Higher order cumulants and factorization breaking in heavy ion collision

Rupam Samanta

PhD Supervisor : Prof. dr. hab. Piotr Bozek

Faculty of Physics and Applied Computer Science
AGH University of Science and Technology, Krakow
NCN grant : 2018/29/B/ST2/00244

Young scientists' workshop and 58. Karpacz Winter School of Theoretical Physics "Heavy Ion Collision: From First to Last Scattering"

19-25 June 2022

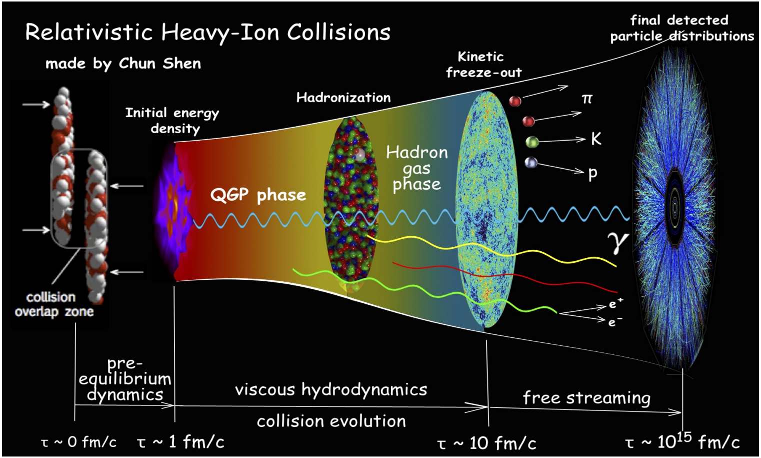
AGH University of Science and Technology

AGH

June 20, 2022
Karpacz, Poland

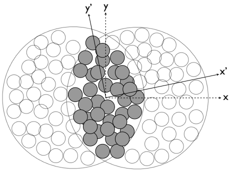
National Science Centre
Poland

High energy heavy ion(HI) collision: "The Little Bang"

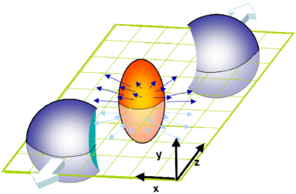


Shen, Heinz, arXiv:1507.01558

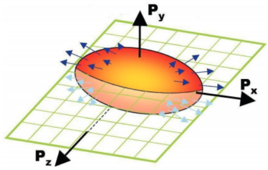
Collective flow in HI Collisions : Momentum anisotropy



PHOBOS arXiv:0711.3724



U. Heinz, arXiv:0810.5529



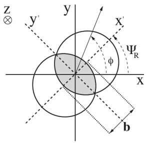
BNL: RHIC

Asymmetry in source distribution $\Longrightarrow$ **Collective expansion of fireball** $\Longrightarrow$ **Momentum anisotropy**

Momentum anisotropy as fourier expansion of flow harmonics

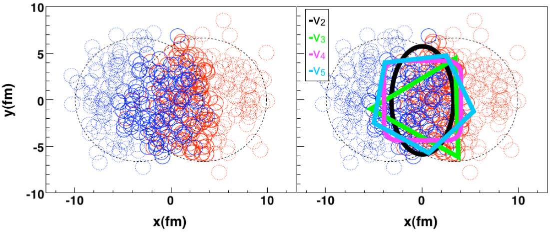
$$\frac{dN}{dp_T\, d\phi} = \frac{dN}{2\pi\, dp_T}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T) cos\left[n\left(\phi - \Psi_n\right)\right]\right)$$

$$\Longrightarrow \frac{dN}{d\phi} \propto 1 + 2v_2 \cos\left[2(\phi - \Psi_2)\right] + 3v_3 \cos\left[3(\phi - \Psi_2)\right] + \ldots$$



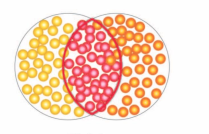
Bilandzic, Snellings and Voloshin arXiv:1010.0233

Harmonic flow coefficients : Elliptic and Triangular flow



Alver, Baker, Loizides, Steinberg arXiv:0805.4411

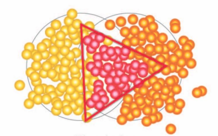
Elliptic flow



Elliptic flow

- $V_2 \longrightarrow$ Elliptic flow
- $V_2 \propto \mathcal{E}_2$
- $\mathcal{E}_2 \longrightarrow$ Elliptic deformation in source

