Net-proton number fluctuations in the presence of the QCD critical point

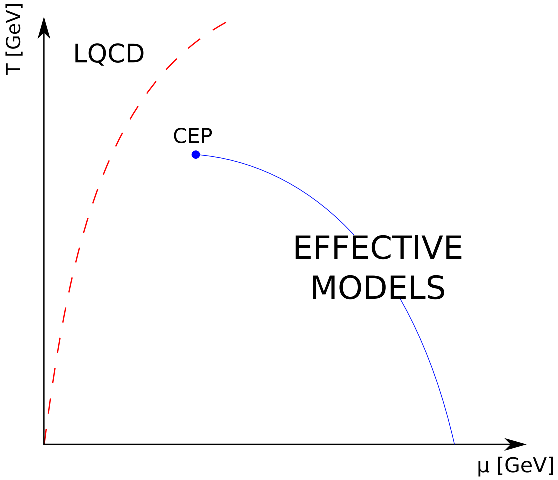
Michał Szymański

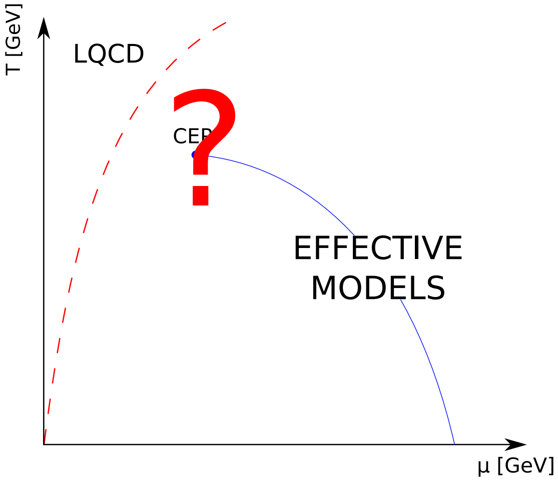
Collaborators: M. Bluhm, K. Redlich, C. Sasaki

Institute of Theoretical Physics, University of Wroclaw

40th Max Born Symposium,
Wrocław, 9-12.10.2019

arXiv:1905.00667


T [GeV]
LQCD
CEP
EFFECTIVE
MODELS
μ [GeV]


T [GeV]
LQCD
CEP
?
EFFECTIVE
MODELS
μ [GeV]

Experimental searches for CP

- Heavy ion collisions $\rightarrow$ Allow to probe different regions of QCD phase diagram
- Baryon number fluctuations $\sim$ Proton number fluctuations
- Non-monotonic $\sqrt{s}$ dependence of higher cumulants observed $\rightarrow$ signature of CP?
- No conclusive results yet $\rightarrow$ Models needed!

This talk

- Ratios of net-proton number cumulants in the presence of CP
- Phenomenological approach $\rightarrow$ HRG model + critical fluctuations

Thermal baseline $\rightarrow$Hadron resonance gas (HRG) model

- QCD pressure $\sim$ Non-interacting gas of hadrons and resonances
  $\rightarrow$ No critical fluctuations

Coupling to critical mode fluctuations $\rightarrow$ No general prescription on modeling this effect

Phenomenological approach[1]:

- Linear sigma models

$$m_p \sim m_0 + g\sigma$$

- σ fluctuations $\rightarrow$ Distribution function modified ($i = p, \bar{p}$)

$$f_i = f_i^0 + \delta f_i,$$

$$\delta f_i = \frac{\partial f_i}{\partial m_p}\delta m_p = -\frac{g}{T}\frac{m_p}{E} f_i^0(1 - f_i^0)\delta\sigma\,,$$

[1] M. Bluhm et al., Eur. Phys. J. C **77**, no. 4, 210 (2017)

nth order cumulant ($i = p, \bar{p}$):

$$C_n^i = VT^3 \frac{\partial^{n-1}(n_i/T^3)}{\partial(\mu_i/T)^{n-1}}\bigg|_T ,$$

