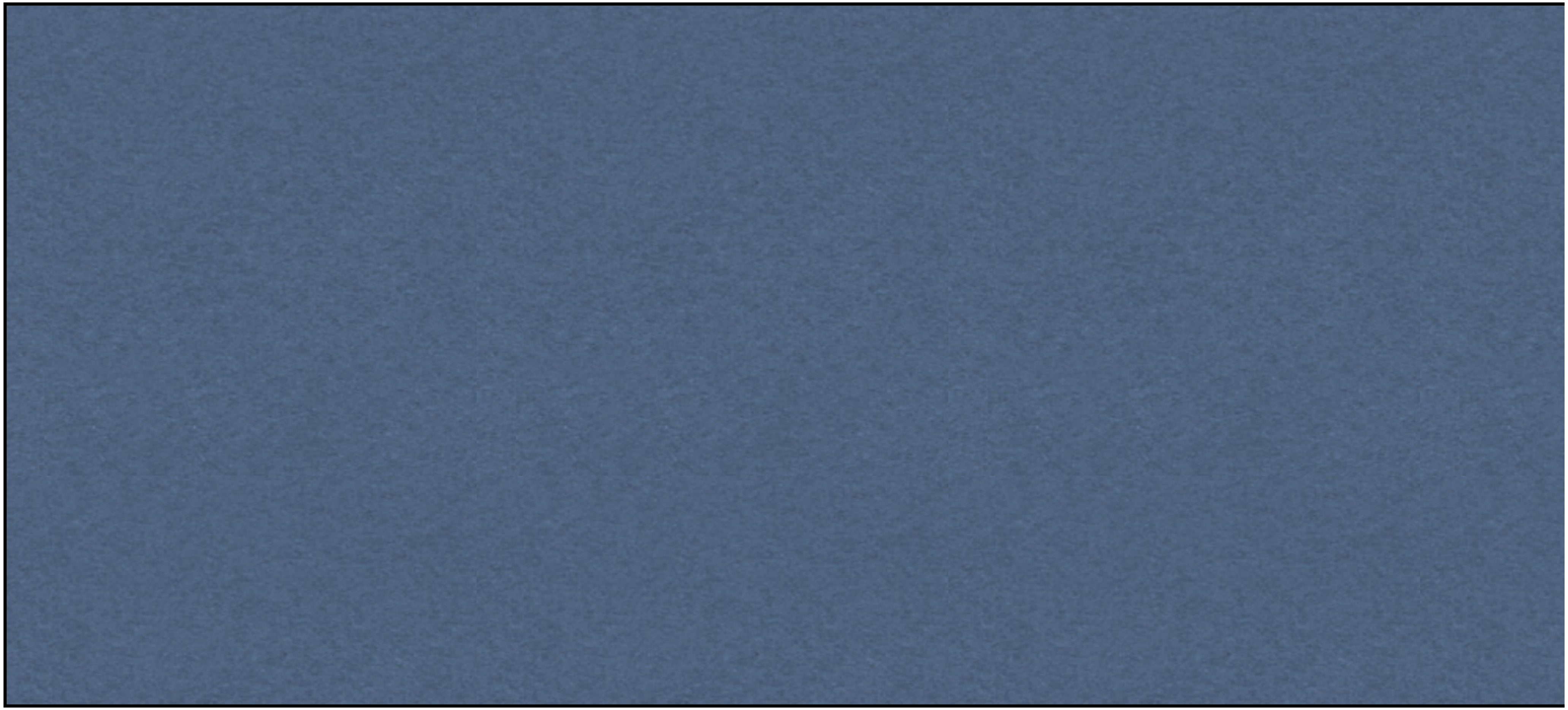
# Finite temperature QFT

$$Z(L) \equiv Tr\exp(-LH) = \int_{bc} D(A,\psi)\exp\left[-\int_0^L dx^0 \int d^3x L_E(A,\psi)\right]$$

$T = L^{-1}$

- compactification of (Euclidean) time
  - bc: $A(x^0 = L/2) = A(x^0 = -L/2)$ Bose fields
    $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields

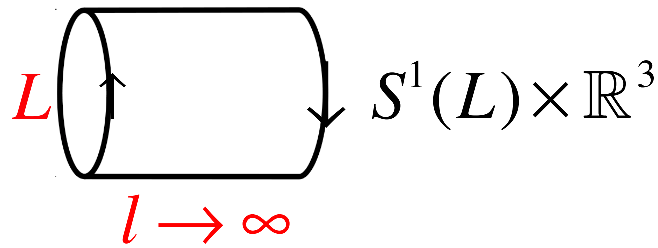


# Finite temperature QFT

$$Z(L) \equiv Tr\exp(-LH) = \int_{bc} D(A,\psi)\exp\left[-\int_0^L dx^0 \int d^3x L_E(A,\psi)\right] \qquad T = L^{-1}$$