Triangular flow



Triangular flow

- $V_3 \longrightarrow$ Triangular flow
- $V_3 \propto \mathcal{E}_3$
- $\mathcal{E}_3 \longrightarrow$ Triangular deformation in source

Mapping initial-state correlation: Higher order cumulants between p_T and v_n^2

P. Bozek, R. Samanta PRC 104, 014905 (2021)

- Flow harmonics $\longrightarrow$ Flow vectors, $V_n = |V_n|\ e^{i\ n\Psi_n}$
  $v_n = |V_n| \rightarrow$ Flow magnitude & $\Psi_n \rightarrow$ Flow angle
- Initial state correlation $\longrightarrow$ correlations between average transverse momentum $[p_T]$ and v_n^2, where $v_n^2 = V_n V_n^*$
- Lowest order correlation, **Pearson's correlation coefficient** :

$$\rho(p_T, v_n^2) = \frac{\langle p_T\ v_n^2\rangle - \langle p_T\rangle\langle v_n^2\rangle}{\sqrt{(\langle p_T^2\rangle - \langle p_T\rangle^2)(\langle (v_n^2)^2\rangle - \langle v_n^2\rangle^2)}}$$

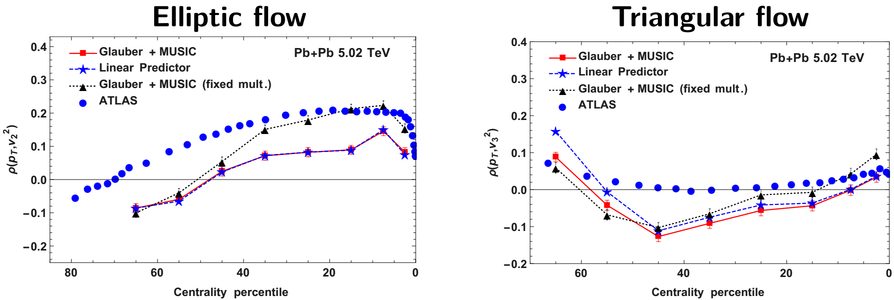
where, and $\langle \ldots \rangle$ = Event average

- Correlation are calculated from final thermal spectra after hydrodynamic evolution
- Correlations predicted from initial state through **linear predictor**:

$$(v_n^2, p_T) \propto (\epsilon_n^2, S, R)$$

where, S = initial entropy density, R = Initial size

Model results



- Correlations are also calculated keeping multiplicity (N) fixed
- $\rho(p_T, v_2^2)$: model result qualitatively describe the data, change of sign occur at peripheral collision !
- $\rho(p_T, v_3^2)$: model result do not reproduce the experimental data
- may indicate the presence of non-flow correlation or missing of the initial flow correlation

Symmetric cumulants

- 2nd order Normalized Symmetric cumulant(NSC) :

$$NSC(p_T, v_n^2) = \frac{\langle p_T \ v_n^2 \rangle - \langle p_T \rangle \langle v_n^2 \rangle}{\langle p_T \rangle \langle v_n^2 \rangle}$$

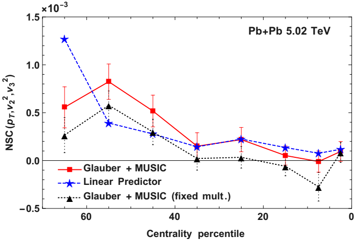
normalization : **Standard deviation** $\longrightarrow$ **Mean**

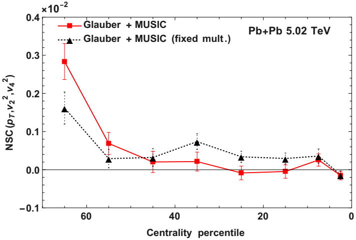
- Third order NSC :

$$SC(A, B, C) = \langle ABC \rangle - \langle AB \rangle \langle C \rangle - \langle AC \rangle \langle B \rangle - \langle BC \rangle \langle A \rangle + 2 \langle A \rangle \langle B \rangle \langle C \rangle$$

$$NSC(A, B, C) = \frac{SC(A, B, C)}{\langle A \rangle \langle B \rangle \langle C \rangle}$$

— similar approach for fourth or higher order NSCs...



