Some large-Nc faces of QCD in the medium

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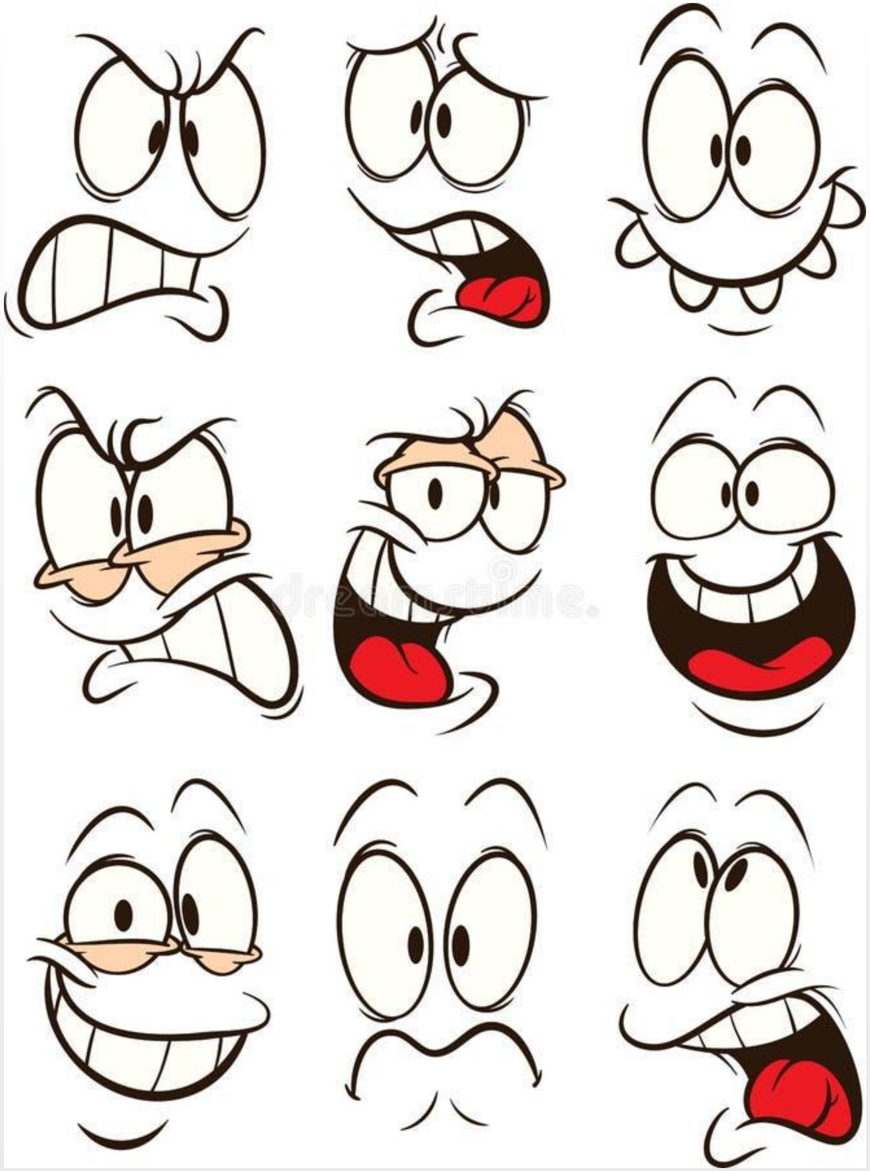
Various faces of QCD

26-28/4/2024 Wrocław Poland

Outline

- QCD: brief review
- Large-Nc: what is that: where it works…it does **not** work:
- Chiral anomaly
- QCD Phase-diagram at large Nc
- Nuclear matter (and neutron stars) at large-Nc
- Conclusions

QCD has many faces



…or phases

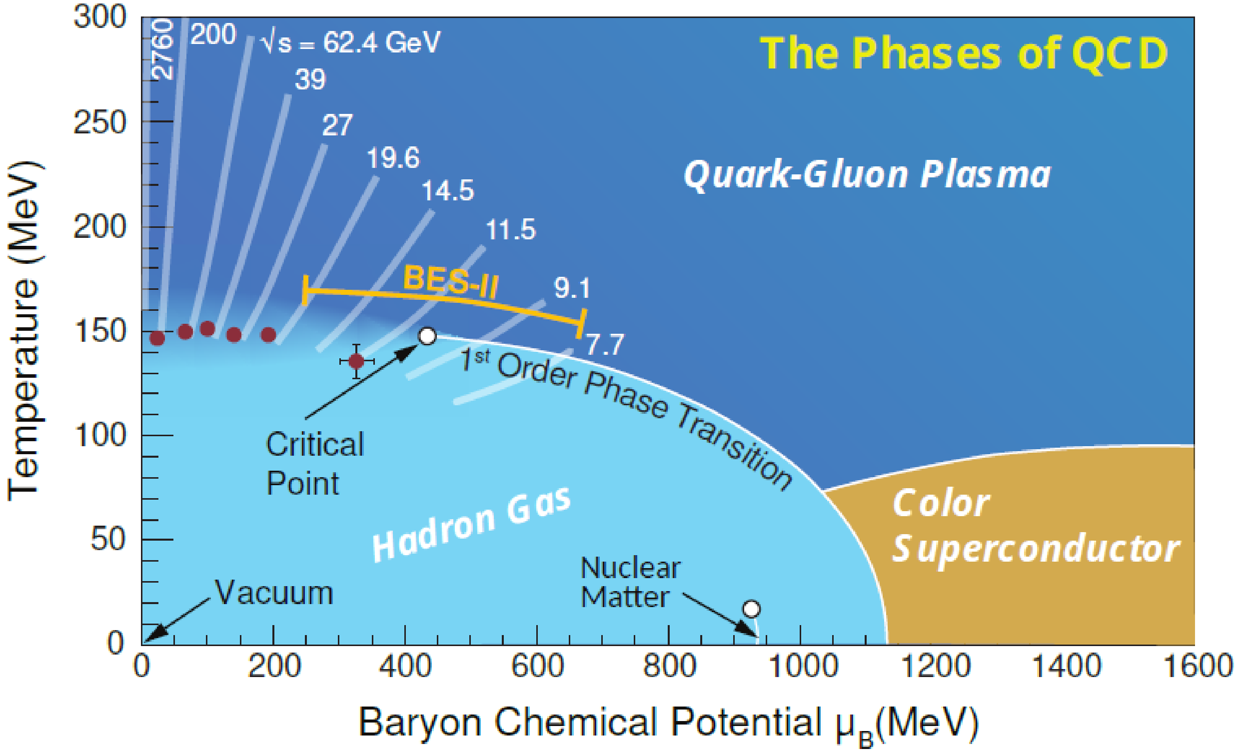


Figure 1: A schematic QCD phase diagram in the thermodynamic parameter space spanned by the temperature T and baryonic chemical potential μ_B. The corresponding (center-of-mass) collision energy ranges for different accelerator facilities, especially the RHIC beam energy scan program, are indicated in the figure. Figure adapted from [40].

Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan
Bzdak et al, Phys. Rept., e-Print: 1906.00936

Three “bad’’ faces of large Nc

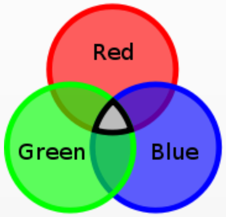
Uniwersytet
Jana Kochanowskiego w Kielcach

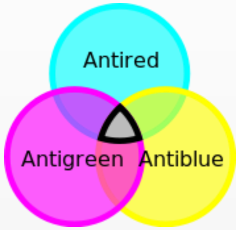
Symmetries of QCD

Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



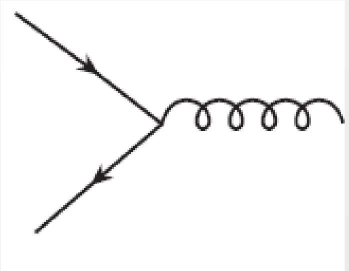


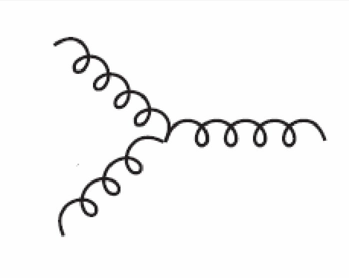
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix};\ i = u,d,s,...$$

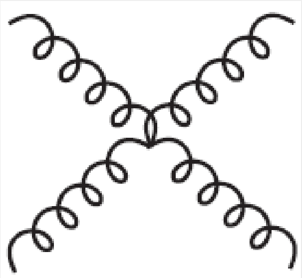
$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

8 type of gluons (R~~G~~,B~~G~~,…)

$$A_\mu^a\ ;\ a = 1,...,8$$







Symmetries of QCD and breakings

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**scale anomaly**) and by quark masses.

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (**chiral anomaly**)

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

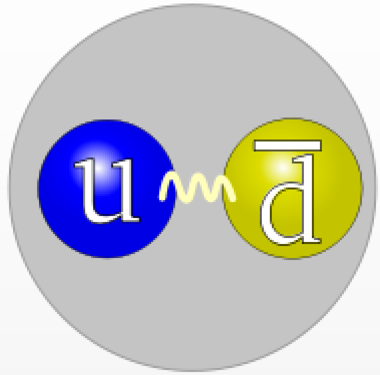
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + ...\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

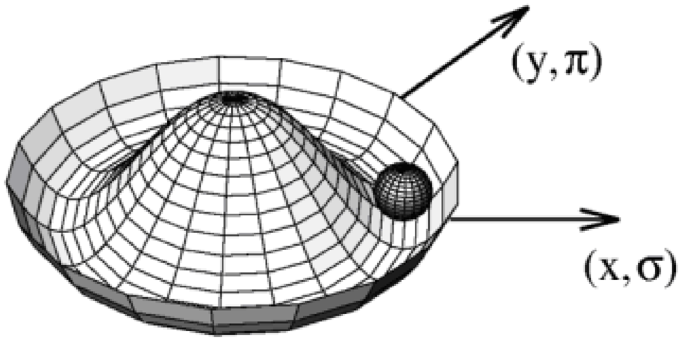
Rho-meson $m_{\rho^+} = 775\,\text{MeV}$

Pion $m_{\pi^+} = 139\,\text{MeV}$

$$m_u + m_d \approx 7\,\text{MeV}$$

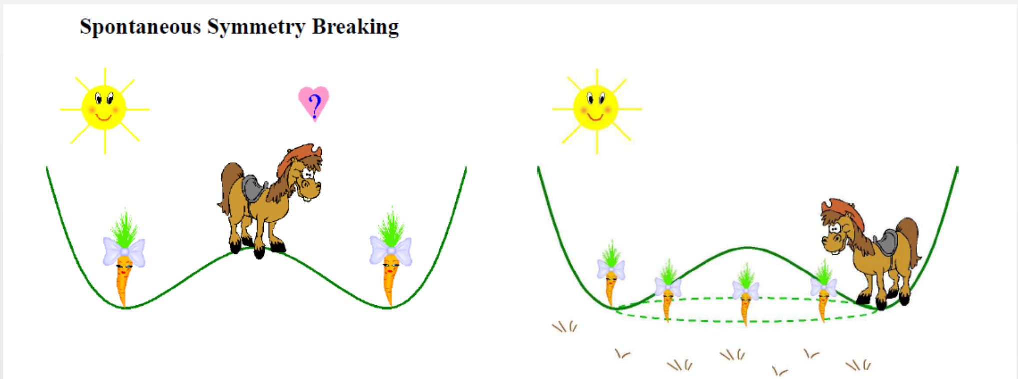
Mass generation in QCD is a nonpert. penomenon based on SSB (mentioned previusly).

SSB and the donkey of Buridan: hadronic approaches



$$\sigma_N \to \sigma_N + \phi$$

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

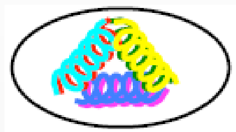
J^{PC}, $^{2S+1}L_J$	$I = 1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$; $I = 1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$; $I = 0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)$**	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_L \times SU(3)_R \times \times U(1)_A$
0^{-+}, $^1S_0$	π; K; $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 q^i$	$\Phi = S + iP$ $(\Phi^{ij} = \bar{q}^j_R q^i_L)$	$\Phi \to e^{-2i\alpha} U_L \Phi U_R^\dagger$
0^{++}, $^3P_0$	$a_0(1450)$; $K_0^*(1430)$; $f_0(1370), f_0(1710)^*$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
1^{--}, $^1S_1$	$\rho(770)$; $K^*(892)$; $\omega(782), \phi(1020)$	$V^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L^{ij}_\mu = \bar{q}^j_L \gamma_\mu q^i_L)$	$L_\mu \to U_L L_\mu U_L^\dagger$
1^{++}, $^3P_1$	$a_1(1260)$; $K_{1,A}$; $f_1(1285), f_1(1420)$	$A^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R^{ij}_\mu = \bar{q}^j_R \gamma_\mu q^i_R)$	$R_\mu \to U_R R_\mu U_R^\dagger$
1^{+-}, $^1P_1$	$b_1(1235)$; $K_{1,B}$; $h_1(1170), h_1(1380)$	$P^{ij}_\mu = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + iP_\mu$ $(\Phi^{ij}_\mu = \bar{q}^j_R i\overleftrightarrow{D}_\mu q^i_L)$	$\Phi_\mu \to e^{-2i\alpha} U_L \Phi_\mu U_R^\dagger$
1^{--}, $^3D_1$	$\rho(1700)$; $K^*(1680)$; $\omega(1650), \phi(?)$	$S^{ij}_\mu = \frac{1}{2}\bar{q}^j i\overleftrightarrow{D}_\mu q^i$		
2^{++}, $^3P_2$	$a_2(1320)$; $K_2^*(1430)$; $f_2(1270), f_2'(1525)$	$V^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L^{ij}_{\mu\nu} = \bar{q}^j_L(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i_L)$	$L_{\mu\nu} \to U_L L_{\mu\nu} U_L^\dagger$
2^{--}, $^3D_2$	$\rho_2(?)$; $K_2(1820)$; $\omega_2(?), \phi_2(?)$	$A^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R^{ij}_{\mu\nu} = \bar{q}^j_R(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_R)$	$R_{\mu\nu} \to U_R R_{\mu\nu} U_R^\dagger$
2^{-+}, $^1D_2$	$\pi_2(1670)$; $K_2(1770)$; $\eta_2(1645), \eta_2(1870)$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(i\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_R(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_L)$	$\Phi_{\mu\nu} \to e^{-2i\alpha} U_L \Phi_{\mu\nu} U_R^\dagger$
2^{++}, $^3F_2$	$a_2(?)$; $K_2^*(?)$; $f_2(?), f_2'(?)$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$		
3^{--}, $^3D_3$	$\rho_3(1690)$; $K_3^*(1780)$; $\omega_3(1670), \phi_3(1850)$	⋮	⋮	⋮

Table from:

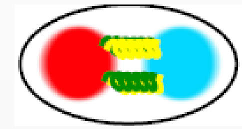
F.G., R. Pisarski,
A. Koenigstein
Phys.Rev.D 97 (2018) 9, 091901
e-Print: 1709.07454

Non-conventional mesons: beyond qq

1) Glueballs



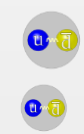
2) Hybrids



Compact diquark-antidiquark states

3) Four-quark states

Molecular states (a type of dynamical generation)



Companion poles (another type of dynamical generation)

Large-Nc: basics/1

- Instead of 3 colors, Nc colors. Then Nc is taken as a large number.
- Why to do that? Certain simplifications appear! (Yet QCD not solvable also in that limit).
- (Some) mesons become stable and slowly interacting.
- Confinement, symmetry breaking, etc…are believed to hold in large-Nc as well.