- compactification of (Euclidean) time




- bc: $A(x^0 = L/2) = A(x^0 = -L/2)$ Bose fields

  $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields

- exploit the O(4)-invariance of the Euclidean Lagrange density to rotate the time axis onto a spatial axis

$x^0 \to x^3 \qquad A^0 \to A^3 \qquad \gamma^0 \to \gamma^3$

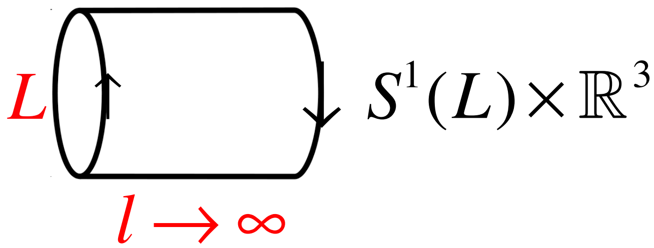
$x^1 \to x^0 \qquad A^1 \to A^0 \qquad \gamma^1 \to \gamma^0$

# Finite temperature QFT

$$Z(L) \equiv Tr\exp(-LH) = \int_{bc} D(A,\psi)\exp\left[-\int_0^L dx^0 \int d^3x L_E(A,\psi)\right]$$

$$T = L^{-1}$$

- compactification of (Euclidean) time



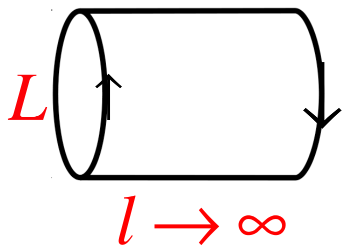

  - bc: $A(x^0 = L/2) = A(x^0 = -L/2)$ Bose fields
    $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields

  - exploit the O(4)-invariance of the Euclidean Lagrange density to rotate the time axis onto a spatial axis

$x^0 \to x^3$ $A^0 \to A^3$ $\gamma^0 \to \gamma^3$

$x^1 \to x^0$ $A^1 \to A^0$ $\gamma^1 \to \gamma^0$

- compactification of one spatial dimension

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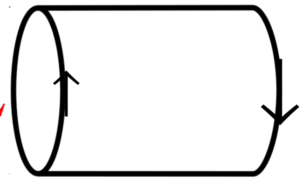
    $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields

[redacted]

# Finite temperature QFT

$$Z(L) \equiv Tr\exp(-LH) = \int_{bc} D(A,\psi)\exp\left[-\int_0^L dx^0 \int d^3x L_E(A,\psi)\right] \qquad T = L^{-1}$$

- compactification of (Euclidean) time

  $L$  $S^1(L)\times\mathbb{R}^3$

  $l \to \infty$

  - bc:
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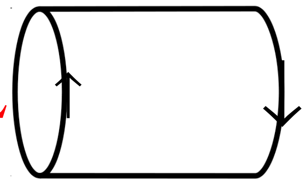
- compactification of one spatial dimension
  - bc:
    $A(x^3 == L/2) = A(x^3 = -L/2)$ Bose fields
    $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields
- canonical quantization on the spatial manifold $S^1(L)\times\mathbb{R}^2$

  $L$ 

  $l \to \infty$

$$Z(L) = \lim_{l\to\infty} Tr\exp(-lH(L)) = \lim_{l\to\infty}\sum_n \exp(-lE_n(L)) = \lim_{l\to\infty}\exp(-lE_0(L))$$

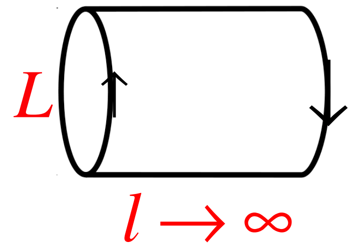
- *temperature is now encoded in a „spatial“ dimension while „time“ has infinite extension independent of the temperature*

# *Hamiltonian approach to finite temperature QFT*

- partition function

*H. R. Phys.Rev.D94(2016)045016*

$$Z(L) = \lim_{l\to\infty} Tr \exp(-lH(L)) = \lim_{l\to\infty} \exp(-lE_0(L))$$




*thermodynamics of a relativistic QFT is completely given given by its vacuum state on the spatial manifold* $\mathbb{R}^2 \times S^1(L)$

- ground state energy on $\mathbb{R}^2 \times S^1(L)$ $\qquad E_0(L) = l^2 L e(L)$

- pressure: $P = -\partial[Ve(L)]/\partial V \qquad V = l^3$ $\qquad P = -e(L)$

- energy density: $\varepsilon = \partial[Le(L)]/\partial L - \mu \partial e/\partial \mu$