Net-proton number cumulants ($n = 1, ..., 4$):

$$C_n = C_n^p + (-1)^n C_n^{\bar{p}} + (-1)^n \langle (V\delta\sigma)^n \rangle_c (m_p)^n (J_p - J_{\bar{p}})^n + \begin{pmatrix} \text{less singular} \\ \text{terms} \end{pmatrix}$$

$$J_i = \frac{gd}{T} \int \frac{d^3k}{(2\pi)^3} \frac{1}{E} f_i^0 (1 - f_i^0)$$

Critical mode cumulants ($n \geq 2$) $\rightarrow$ Universality

$$\begin{aligned} \text{QCD} &\longleftrightarrow \text{3D Ising model} \\ \sigma &\longleftrightarrow M_I \\ (T, \mu) &\longleftrightarrow (r, h) \end{aligned}$$

$$\langle (V\delta\sigma)^n \rangle_c \propto \frac{\partial^{n-1} M_I}{\partial h^{n-1}}\bigg|_r$$

Problem with this approach

$$C_2^{\text{sing.}} \sim \frac{\partial M_I}{\partial h} = \chi_I \longleftrightarrow \chi_{\text{chiral}} \text{ in QCD}$$

$$\chi_{\mu\mu}^{\text{sing.}} \approx C_2^{\text{sing.}} \sim \chi_{\text{chiral}}^{\text{sing.}} \Rightarrow \text{Too strong divergence of } C_2!$$

χ_{chiral} and $\chi_{\mu\mu}$ in the mean-field NJL model[1]:

$$\chi_{\mu\mu} \simeq \chi_{\mu\mu}^{reg} + \sigma^2 \chi_{\text{chiral}}$$

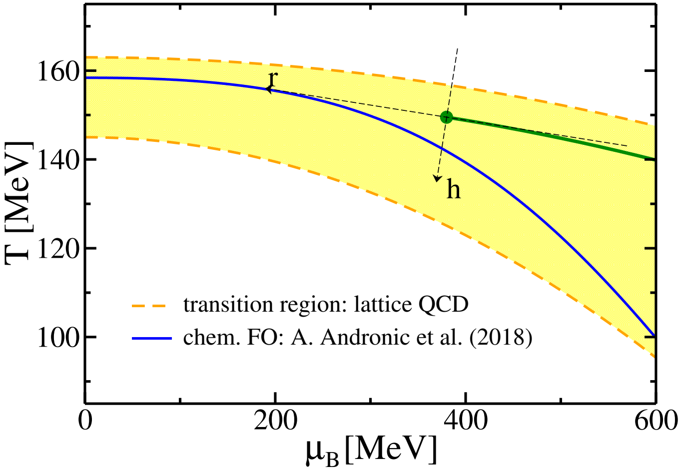
Refined cumulants:

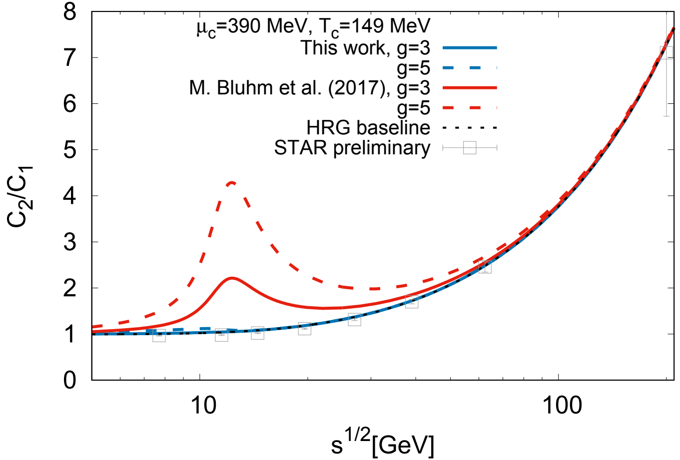
$$C_2 = C_2^p + C_2^{\bar{p}} + g^2\sigma^2 \langle (V\delta\sigma)^n \rangle (J_p - J_{\bar{p}})^2$$
$$C_3 = C_3^p - C_3^{\bar{p}} - g^3\sigma^3 \langle (V\delta\sigma)^n \rangle (J_p - J_{\bar{p}})^3$$
$$C_4 = C_4^p + C_4^{\bar{p}} + g^4\sigma^4 \langle (V\delta\sigma)^n \rangle (J_p - J_{\bar{p}})^4$$