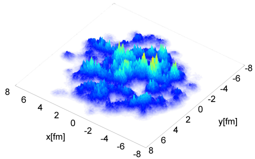


Mapping initial state fluctuation: Factorization breaking for higher moments of harmonic flow

P. Bozek, R. Samanta PRC 105, 034904 (2022)

- Flow vector, $V_n = |V_n|\ e^{i\ n\Psi_n}$
  $|V_n| \rightarrow$ Flow magnitude & $\Psi_n \rightarrow$ Flow angle
- Event by event fluctuation of initial state $\rightarrow$ event by event fluctuation of flow vectors V_n

lumpy structure of the initial density



Schenke, Tribedy, Venugopalan arXiv: 1206.6805

- Can we map the flow $V_n(p_T, \eta)$ with the flow fluctuation **event by event ?** No !
- We could map the fluctuation through covariance : $\langle V_n(p_1, \eta_1) V_n^*(p_2, \eta_2) \rangle$

  or the correlation :

$$\frac{\langle V_n(p_1, \eta_1) V_n^*(p_2, \eta_2) \rangle}{\sqrt{\langle V_n(p_1, \eta_1) V_n^*(p_1, \eta_1) \rangle \langle V_n(p_2, \eta_2) V_n^*(p_2, \eta_2) \rangle}}$$

  , which now we call factorization breaking coefficients !

Flow factorization coefficient and experimental difficulties

Flow correlations in p_T bins

- In first order flow vector - flow vector factorization coefficient :

$$r_n(p_1, p_2) = \frac{\langle V_n(p_1) V_n^*(p_2) \rangle}{\sqrt{\langle V_n(p_1) V_n^*(p_1) \rangle \ \langle V_n(p_2) V_n^*(p_2) \rangle}}$$

  can be measured experimentally !

- flow magnitude decorrelations or flow angle decorrelation cannot be measured ! in first moment

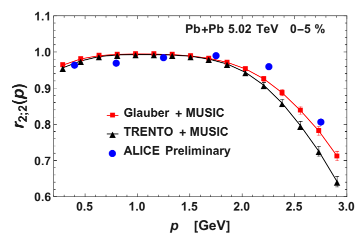
- So, one needs to go for the second moment :
  (flow vec)^2-(flow vec)^2 decor : $\frac{\langle V_n(p_1)^2 V_n^*(p_2)^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \ \langle |V_n(p_2)|^4 \rangle}}$
  (flow mag)^2-(flow mag)^2 decor: $= \frac{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}{\sqrt{\langle |V_n(p_1)|^4 \rangle \ \langle |V_n(p_2)|^4 \rangle}}$
  flow angle decor $= \frac{flow \ \ vector \ \ decor}{flow \ \ mag \ \ decor}$: $\frac{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 cos[2n(\Psi_n(p1) - \Psi_n(p2))] \rangle}{\langle |V_n(p_1)|^2 |V_n(p_2)|^2 \rangle}$
  — Could be measured experimentally, but difficult due to poor statistics !

- Alternative!: $\longrightarrow$ one of the flow at a fixed p_T ($V_n(p)$) and another as global (momentum averaged)(V_n).
  —- statistically preferable and accessible ! proposed by ALICE !

$V_n^2 - V_n^2(p)$ factorization breaking coefficients

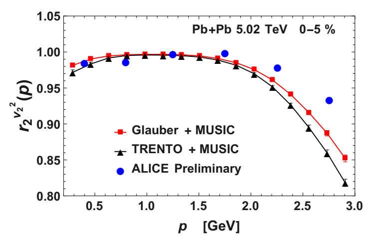
(flow vector)^2-(flow vector)^2 decor.

$$r_{n;2}(p) = \frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\sqrt{\langle |V_n|^4 \rangle \ \langle |V_n(p)|^4 \rangle}}$$



(flow magnitude)^2-(flow magnitude)^2

$$r_n^{v_n^2}(p) = \frac{\langle |V_n|^2 |V_n(p)|^2 \rangle}{\sqrt{\langle |V_n|^4 \rangle \ \langle |V_n(p)|^4 \rangle}}$$



Observations

- The flow magnitude decorrelation is **approximately one half** of the flow vector decorrelation:

$$[1 - r_n^{v_n^2}(p)] \simeq \frac{1}{2}[1 - r_{n;2}(p)]$$

- for central collision our model results reproduce the data.