- Dirac fermions with finite chemical potential

$$h = \vec{\alpha}\cdot\vec{p} + \beta m \to h + i\mu\alpha^3$$

# Relativistic Bose gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p\,\frac{p^2}{\omega(p)}\,n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)}-1} \qquad \omega(p) = \sqrt{p^2+m^2}$$

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- (pseudo-)energy density on $\mathbb{R}^2 \times S^1(L)$

$$e(L) = \frac{1}{2}\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty} \sqrt{m^2 + p_\perp^2 + \omega_n^2} \qquad \omega_n = \frac{2\pi n}{L}$$

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$$\sqrt{A} = \frac{1}{\Gamma(-\frac{1}{2})}\lim_{\Lambda\to\infty}\int_{1/\Lambda^2}^{\infty} d\tau \exp(-\tau A)$$

- proper-time regularization

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$$\tfrac{1}{2\pi}\sum_{k=-\infty}^{k=\infty} e^{ikx} = \sum_{n=-\infty}^{n=\infty}\delta(x - 2\pi n)$$

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modified Bessel function

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**modified Bessel function**

- massles bosons: m=0

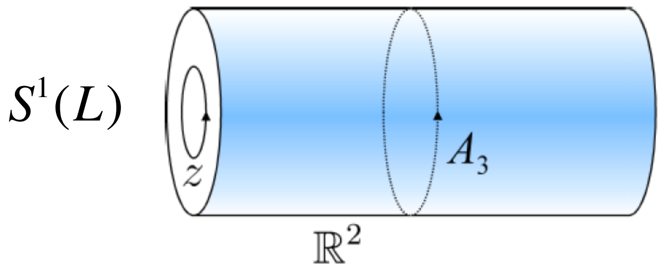
*Stephan – Boltzmann – law*

$$P = \frac{\zeta(4)}{\pi^2}T^4 = \frac{\pi^2}{90}T^4$$

# QCD at finite T

- Hamiltonian approach in Coulomb gauge on the partially compactified spatial manifold $\mathbb{R}^2 \times S^1(L)$

H. R. Phys.Rev.D94(2016)045016




- finite temperature is fully encoded in the vacuum
- variational solution of the Schrödinger equation for the vacuum

*chiral phase transition*

>quark condensate

M.Quandt, E.Ebadati, H.R. & P.Vastag
Phys. Rev D98,arXiv:1806.04493

*deconfinement phase transition*

>Polyakov loop

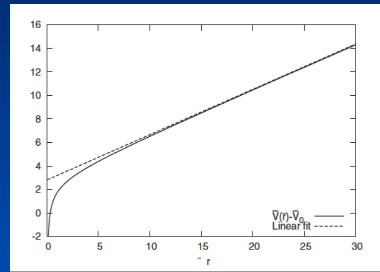
H. R. & J. Heffner, PRD88

M.Quandt & H.R. to be published

# The QCD Hamiltonian in Coulomb gauge: Adler-Davis model

*-neglect coupling of quarks to the spatial gluons*

*-keep only IR part of the Coulomb potential*




$$H_{AD} = \int d^3x \Psi^\dagger(x)\vec{\alpha}\vec{p}\Psi(x) + \tfrac{1}{2}\int d^3x d^3y \rho(\vec{x}) V_C(\vec{x}-\vec{y})\rho(\vec{y})$$