Large-Nc: basics/2

Running coupling and the ‘t Hooft limit

$$N_c \longrightarrow \infty, \qquad g^2_{\mathrm{QCD}} N_c \longrightarrow \text{finite.}$$

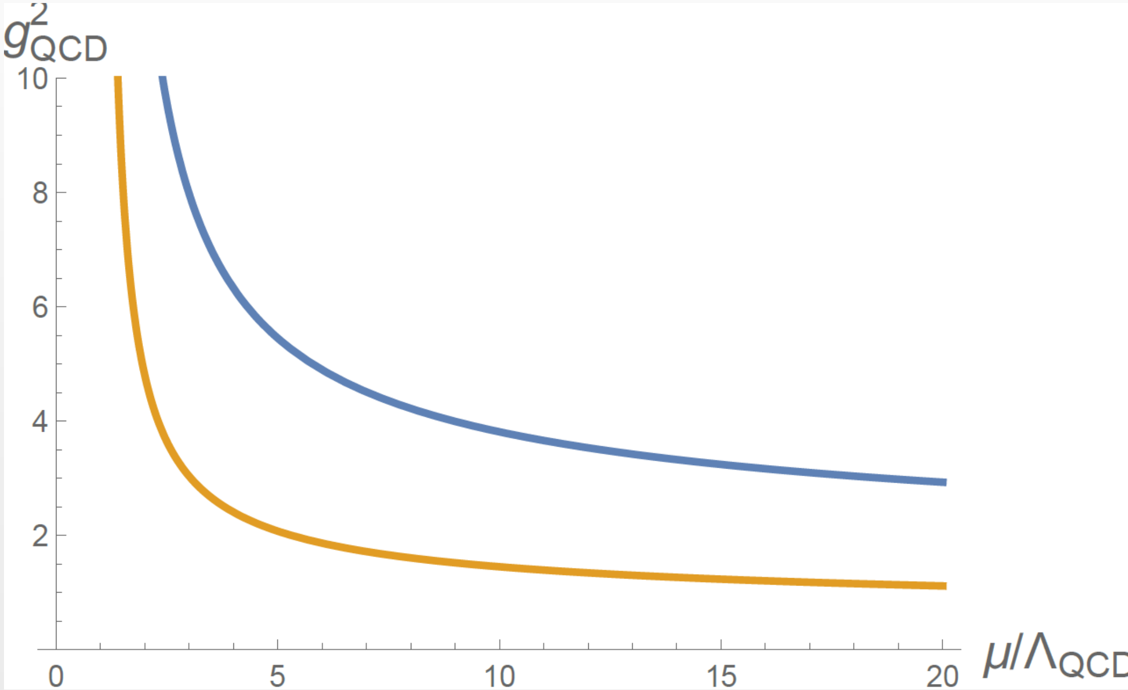
$$\mu \frac{dg}{d\mu} = -b g^3$$

$$b = \frac{1}{2}\frac{1}{8\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

For large-N_c we get:

$$g^2(\mu) = \frac{8\pi^2}{\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)} \frac{1}{\ln\frac{\mu}{\Lambda_{QCD}}}$$

$$g^2(\mu) = \frac{8\pi^2}{\left(\frac{11}{3}N_c\right)} \frac{1}{\ln\frac{\mu}{\Lambda_{QCD}}} \propto \frac{1}{N_c}$$



Large-Nc: consequences

- Constituent quark mass Nc^0
- Masses of conventional quark-antiquark states mesons and glueballs (and hybrids): Nc^0

  (with one important exception…)
- Decay width of these states decreases with Nc
- Masses of baryons proprtional to Nc; meson-baryon coupling proportional to Nc^(1/2)

Recent lectures

Introductory visual lecture on QCD at large-N_c: bound states, chiral models, and phase diagram

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- e-Print: 2402.14097 [hep-ph]

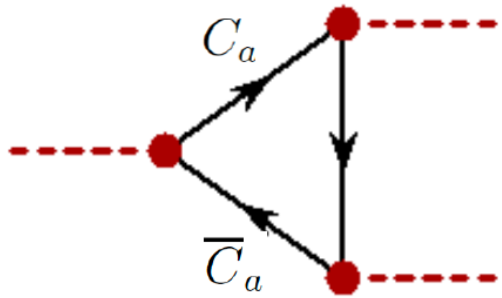
- Note: 114 pages, 52 figures. Lectures prepared for the 63. Cracow School of Theoretical Physics, September 17-23, 2023 Zakopane, Tatra Mountains, Poland

Is 3 a large number?

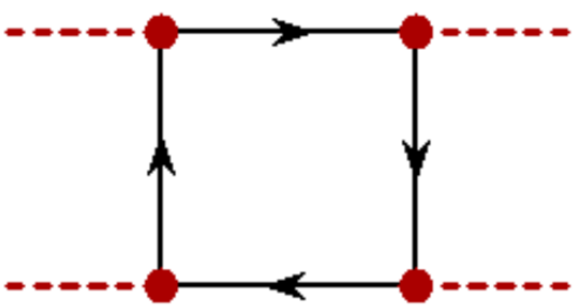
Spoiler: in most cases yes,

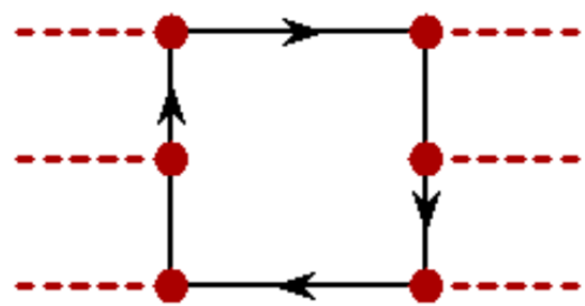
but in some selected interesting cases no!