Cumulant ratios

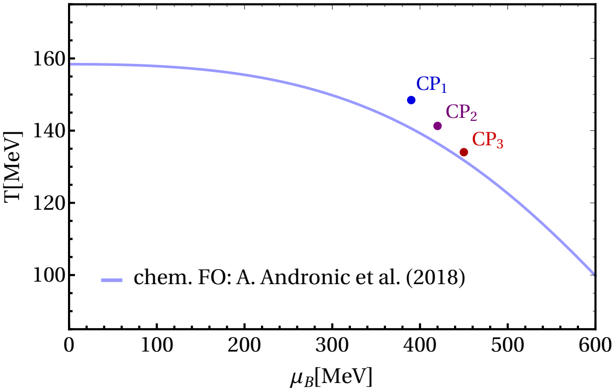
$$\frac{C_2}{C_1} = \frac{\sigma^2}{M}, \qquad \frac{C_3}{C_2} = S\sigma, \qquad \frac{C_4}{C_2} = \kappa\sigma^2,$$

[1] Y. Hatta, T. Ikeda, Phys. Rev. D **67**, 014028 (2003); C. Sasaki et al., Phys. Rev. D **77**, 034024 (2008)

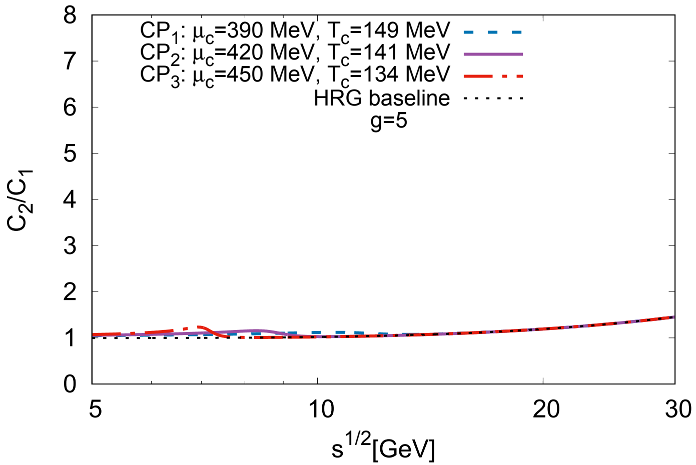

160
140
120
100
T [MeV]
0
200
400
600
μ_B [MeV]
r
h
transition region: lattice QCD
chem. FO: A. Andronic et al. (2018)



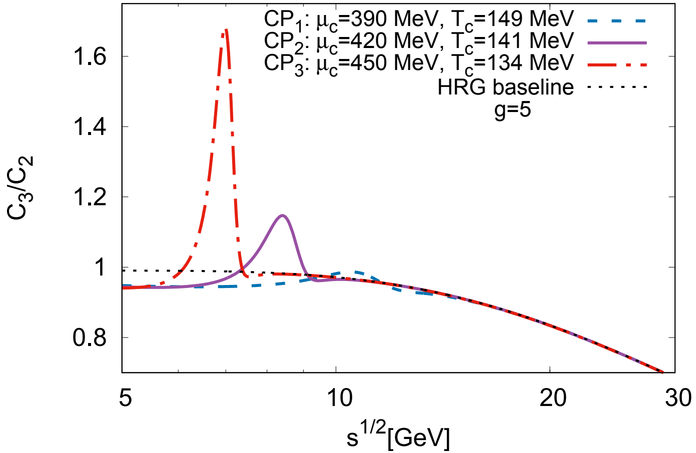
μ_c=390 MeV, T_c=149 MeV
This work, g=3
g=5
M. Bluhm et al. (2017), g=3
g=5
HRG baseline
STAR preliminary
C_2/C_1
0
1
2
3
4
5
6
7
8
10
100
$s^{1/2}$[GeV]