$$V_C(\mathbf{p}) = \frac{8\pi\sigma_C}{|\mathbf{p}|^4}$$

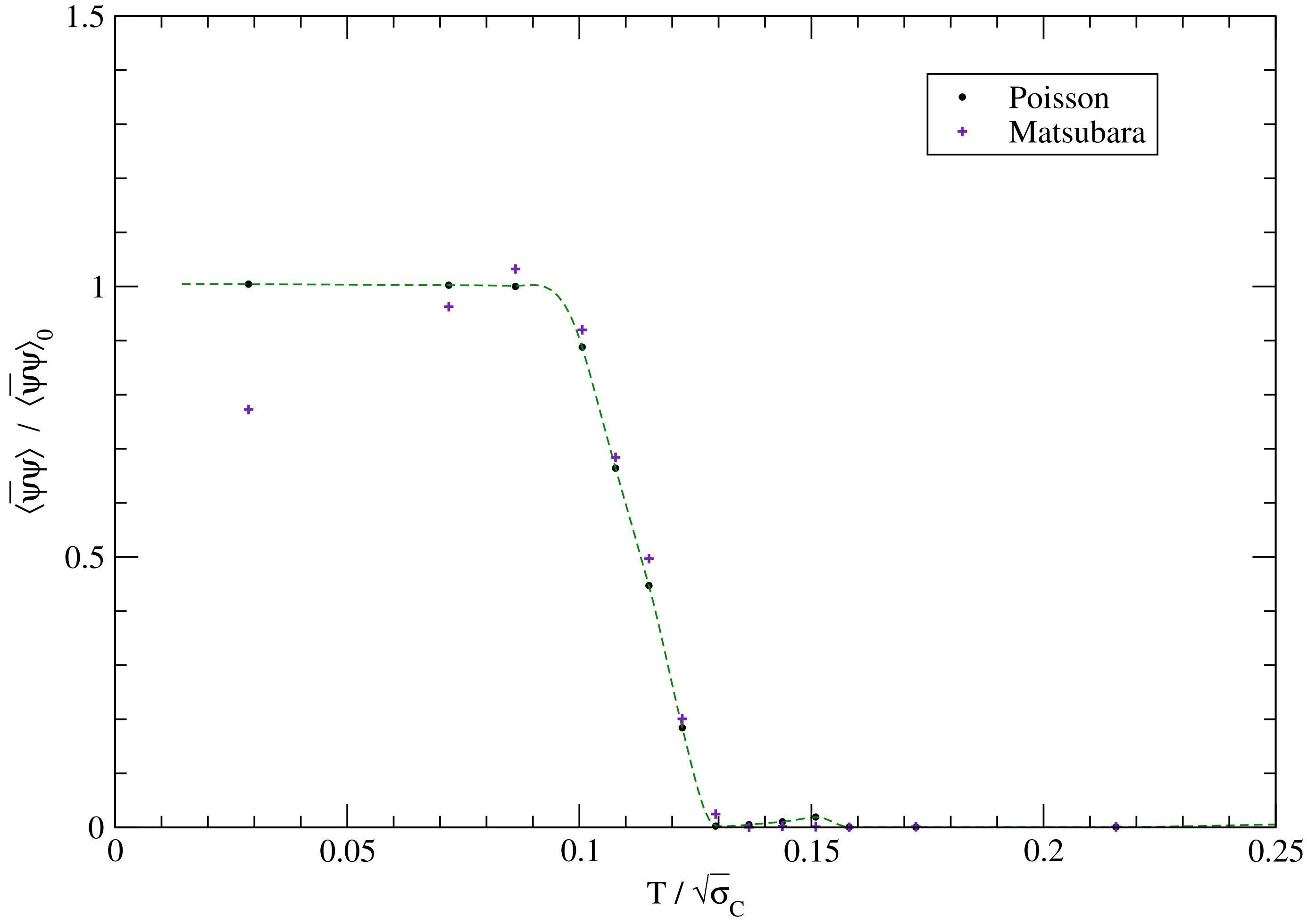
*color charge density* $\rho^a(x) = \Psi^\dagger(x) t^a \Psi(x)$

*wave functional* $|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger \beta s \Psi_-\right]|0\rangle$

$s$ – variational kernel

*UV-finite*

# Quark condensate



# Adler-Davis model on $\mathbb{R}^2 \times S^1(L)$

*-2. order transition*

*critical temperature:*

$$T_\chi = 0.13\sqrt{\sigma_C}$$

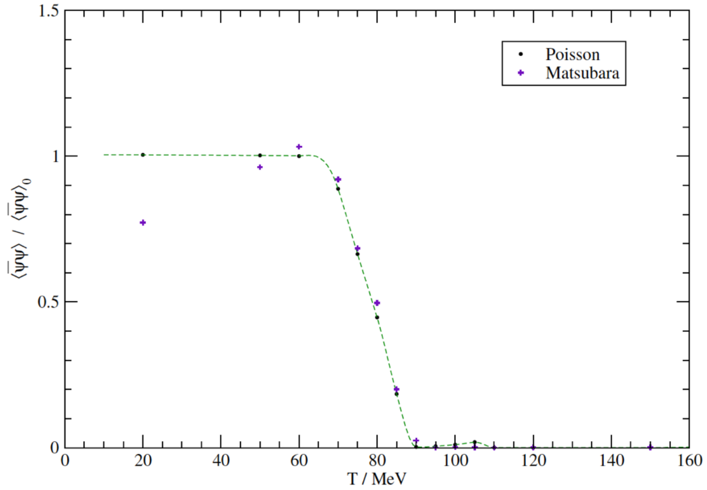



FIG. 7: Chiral condensate as a function of the temperature, from both the Matsubara and Poisson ormulation. The dashed line indicates a fit to the Poisson data from which the critical temperature is determined.