Mesonic decays and interactions



$$g_{Q\bar{q}q}^{3}\,N_c \sim \frac{1}{\sqrt{N_c}} \rightarrow \Gamma \sim \frac{1}{N_c}$$

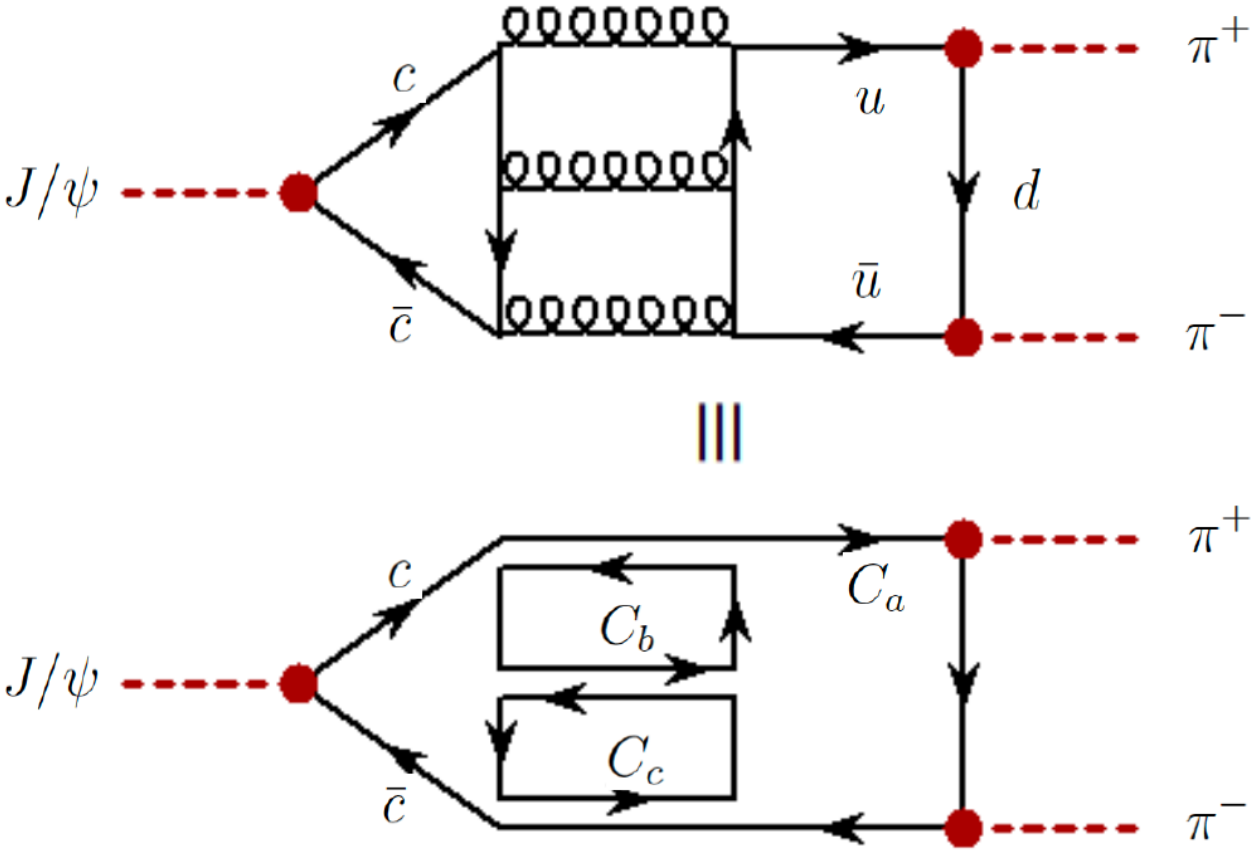




$$g_{Q\bar{q}q}^{4}\,N_c \sim \frac{1}{N_c}$$

$$g_{Q\bar{q}q}^{6}\,N_c \sim \frac{1}{N_c^2}$$

A suppressed decay



$$g_{\psi\bar{c}c}\, g^6\, N_c^3\, g^2_{\pi\bar{q}q} \sim \frac{1}{N_c^{3/2}} \to \Gamma_{J/\psi\to\pi^+\pi^-} \sim \frac{1}{N_c^3}$$

In well agreement with the experiment!

The chiral anomaly

There are 8 but not 9 Goldstone bosons:
3 pions, 4 kaons, and one $\eta(547)$ meson.

The $\eta'(958)$ meson has a mass of almost 1 GeV.

$$m_{\eta'}^2 \sim 1/N_c$$

E. Witten, Current Algebra Theorems for the U(1) Goldstone Boson, Nucl. Phys. B 156 (1979), 269-283

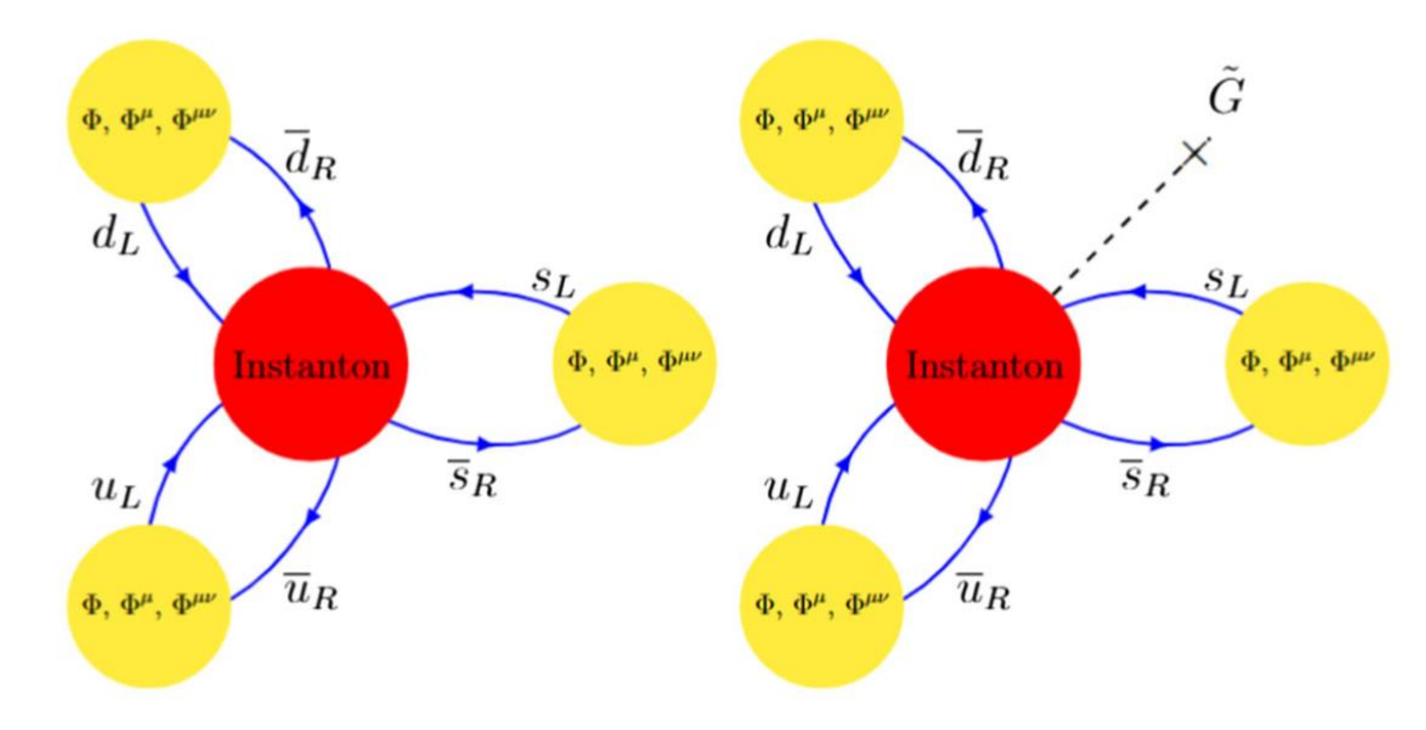
G. 't Hooft, Computation of the quantum effects due to a four-dimensional pseudoparticle, Phys. Rev. D 14, 3432 (1976).