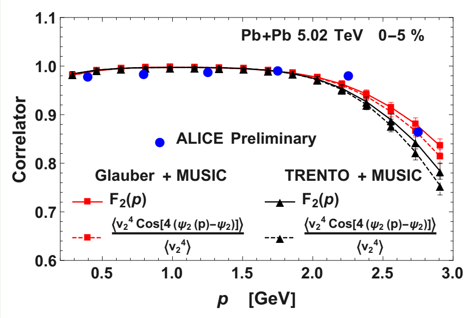
flow angle decorrelation

- Flow angle factorization coefficient,

$$F_n(p) = \frac{flow\ \ vec.\ decor}{flow\ \ mag.\ decor} = \frac{\langle V_n^2 V_n^*(p)^2 \rangle}{\langle |V_n|^2 |V_n(p)|^2 \rangle}$$

$$= \frac{\langle |V_n|^2 |V_n(p)|^2 cos[2n(\Psi_n - \Psi_n(p))] \rangle}{\langle |V_n|^2 |V_n(p)|^2 \rangle} \simeq \frac{\langle |V_n|^4 cos[2n(\Psi_n - \Psi_n(p))] \rangle}{\langle |V_n|^4 \rangle}$$

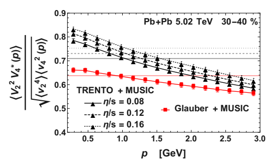
- Strong correlation between flow angle and flow magnitude → can't be factorized.



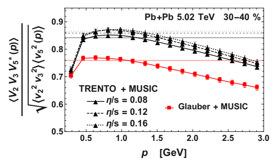
Factorization breaking between mixed harmonics (New!)

First order (flow vector factorization) : $V_2^2 - V_4(p)$ and $V_2 V_3 - V_5(p)$ *corr*.

$$\frac{\langle V_2^2 V_4^*(p) \rangle}{\sqrt{\langle |V_2|^4 \rangle \; \langle |V_4(p)|^2 \rangle}}$$

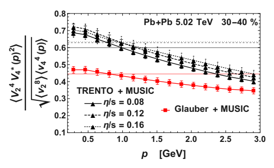


$$\frac{\langle V_2 V_3 V_5^*(p) \rangle}{\sqrt{\langle |V_2|^2 |V_3^2| \rangle \; \langle |V_5(p)|^2 \rangle}}$$

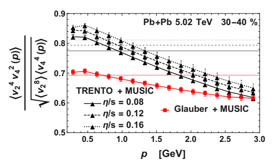


Second order: $V_2^4 - V_4^2(p)$ *corr* : (flow vec)^4-(flow vec)^2 decor. and (flow mag)^4-(flow mag)^2 decor.

$$\frac{\langle V_2^4 V_4^*(p)^2 \rangle}{\sqrt{\langle |V_2|^8 \rangle \; \langle |V_4(p)|^4 \rangle}}$$

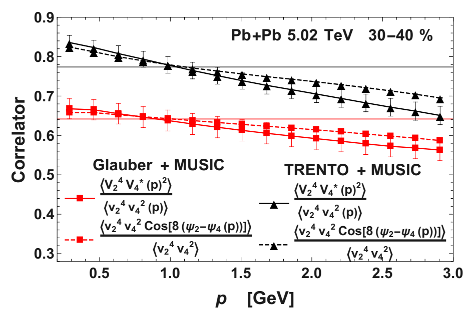


$$\frac{\langle |V_2|^4 |V_4(p)|^2 \rangle}{\sqrt{\langle |V_2(p)|^8 \rangle \; \langle |V_4(p)|^4 \rangle}}$$



$V_2^4 - V_4^2(p)$ flow angle decorrelation :

$$\frac{\langle V_2^4 V_4^*(p)^2\rangle}{\langle |V_2|^4 |V_4(p)|^2\rangle} = \frac{\langle |V_2|^4 |V_4(p)|^2 cos[8(\Psi_2 - \Psi_4(p))]\rangle}{\langle |V_2|^4 |V_4(p)|^2\rangle} \simeq \frac{\langle |V_2|^4 |V_4|^2 cos[8(\Psi_2 - \Psi_4(p))]\rangle}{\langle |V_2|^4 |V_4|^2\rangle}$$



Study of the factorization breaking of mixed flow puts additional constraints on the initial state models !