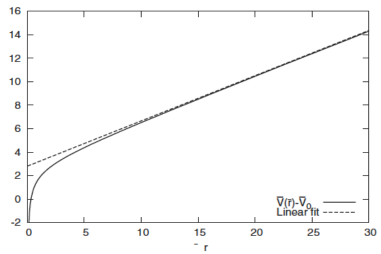
-neglect of spatial gluons

$$adjust \quad \sigma_C = 4.1\sigma$$

$$\langle \bar{q}q \rangle = (-235\,MeV)^3$$

$$T_\chi = 115\,MeV$$

$$T^{lat}_\chi = 155\,MeV$$



-neglect of UV-part of the Coulomb potential
-quenched: T=0 gluon vacuum: $\sigma_C$ increases with T

*canonical finite temperature Hamiltonian approach with quasiparticle approx. to the density operator exp(-H/T):*

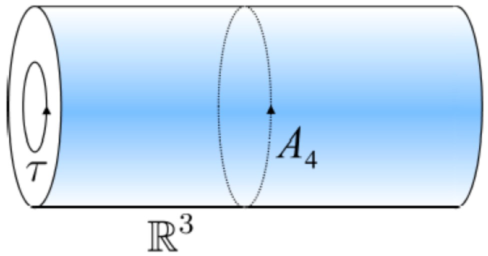
$$T_\chi = 0.091\sqrt{\sigma_C}$$

# The Polyakov loop in the Hamiltonian approach

$A_0 = 0$

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[ i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$$T^{-1} = L$$

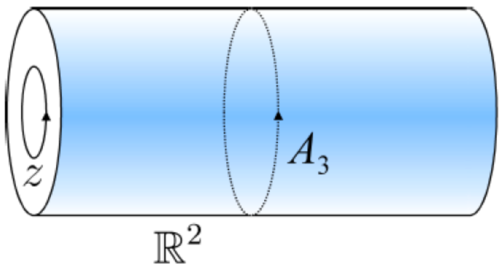



**canonical Hamiltonian approach to finite T:**
**Polyakov loop - not accessible**

**alternative Hamiltonian approach to finite T**
**with a compactified spatial dimension:**
**Polyakov loop - accessible**

$$P[A_3](\vec{x}_\perp) = \tfrac{1}{d_r} tr P \exp\left[ i \int_0^L dx_3 A_3(\vec{x}_\perp, x_3) \right]$$

$$T^{-1} = L$$



# The Polyakov loop- order parameter of confinement

$$P[A_0](\vec{x}) = \tfrac{1}{d_r}\, tr P \exp\left[ i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

*Polyakov gauge:* $\partial_0 A_0 = 0, \quad A_0 = diagonal$

$SU(2): P[A_0](\vec{x}) = \cos(A_0(\vec{x})L)$ unique function of $A_0$

$$\langle P[A_0](\vec{x}) \rangle \qquad P[\langle A_0 \rangle](\vec{x}) \qquad \langle A_0(\vec{x}) \rangle$$

*J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*

# effective potential of the Polyakov loop in the Hamiltonian approach on $\mathbb{R}^2 \times S^1(L)$

$$\langle H \rangle_a = \min \langle H \rangle \text{ under the constraint: } \langle A \rangle = a$$

$$\langle H \rangle_a = (\text{"}spatial\text{" } volume)\ \ e[a]$$

$e[a_3]$ (pseudo-) energie density on $\mathbb{R}^2 \times S^1(L)$

*background gauge:* $[\partial + a, A] = 0$

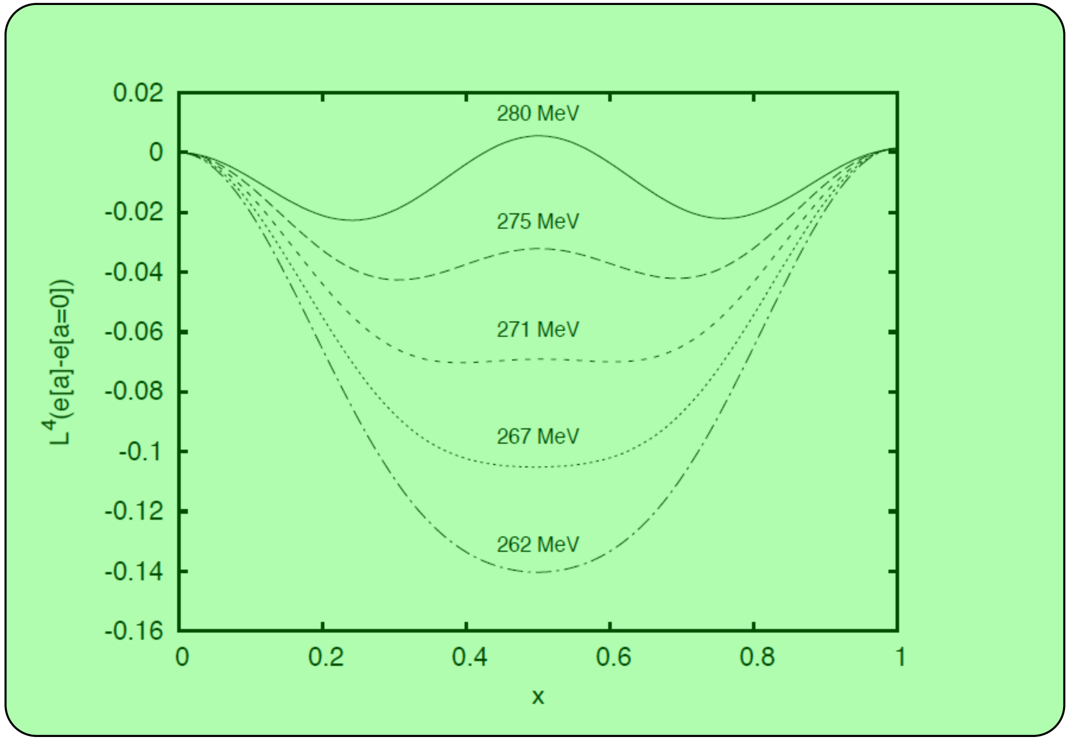
$$G_{a\,,\,background\ \ gauge}(\vec{p}) = G_{a=0\,,\,Coulomb\ \ gauge}(\tilde{p})$$