Anomalous interactions between mesons with nonzero spin and glueballs

Phys.Rev.D 109 (2024) 7, L071502

e-Print: 2309.00086 [hep-ph]

$$\mathcal{L}_{\text{eff}}^{J=0} = -a_0 (\det \Phi + \det \Phi^\dagger)$$



Extension to other mesons with higher spin

$$\mathcal{L}_{\text{eff}}^{J=1} = -\frac{k_1}{3!}\left(\epsilon\left[(\bar{q}_L q_R)(\bar{q}_L \overleftrightarrow{D}_\mu q_R)^2\right] + R \leftrightarrow L\right)$$
$$= a_1(\epsilon[\Phi\Phi_\mu\Phi^\mu] + \text{c.c.}),$$

where we introduce the symbol [44]

$$\epsilon[ABC] = \epsilon^{ijk}\epsilon^{i'j'k'}A_{ii'}B_{jj'}C_{kk'}/3!,$$

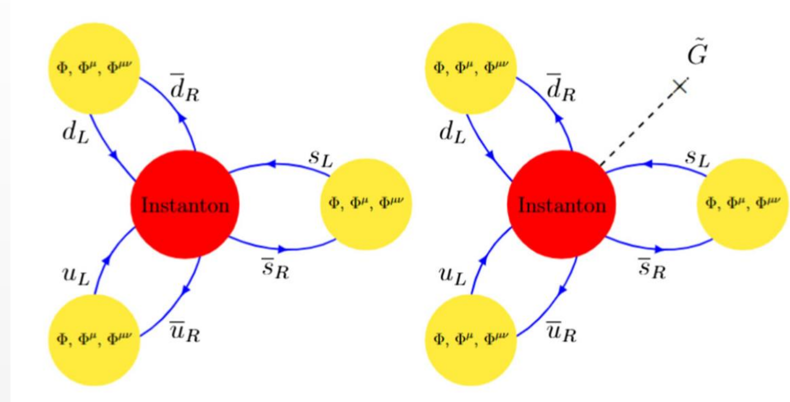
Indeed, it turns out that it the chiral anomaly effects for spin 1,2 mesons is Quite small…

Large Nc works!!!!

But…

Pseudoscalar glueball!

$$\mathcal{L}_{c_g} = -ic_g\tilde{G}_0(\det\Phi - \det\Phi^\dagger).$$



$$\Gamma(\tilde{G}_0 \to K\bar{K}\pi) \approx 0.24\ \text{GeV} \ \text{ and } \ \Gamma(\tilde{G}_0 \to \pi\pi\eta') \approx 0.05\ \text{GeV}$$

PHYSICAL REVIEW LETTERS **129,** 042001 (2022)

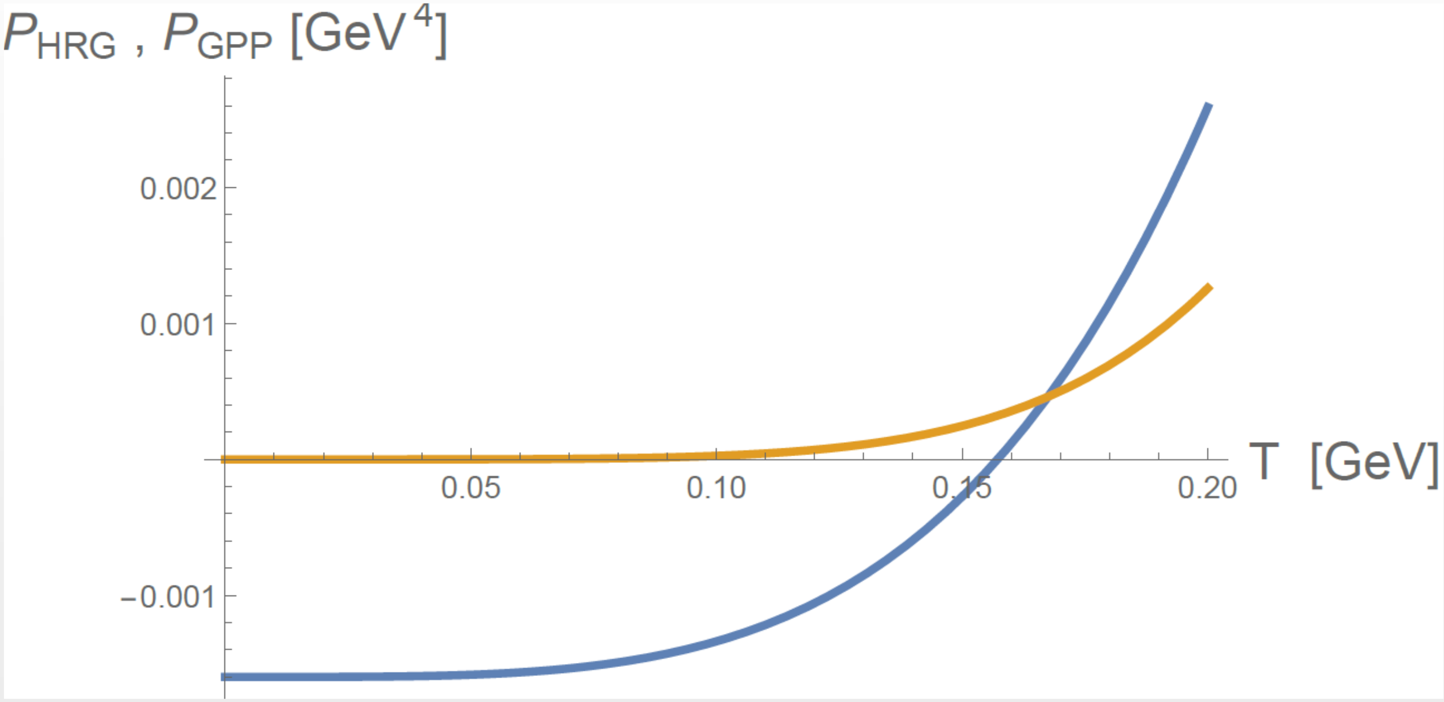
Observation of a State $X(2600)$ in the $\pi^+\pi^-\eta\prime$ System in the Process $J/\psi \to \gamma\pi^+\pi^-\eta\prime$

Original Lagrangian presented long ago in:
W. Eshraim, S. Janowski, F.G., D. Rischke,
Phys.Rev. D87 (2013) 054036. arxiv: 1208.6474

Finite T: the simple picture

$$P_{HRG}(T) = \sum_n P_n(T)$$

$$P_n(T) = -T\varsigma_n \int_k \ln\left[1 - \frac{\sqrt{k^2 + M_n^2}}{T}\right] \quad \text{if } n \text{ is a meson}$$



$$P_{QGP}(T) = 2N_c^2 \frac{\pi^2}{90} T^4 + 2N_c N_f \frac{\pi^2}{90} T^4 - P_{G,vac}$$
$$= 2N_c^2 \frac{\pi^2}{90} T^4 + 2N_c N_f \frac{\pi^2}{90} T^4 - B_G N_c^2 - B_G N_c$$

Finite chemical potential (with stiff matter as an example)

$$P_B(\mu_q) = N_c \bar{a}_B \mu_q^2$$

$$P_{QGP}(\mu_q) = P_q(\mu_q) = \frac{N_c N_f}{12\pi^2}\mu_q^4 + P_{QCD,vac} = \frac{N_c N_f}{12\pi^2}\mu_q^4 - B_G N_c^2 - B_G N_c.$$

$$\mu_{q,dec} \sim N_c^{1/4}$$

Large Nc at nonzero T

PHYSICAL REVIEW D **85**, 056005 (2012)