Conclusions and outlook

- Mapping initial state correlations $\Longrightarrow$ Correlation ρ or NSC between p_T, and v_n^2
- Mapping the initial state fluctuation $\Longrightarrow$ Factorization Breaking coefficients
- We propose new correlations between mixed flow harmonics : measure of non-linearity!
- Future plan:
  - Full 3+1D hydro simulations and longitudinal correlations
  - Effect of nucleon size on the NSC and factorization breaking coefficients
  - Quadrupole and octupole deformation effect in those observables in the collision of deformed nuclei

Thank you !

for your attention

Back up

Calculating averages from particle spectra

- One can calculate the average of the final state observables from particle spectra :

$$\frac{dN}{dpd\phi} = \frac{dN}{2\pi dp}\left(1 + 2\sum_{n=1}^{\infty} V_n(p)e^{in\phi}\right)$$

- Average transverse momentum :

$$[p_T] = \frac{1}{N}\int_{p_{min}}^{p_{max}} p\frac{dN}{dp}dp$$

- Average multiplicity :

$$N = \int_{p_{min}}^{p_{max}} \frac{dN}{dp}dp$$

- Harmonic flow coefficient (momentum averaged) :

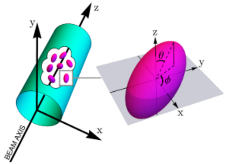
$$V_n = \frac{1}{N}\int_{p_{min}}^{p_{max}} V_n(p)\frac{dN}{dp}dp$$

Deformed nuclei and Scaled Symmetric Cumulant(SSC)

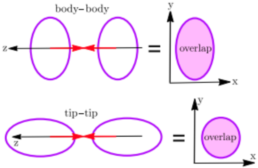
- Collision between the nuclei which are deformed due to quadrupole deformation e.g. U + U at 193 GeV.
- tip-on-tip collision $\longrightarrow$ larger p_T $\longrightarrow$ smaller V_2
  body-on-body collision $\longrightarrow$ smaller p_T $\longrightarrow$ larger V_2
- ρ and NSC between p_T and v_n's might be more interesting for deformed nuclei collision
- Scaled symmetric cumulants (SSC) :

$$SSC(A, B, C) = \frac{SC(A, B, C)}{\sqrt{Var(A)Var(B)Var(C)}}$$

  where, $Var(A) = \langle A^2 \rangle - \langle A \rangle^2$
  Alternative normalization: **Mean** $\longrightarrow$ **Standard deviation**
  $SSC(A, B) = \rho(A, B)$
- advantage of SSC $\longrightarrow$ prediction from inital state doesn't required additional input of p_T



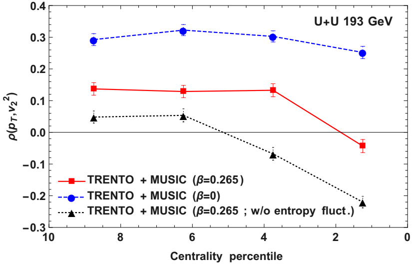
Deformed nuclus

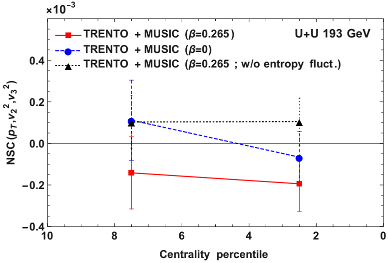


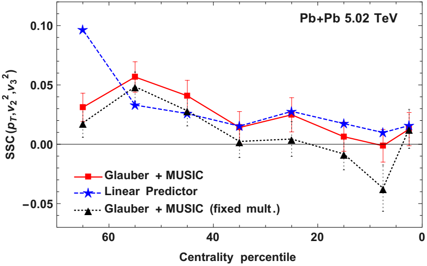
Tip-on-tip and body-on-body collision

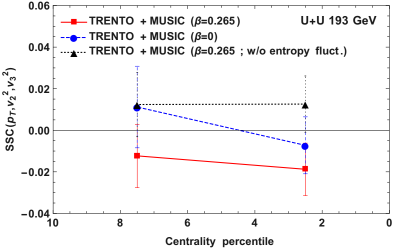
G. Giacalone arXiv:1910.04673

Model results









4th order cumulants NSC

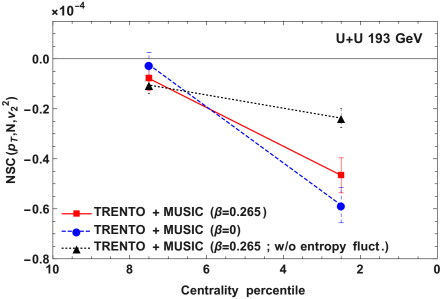
Fourth order NSC :