*fermionic line:* $\tilde{p} = \vec{p} - \vec{a}\mu$ *weights*

*bosonic line:* $\tilde{p} = \vec{p} - \vec{a}\sigma$ *roots*

# The gluon effective potential *SU(2)*

variational calculation in Coulomb gauge




$$x = \frac{aL}{2\pi}$$

second order phase transition:

$$input: \quad M = 880\,MeV \qquad\qquad T_C \simeq 269\,MeV$$

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013)

# The effective potential for SU(3)

SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive rooots

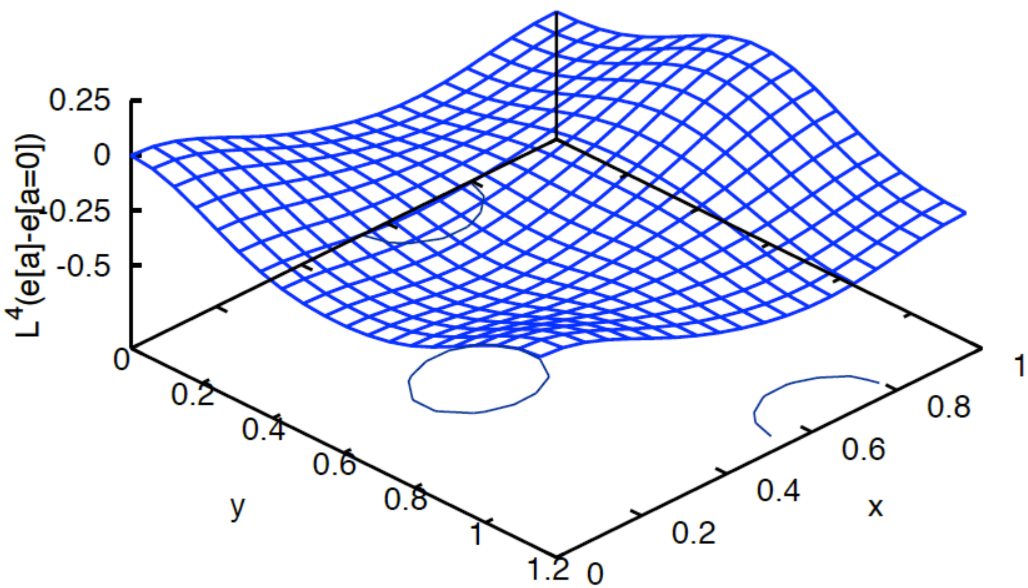
$$\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

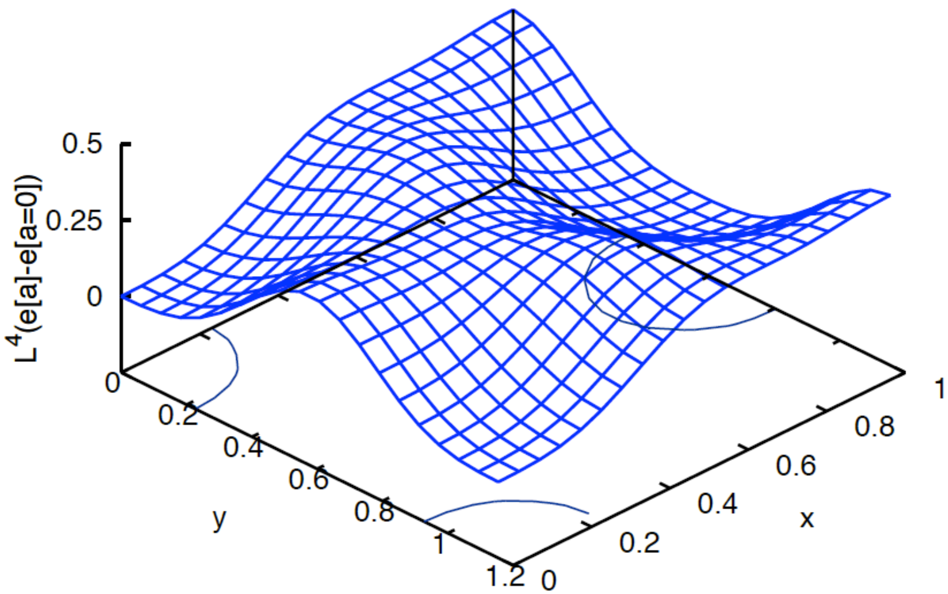
# The full effective potential for SU(3)