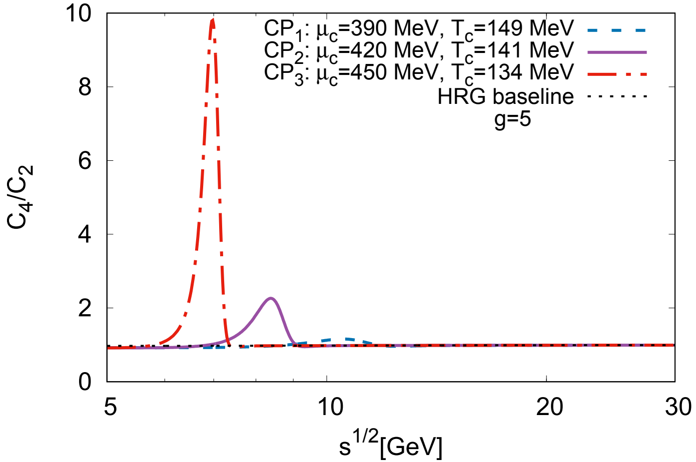
CP_i	μ_{cp} [MeV]	T_{cp} [MeV]
1	390	149
2	420	141
3	450	134



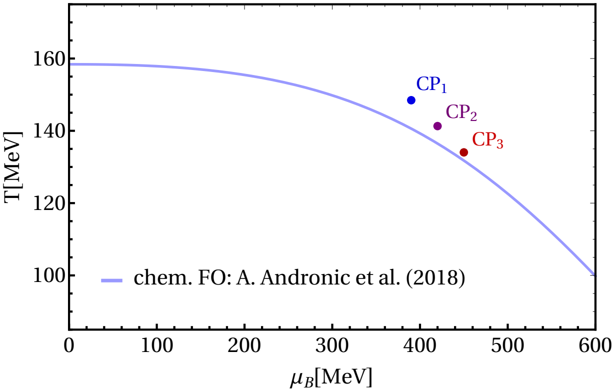
Distance to the FO curve: CP_1 - farthest, CP_3 - closest



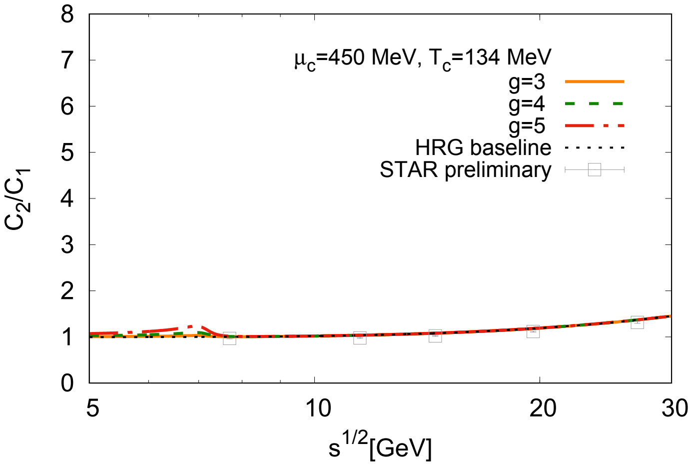
Distance to the FO curve: CP_1 - farthest, CP_3 - closest



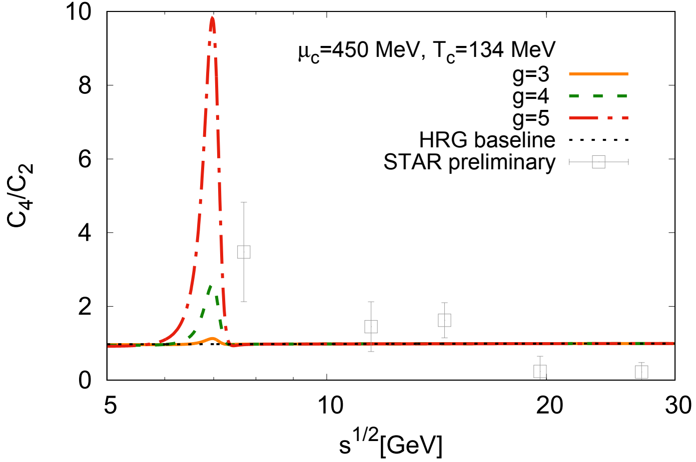
Distance to the FO curve: CP_1 - farthest, CP_3 - closest