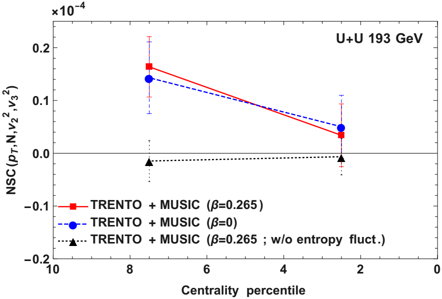
$$
\begin{aligned}
SC(A,B,C,D) = & \langle ABCD\rangle - \langle ABC\rangle\langle D\rangle - \langle ABD\rangle\langle C\rangle - \langle BCD\rangle\langle A\rangle \\
& - \langle ACD\rangle\langle B\rangle - \langle AB\rangle\langle CD\rangle - \langle AC\rangle\langle BD\rangle - \langle BC\rangle\langle AD\rangle \\
& + 2(\langle AB\rangle\langle C\rangle\langle D\rangle + \langle AC\rangle\langle B\rangle\langle D\rangle + \langle AD\rangle\langle B\rangle\langle C\rangle \\
& + \langle BC\rangle\langle A\rangle\langle D\rangle + \langle BD\rangle\langle A\rangle\langle C\rangle + \langle CD\rangle\langle A\rangle\langle B\rangle) \\
& - 6\langle A\rangle\langle B\rangle\langle C\rangle\langle D\rangle
\end{aligned}
$$

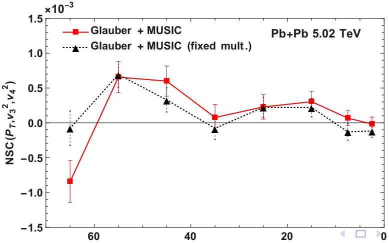
and

$$
NSC(A,B,C,D) = \frac{SC(A,B,C,D)}{\langle A\rangle\langle B\rangle\langle C\rangle\langle D\rangle}
$$

NSC's for other observables







Measuring moments of the flow harmonics

Cumulant method

- Experimentally, $q_n = \frac{1}{N}\sum_{i=1}^{N} e^{i\ n\phi_i}$,

  N = the number of particles

  ϕ_i = the azimuth of i^{th} particle
- The two particle cumulant, $v\{2\}^2 = \langle q_n q_n^* \rangle_{without\ \ self-correlation}$
- where,

$$\langle q_n q_n^* \rangle_{without\ \ self-correlation} = \langle \frac{1}{N(N-1)} \sum_{i \neq j} e^{i\ n(\phi_i - \phi_j)} \rangle$$

- **Scalar product** of the flows is used to measure the cumulants.

Only even moments of the flow can be measured !

The other definition

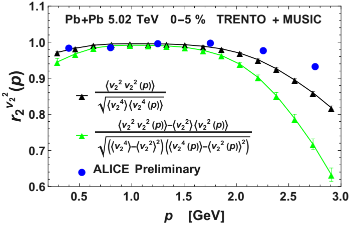
The usual correlation (ρ)

One could use a normalization :

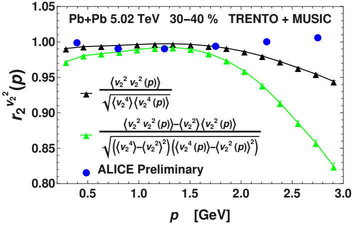
$$r_n^{v_n^2}(p) = \frac{\langle |V_n|^2 |V_n(p)|^2\rangle - \langle |V_n|^2\rangle \langle |V_n(p)|^2\rangle}{\sqrt{\left(\langle |V_n|^4\rangle - \langle |V_n|^2\rangle^2\right)\left(\langle |V_n(p)|^4\rangle - \langle |V_n(p)|^2\rangle^2\right)}}$$

But that gives quite different result !

flow vec. decor.



flow mag. decor.



Why magnitude decorrelation $\simeq$ angle direction ?