variational calculation in Coulomb gauge

$T < T_C$



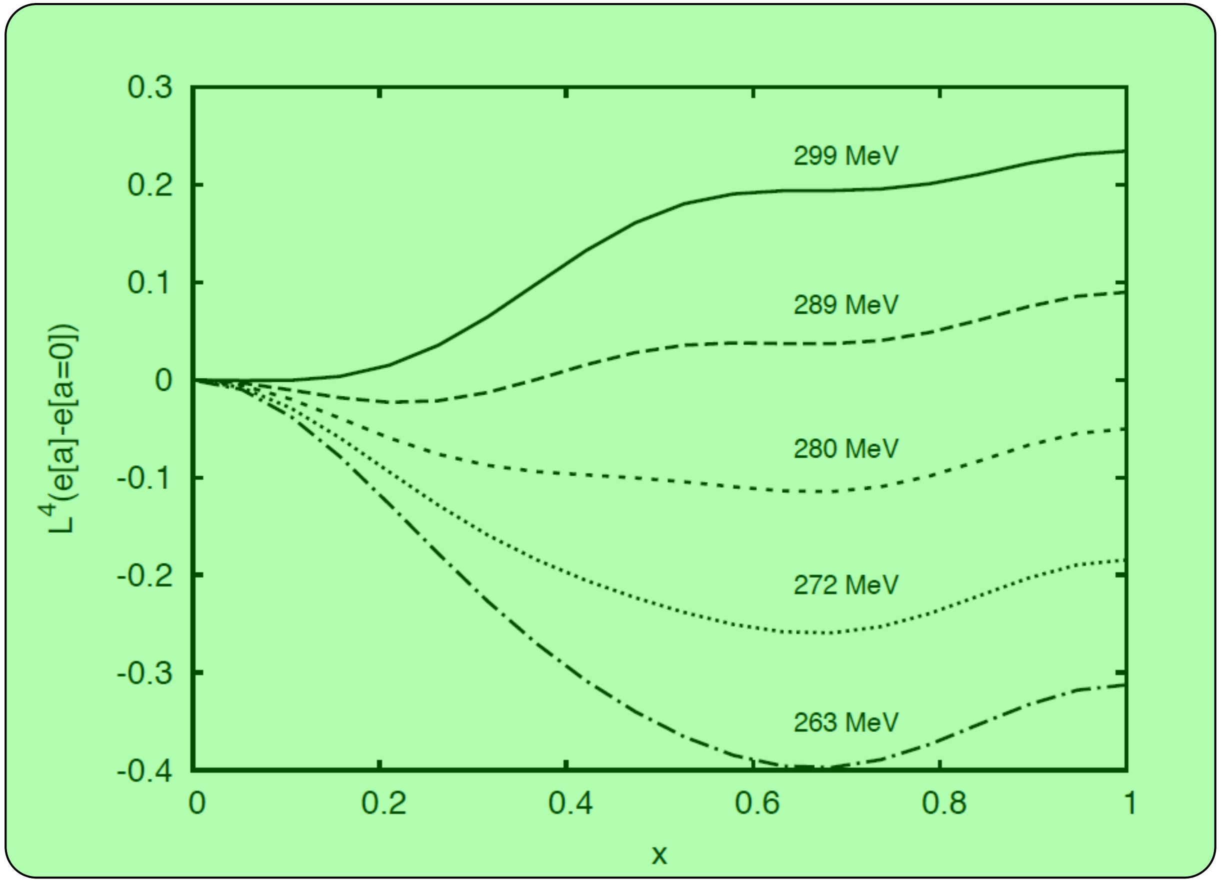

$T > T_C$




$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi}$$

H. Reinhardt & J. Heffner,Phys.Rev.D88(2013)

# Polyakov loop potential for SU(3)



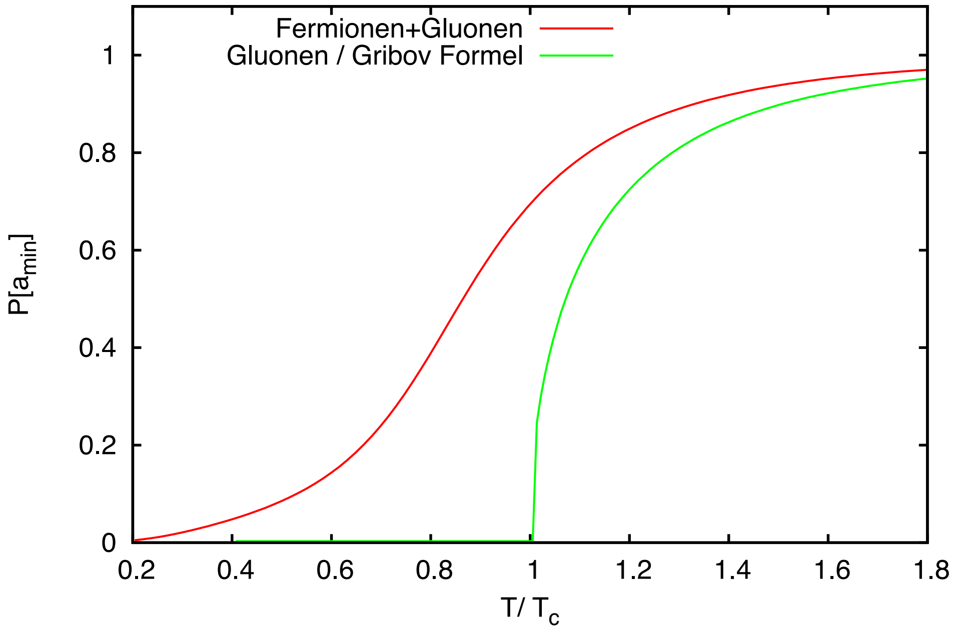

$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi} = 0$$

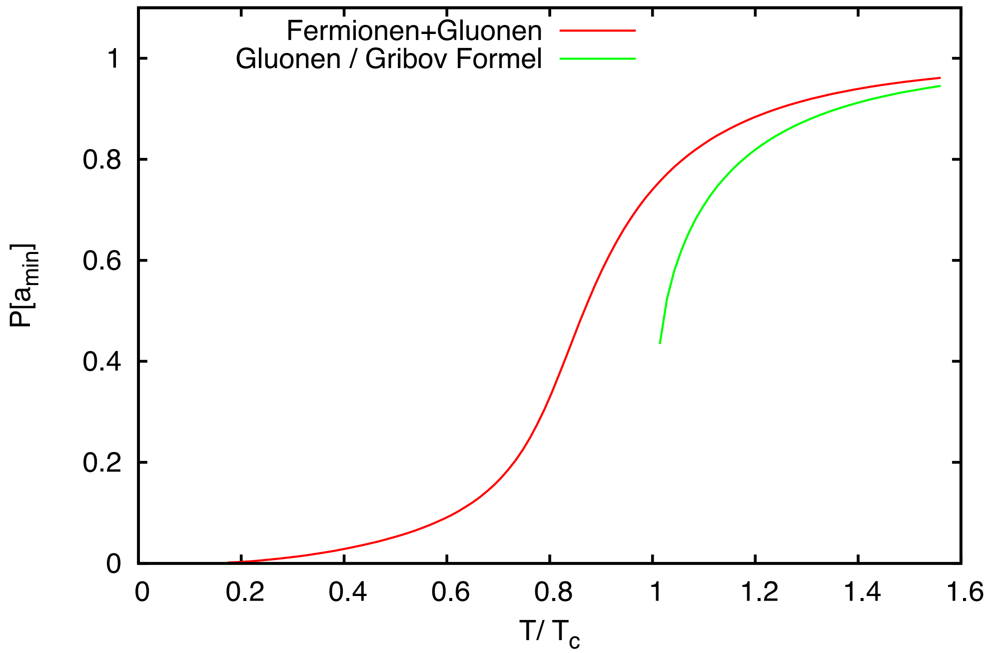
$$input: \; SU(2) - data:$$

$$M = 880\,MeV$$

$$T_C = 283 MeV$$

# The Polyakov loop




SU(2)

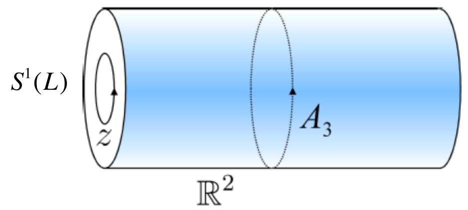
SU(3)

- *no ghost loop*
- *no Coulomb term*

*M. Quandt & H. R, to be published*

# Conclusions

- Hamiltonian approach to QCD in Coulomb gauge at T=0
  - decent description of the IR sector
    - confinement
    - chiral symmetry breaking
  - satisfactory agreement with lattice
- QCD at finite temperature
  - compactification of a spatial dimension




  - chiral phase transition
    - weak second order
  - effective potential of the Polyakov loop
    - deconfinement phase transition in YMT
    - SU(2): 2.order
    - SU(3): 1.order
    - inclusion of quarks:
    - deconfinement phase transition is turned into a crossover
  - dual quark condensate
- outlook: -Polyakov loop with Coulomb term(2-loop)
- -finite chemical potential

Thanks for your attention