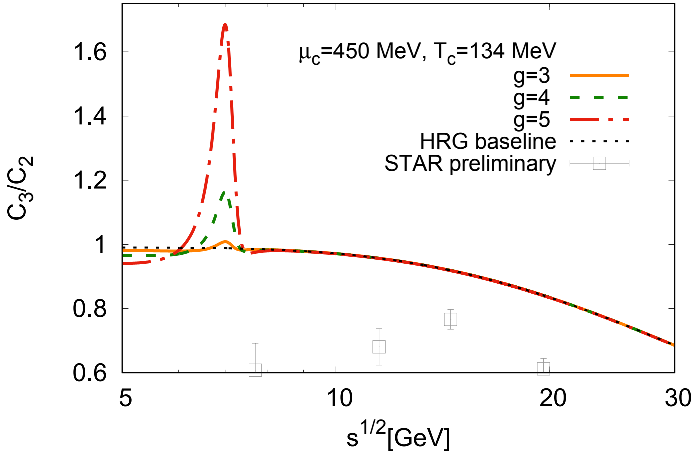


CP_i	μ_{cp} [MeV]	T_{cp} [MeV]
1	390	149
2	420	141
3	450	134



μc=450 MeV, Tc=134 MeV
g=3
g=4
g=5
HRG baseline
STAR preliminary
C2/C1
s1/2[GeV]
0
1
2
3
4
5
6
7
8
5
10
20
30


μ_c=450 MeV, T_c=134 MeV
g=3
g=4
g=5
HRG baseline
STAR preliminary
C_4/C_2
$s^{1/2}$[GeV]
0
2
4
6
8
10
5
10
20
30

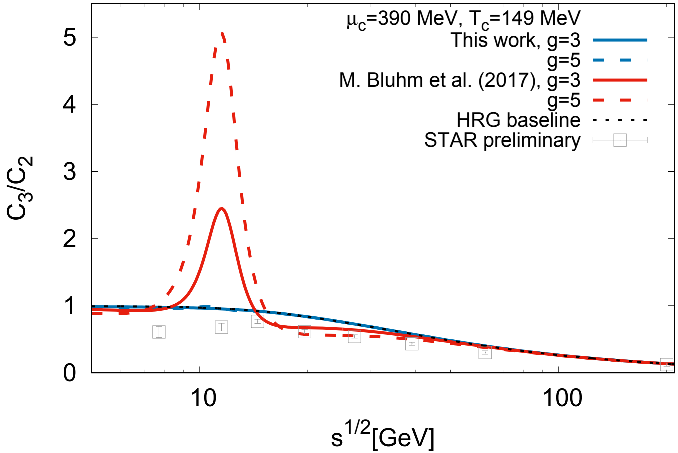

μ_c=450 MeV, T_c=134 MeV
g=3
g=4
g=5
HRG baseline
STAR preliminary
C_3/C_2
$s^{1/2}$[GeV]
1.6
1.4
1.2
1
0.8
0.6
5
10
20
30

Conclusions:

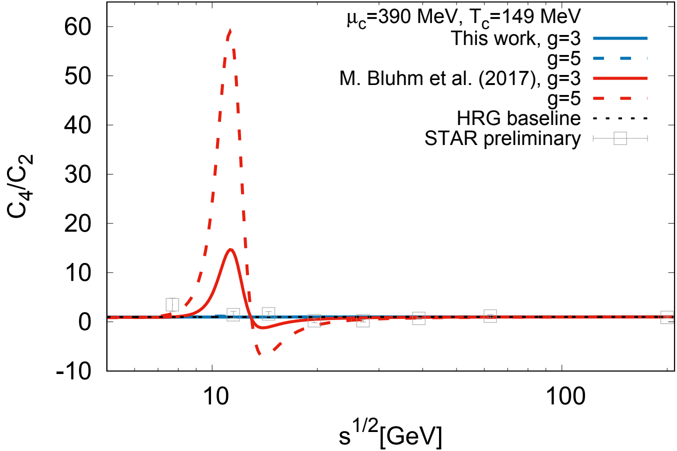
- This talk $\rightarrow$ ratios of net-proton number cumulants obtained with an effective model
- Identification of CP from the data $\rightarrow$ Need to know the systematics of cumulants expected from the Z_2 scaling
  - Systematics in the data $\neq$ Systematics expected from Z_2
  - CP close to FO curve $\rightarrow$ Rather unlikely but more study needed
    1. Role of less critical contributions to cumulants[1]
    2. Impact of resonance decays on net-proton number fluctuations[2]
    3. Out of equilibrium effects

[1] A. Bzdak et al. Phys. Rev. C **95** no. 5, 054906 (2017)
[2] M. Bluhm et al., Eur. Phys. J. C **77**, no. 4, 210 (2017)

Appendix



μc=390 MeV, Tc=149 MeV
This work, g=3
g=5
M. Bluhm et al. (2017), g=3
g=5
HRG baseline
STAR preliminary
C3/C2
s1/2[GeV]
0
1
2
3
4
5
10
100



μc=390 MeV, Tc=149 MeV
This work, g=3
g=5
M. Bluhm et al. (2017), g=3
g=5
HRG baseline
STAR preliminary
C4/C2
60
50
40
30
20
10
0
-10
10
100
s1/2[GeV]

The recent[1] parametrization of chemical freeze-out conditions reads:

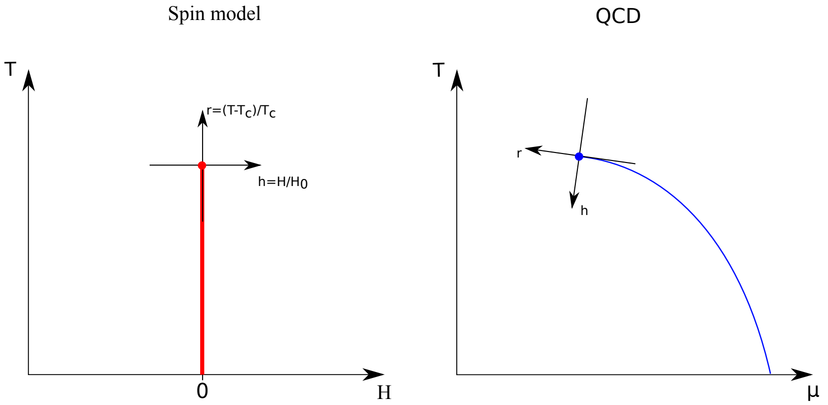
$$\mu_{fo}(\sqrt{s}) = \frac{a}{1 + 0.288\sqrt{s}}, \quad a = 1307.5\,\mathrm{MeV}$$

$$T_{fo}(\sqrt{s}) = \frac{T_{CF}^{lim}}{1 + \exp(2.60 - \ln(\sqrt{s})/0.45)}, \quad T_{CF}^{lim} = 158.4\,\mathrm{MeV}$$

[1] A. Andronic et al., Nature **561**, no. 7723, 321 (2018)

Mapping from QCD to the spin model phase diagram:

$$\tilde{r} = \frac{\mu_B - \mu_{cp}}{\Delta\mu_{cp}}, \qquad \tilde{h} = \frac{T - T_{cp}}{\Delta T_{cp}},$$



Spin model equation of state reads

$$M_l = M_0 R^{\beta} \theta$$

where (R, θ) is obtained from

$$\begin{aligned} r &= R(1-\theta^2), \\ h &= R^{\beta\delta} w(\theta), \end{aligned}$$

β and δ are critical exponents and

$$w(\theta) = c\theta(1 + a\theta^2 + b\theta^4).$$