Restoration of chiral symmetry in the large-N_c limit

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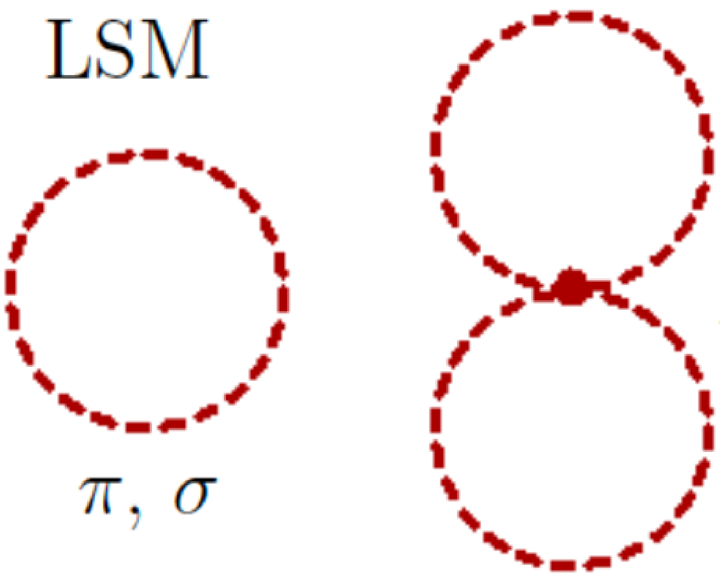
- NJL model
- Sigma model(s)
- Comparison and improvements

FInite T, sigma model

$$\mathcal{L}_{\sigma}(N_c) = \frac{1}{2}(\partial_{\mu}\Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4;$$

$$\Phi^t = (\sigma, \vec{\pi})$$



$$\lambda \sim \frac{1}{N_c}$$

$$P \sim N_c^0 \qquad P \sim \lambda \sim \frac{1}{N_c}$$

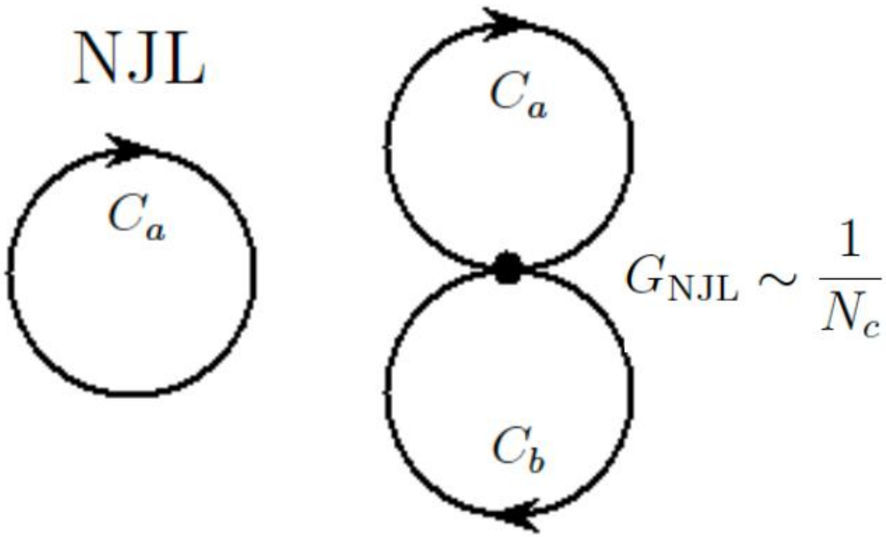
$$T_c \sim f_\pi \sim N_c^{1/2}$$

$$T_c(N_c) = \sqrt{2} f_{\pi} \sqrt{\frac{N_c}{3}} \propto N_c^{1/2}.$$

Finite T, NJL model

Uniwersytet
Jana Kochanowskiego w Kielcach

$$\mathcal{L}_{\rm NJL}(N_c) = \bar{\psi}(\imath\gamma^{\mu}\partial_{\mu} - m_q)\psi + \frac{3G}{N_c}[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\psi)]^2$$



$$P \sim N_c \qquad P \sim G_{\rm NJL} N_c^2, \sim N_c$$

$$T_c \sim N_c^0$$

$$T_c(N_c) \simeq \Lambda\sqrt{\frac{3}{\pi^2}}\sqrt{1 - \frac{\pi^2}{6\Lambda^2 G}} \propto N_c^0.$$

How to cure the problem of the LSM?

- Modify the mass term: $$\mu^2 \to \mu(T)^2 = \mu^2\left(1 - \frac{T^2}{T_0^2}\right)$$

- Use a quark-meson model

- Introduce the Polyakov loop

$$l(x) = N_c^{-1}\,\mathrm{Tr}\left[\mathcal{P}\exp\left(\imath g_{\mathrm{QCD}}\int_0^{1/T} A_0(\tau, x)d\tau\right)\right],$$

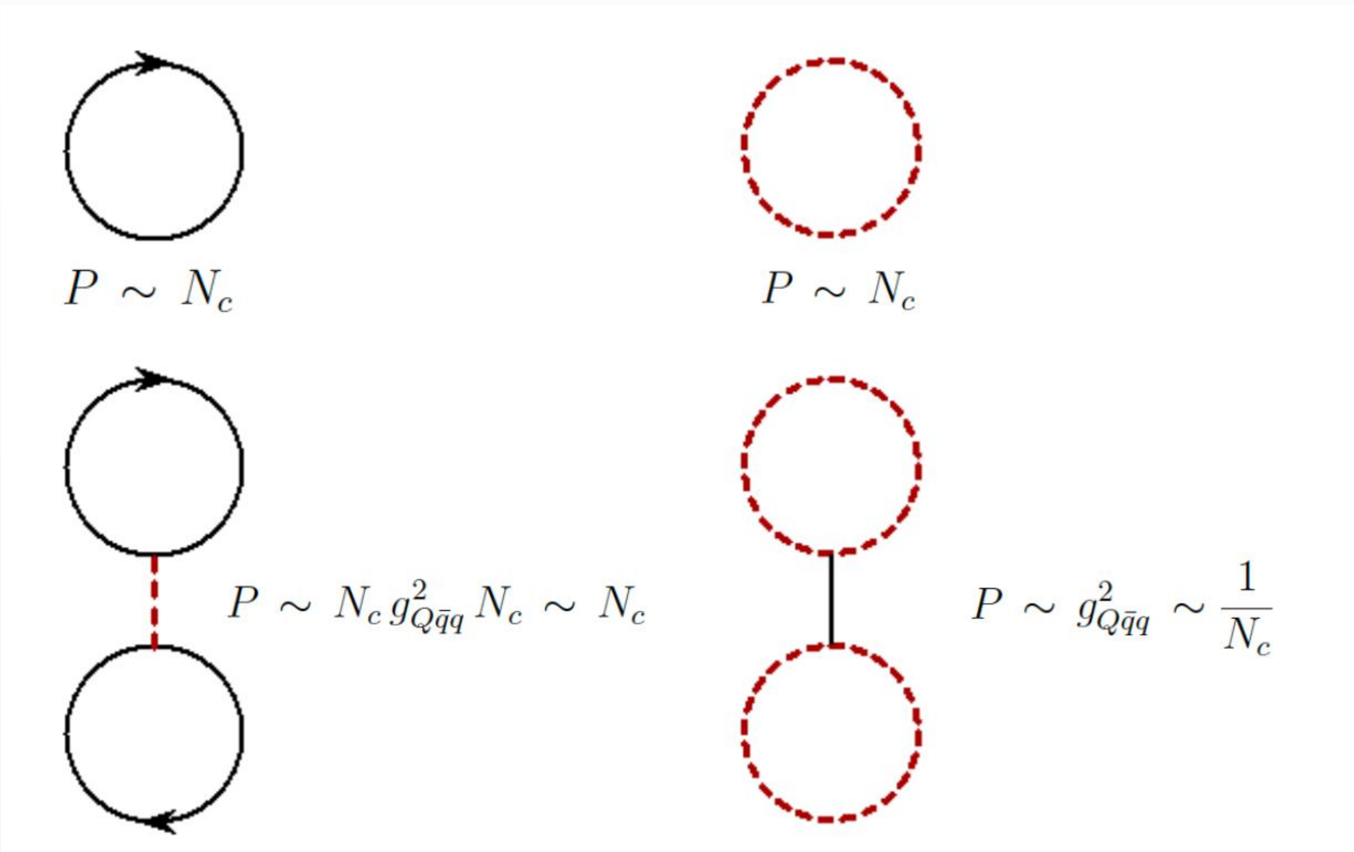
For l =0 conf, l =1 deconf.