Simple model of vector decorrelation Bozek, Mehrabpour

Let's consider two vector: $\vec{X}_n = \vec{V}_n + \vec{\delta}_n$ and $\vec{Y}_n = \vec{V}_n - \vec{\delta}_n$

- It can be shown that, factorization breaking of flow vector:

$$\frac{\langle X_n Y_n^\star \rangle}{\sqrt{\langle X_n^2 \rangle \langle Y_n^2 \rangle}} \simeq 1 - 2\frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

- factorization breaking of flow magnitude:

$$\frac{\langle X_n Y_n \rangle}{\sqrt{\langle X_n^2 \rangle \langle Y_n^2 \rangle}} \simeq 1 - \frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

- and flow angle decorrelation :

$$\frac{\langle V_n^2 \cos(n\Delta\Psi) \rangle}{\langle V_n^2 \rangle} \simeq 1 - \frac{\langle \delta_n^2 \rangle}{\langle V_n^2 \rangle}$$

Removing non-flow correlation

Forming correlation in η bins along with p$_T$

- four well-separated pseudorapidty bins; $-\eta_F$,$-\eta$,η and η_F
- Flow vector factorization coefficient :

$$r_{n;2}(p) \simeq \frac{\langle V_n(-\eta_F) V_n^*(-\eta, p) V_n^*(\eta, p) V_n(\eta_F)\rangle \langle V_n^*(-\eta) V_n(\eta)\rangle}{\langle V_n(-\eta_F) V_n^*(-\eta) V_n^*(\eta) V_n(\eta_F)\rangle \langle V_n^*(-\eta, p) V_n(\eta, p)\rangle}$$

- Flow magnitude factorization coefficient :

$$r_{n;2}^{v_n^2}(p) \simeq \frac{\langle V_n(-\eta_F) V_n(-\eta, p) V_n^*(\eta, p) V_n^*(\eta_F)\rangle \langle V_n^*(-\eta) V_n(\eta)\rangle}{\langle V_n(-\eta_F) V_n(-\eta) V_n^*(\eta) V_n^*(\eta_F)\rangle \langle V_n^*(-\eta, p) V_n(\eta, p)\rangle}$$

- Flow angle decorrelation (ratio of the above two) :

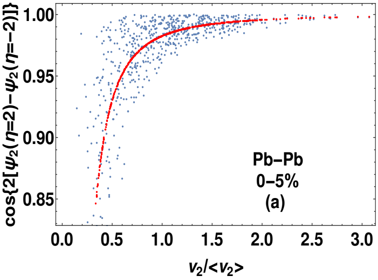
$$F_n(p) \simeq \frac{\langle V_n(-\eta_F) V_n^*(-\eta, p) V_n^*(\eta, p) V_n(\eta_F)\rangle \langle V_n(-\eta_F) V_n(-\eta) V_n^*(\eta) V_n^*(\eta_F)\rangle}{\langle V_n(-\eta_F) V_n(-\eta, p) V_n^*(\eta, p) V_n^*(\eta_F)\rangle \langle V_n(-\eta_F) V_n^*(-\eta) V_n^*(\eta) V_n(\eta_F)\rangle}$$

Equivalence of different normalization

Scaling of data

- The ALICE collaboration use different normalization in their data, namely for vector and magnitude correlation :
$\frac{\langle V_n^2 V_n^*(p)^2\rangle}{\langle |V_n|^2\rangle\ \langle |V_n(p)|^2\rangle}$ and $\frac{\langle |V_n|^2 |V_n(p)|^2\rangle}{\langle |V_n|^2\rangle\ \langle |V_n(p)|^2\rangle}$ respectively
- We divide them by a factor $\frac{\langle |V_n^4|\rangle}{\langle |V_n^2|\rangle^2}$, the **baseline** of the plots, and we have:
$\frac{\langle V_n^2 V_n^*(p)^2\rangle\ \langle |V_n^2|\rangle}{\langle |V_n|^4\rangle\ \langle |V_n(p)|^2\rangle}$ and $\frac{\langle |V_n|^2 |V_n(p)|^2\rangle\ \langle |V_n^2|\rangle}{\langle |V_n|^4\rangle\ \langle |V_n(p)|^2\rangle}$
- But we use the definitions: $\frac{\langle V_n^2 V_n^*(p)^2\rangle}{\sqrt{\langle |V_n|^4\rangle\ \langle |V_n(p)|^4\rangle}}$ and $\frac{\langle |V_n|^2 |V_n(p)|^2\rangle}{\sqrt{\langle |V_n|^4\rangle\ \langle |V_n(p)|^4\rangle}}$
- The difference between the two normalization is a factor :
$\sqrt{\frac{\langle |V_n^4(p)|\rangle \langle |V_n^2|\rangle^2}{\langle |V_n^4|\rangle \langle |V_n^2(p)|\rangle^2}} \simeq 1 \implies \frac{\sqrt{\langle |V_n^4(p)|\rangle}}{\langle |V_n^2(p)|\rangle} \simeq \frac{\sqrt{\langle |V_n^4|\rangle}}{\langle |V_n^2|\rangle}$