$$\mathcal{L}_{\sigma\text{-Pol}}(N_c) = \mathcal{L}_\sigma(N_c) + \frac{\alpha N_c}{4\pi}|\partial_\mu l|^2 T^2 - \mathcal{V}(l) - \frac{h^2}{2}\Phi^2|l|^2T^2.$$

$$T_c = \frac{\mu}{\sqrt{h^2|l(T_c)|^2 + \frac{6\lambda}{N_c}}}$$

Quark-meson models

$$V_{LSM,quarks}(\sigma,\pi,l) = g_\sigma \sigma \left(\bar{\psi}\psi\right) + g_\pi \pi \left(\bar{\psi} i \gamma^5 \psi\right)$$



$$T_c \sim N_c^0$$

PHYSICAL REVIEW D **106**, 116016 (2022)

Editors' Suggestion

Fate of the critical endpoint at large N_c

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- Using a model a sigma-model that is as complete as possible (with (psuedo)scalar, (axial-)vector) d.o.f.)
- Linear realization of chiral symmetry
- Vacuum: D. Parganlija et al., Phys.Rev.D 87 (2013) 1, 014011 • e-Print: 1208.0585 [hep-ph]
- Extension to the medium: P. Kovacs, Phys.Rev.D 93 (2016) 11, 114014 • e-Print: 1601.05291 [hep-ph]: **coupling to quarks and to the Polyakov loop.**

eLSM Lagrangian, etc. Actually just a complicated vs of the Mexican hat ☺

$$\mathcal{L}_m = \mathrm{Tr}[(D_\mu M)^\dagger (D^\mu M)] - m_0 \mathrm{Tr}(M^\dagger M) - \lambda_1 [\mathrm{Tr}(M^\dagger M)]^2 - \lambda_2 [\mathrm{Tr}(M^\dagger M)^2] + c(\det M + \det M^\dagger) + \mathrm{Tr}[H(M + M^\dagger)]$$
$$- \frac{1}{4}\mathrm{Tr}[L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}] + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu L^\mu + R_\mu R^\mu)\right] + \frac{h_1}{2}\mathrm{Tr}(\phi^\dagger \phi)\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu]$$
$$+ h_2 \mathrm{Tr}[(M R_\mu)^\dagger (M R^\mu) + (L_\mu M)^\dagger (L^\mu M)] + 2h_3 \mathrm{Tr}[R_\mu M^\dagger L^\mu M] - 2g_2 \mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\},$$

$$\mathcal{L}_Y = \bar{\psi}(i\gamma_\mu \partial^\mu - g_F(S + i\gamma_5 P))\psi.$$

$$M = S + iP = \sum_a (S_a + iP_a) T_a,$$
$$L^\mu = V^\mu + A^\mu = \sum_a (V_a^\mu + A_a^\mu) T_a,$$
$$R^\mu = V^\mu - A^\mu = \sum_a (V_a^\mu - A_a^\mu) T_a,$$

$$D^\mu = \partial^\mu M - ig_1(L_\mu M - M R_\mu) - ieA^\mu [T_3, M],$$
$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu [T_3, L^\mu]\},$$
$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu [T_3, R^\mu]\},$$

- Polyakov loop potential.

$$\Omega(T, \mu_q) = U_{Cl} + \Omega_{\bar{q}q}(T, \mu_q) + U_{\mathrm{Pol}}(T, \mu_q) \qquad (2)$$

$$\Omega_{\bar{q}q}^{\mathrm{v}} = -2N_c \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_f(p),$$
$$\Omega_{\bar{q}q}^{\mathrm{T}}(T, \mu_q) = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}_c \left[\ln\left(1 + L^\dagger e^{-\beta(E_f(p) - \mu_q)}\right) + \ln\left(1 + L e^{-\beta(E_f(p) + \mu_q)}\right)\right]$$

$$\Omega(T, \mu_q) = U(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)}(T, \mu_q) + U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle)$$

Parameters and large-Nc scaling

TABLE I. Parameter sets. Left column is taken from [11] (set A) and right column is taken from [38] (set B).

Parameter	Set A	Set B
ϕ_N [GeV]	0.1411	0.1290
ϕ_S [GeV]	0.1416	0.1406
m_0^2 [GeV^2]	2.3925_{E-4}	-1.2370_{E-2}
m_1^2 [GeV^2]	6.3298_{E-8}	0.5600
λ_1	−1.6738	−1.0096
λ_2	23.5078	25.7328
c_1 [GeV]	1.3086	1.4700
δ_S [GeV^2]	0.1133	0.2305
g_1	5.6156	5.3295
g_2	3.0467	−1.0579
h_1	37.4617	5.8467
h_2	4.2281	−12.3456
h_3	2.9839	3.5755
g_F	4.5708	4.9571
M_0 [GeV]	0.3511	0.3935

TABLE II. N_c dependence of the parameters.

m_0^2, m_1^2, δ_S	N_c^0
g_1, g_2, g_f	$1/\sqrt{N_c}$
$\lambda_2,\ h_2,\ h_3$	N_c^{-1}
$\lambda_1,\ h_1$	N_c^{-2}
c_1	$N_c^{-3/2}$
$h_{N/S}$	$\sqrt{N_c}$
g_F	$1/\sqrt{N_c}$