Twist angle - flow magnitude correlation in η Bozek, Broniowski arXiv: 1711.03325



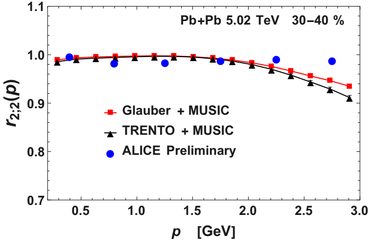
Observations

- **Strong correlation** exists between flow magnitude and twist angle
- Correct measure of angle decor.:

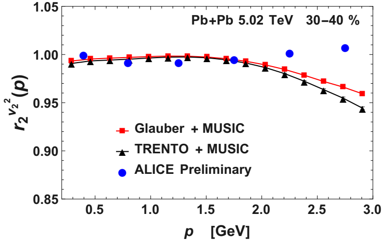
$$ang\ decor. = \frac{flow\ \ vec.\ decor.}{flow\ \ mag.\ decor.}$$
$$= \frac{\langle v_n^2 cos(n(\Delta\Psi))\rangle}{\langle v_n^2\rangle}$$
$$\neq \langle cos(n(\Delta\Psi))\rangle$$

Semi-peripheral collision (30-40 %)

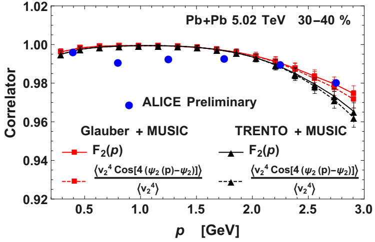
flow vec. decor.



flow mag. decor.



flow angle decor.

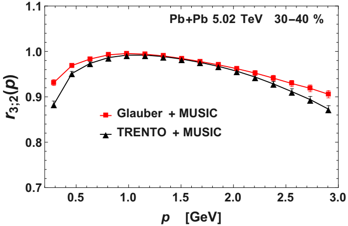


Observations

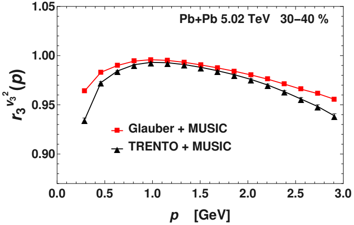
- For semi-peripheral collision our model results do not reproduce the data
- For 30-40 % the data go slightly above 1 at high $p_T \longrightarrow$ may indicate a significant non-flow contribution.

Same for triangular flow (v_3)

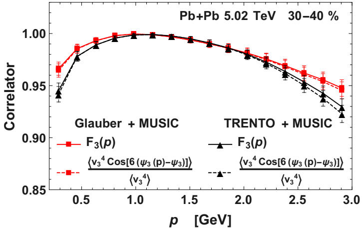
flow vec. decor.



flow mag. decor.



flow angle decor.



Factorization breaking between mixed harmonics (Non-linearity)

- Serves as a measure of non linear response of the hydrodynamic expansion
- General definition (1st order):

$$\frac{\langle V_m^*(p) V_k V_n \rangle}{\sqrt{\langle |V_m(p)|^2 \rangle \langle |V_k|^2 |V_n|^2 \rangle}}$$

with the constraint: $m = k + n$

- For example, we have,

$$V_4 = V_4^L + V_4^{NL} \ , \ where \ V_4^{NL} \propto V_2^2$$

$$V_5 = V_5^L + V_5^{NL} \ , \ where \ V_5^{NL} \propto V_2 V_3$$

So, V_2^2-$V_4(p)$ and $V_5(p)$-$V_2 V_3$ correlations measures the **non-linear coupling**