Chiral condensate vs T

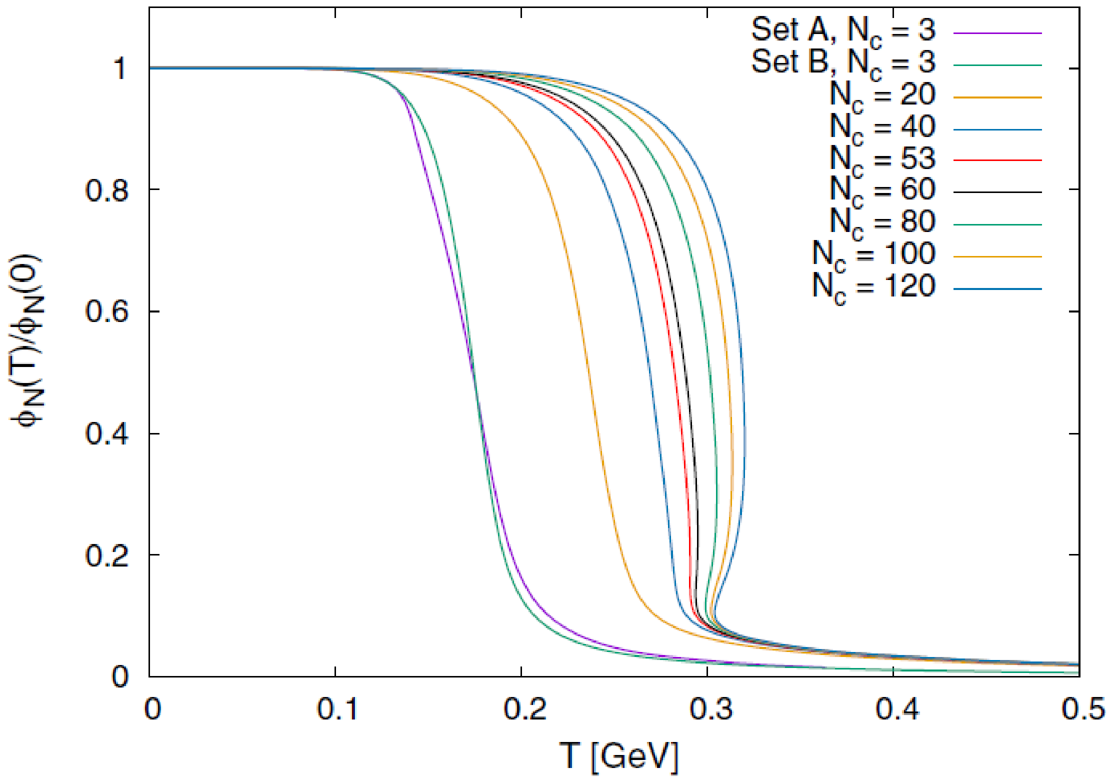


FIG. 4. The temperature dependence of the normalized chiral condensate ϕ_N.

Chiral condensate vs chemical potential

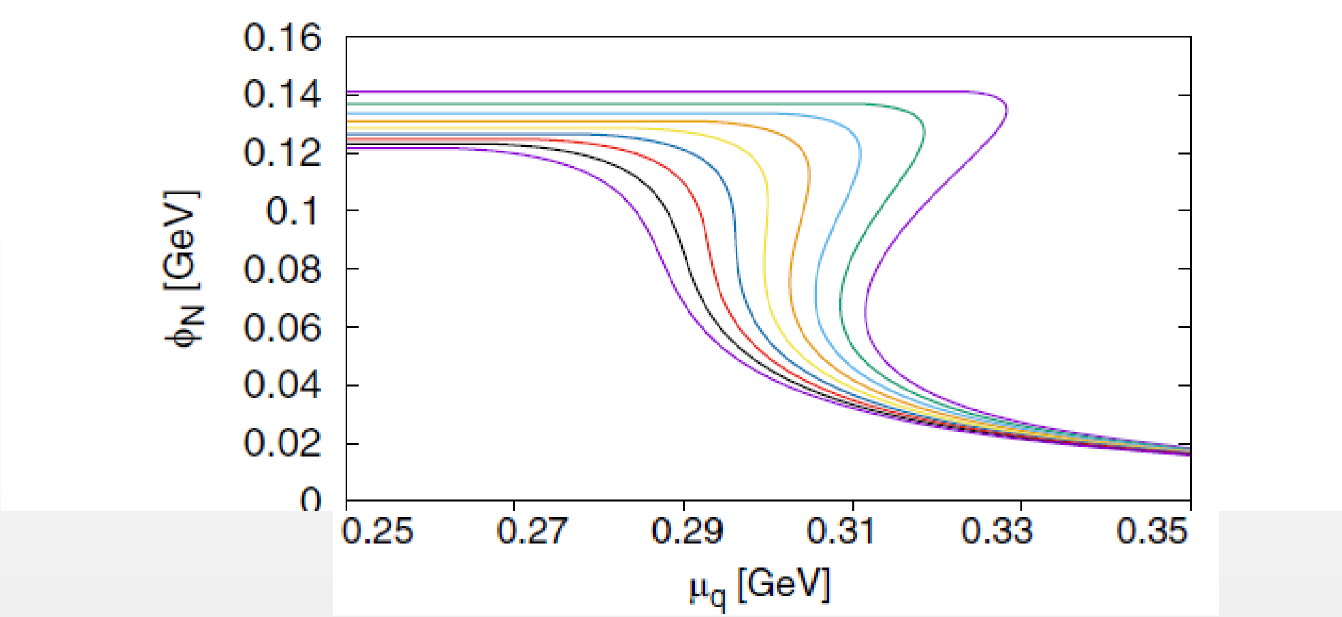
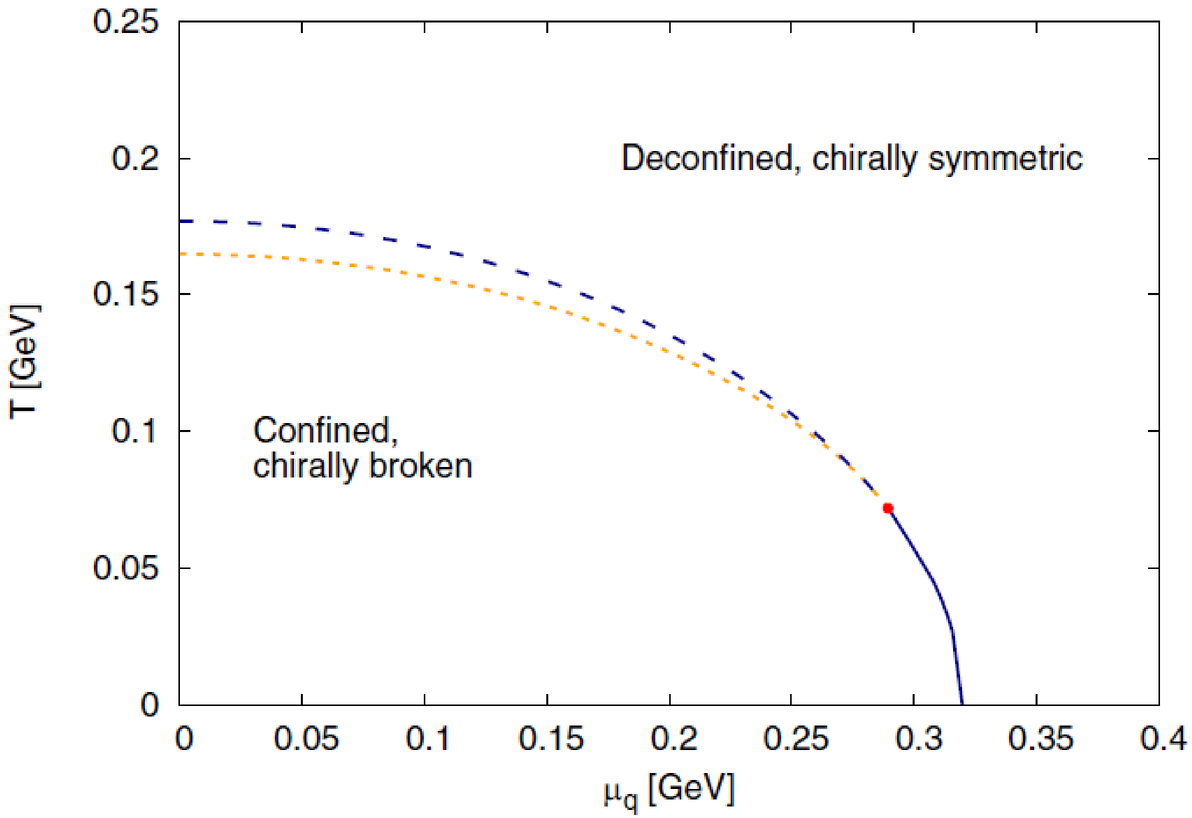
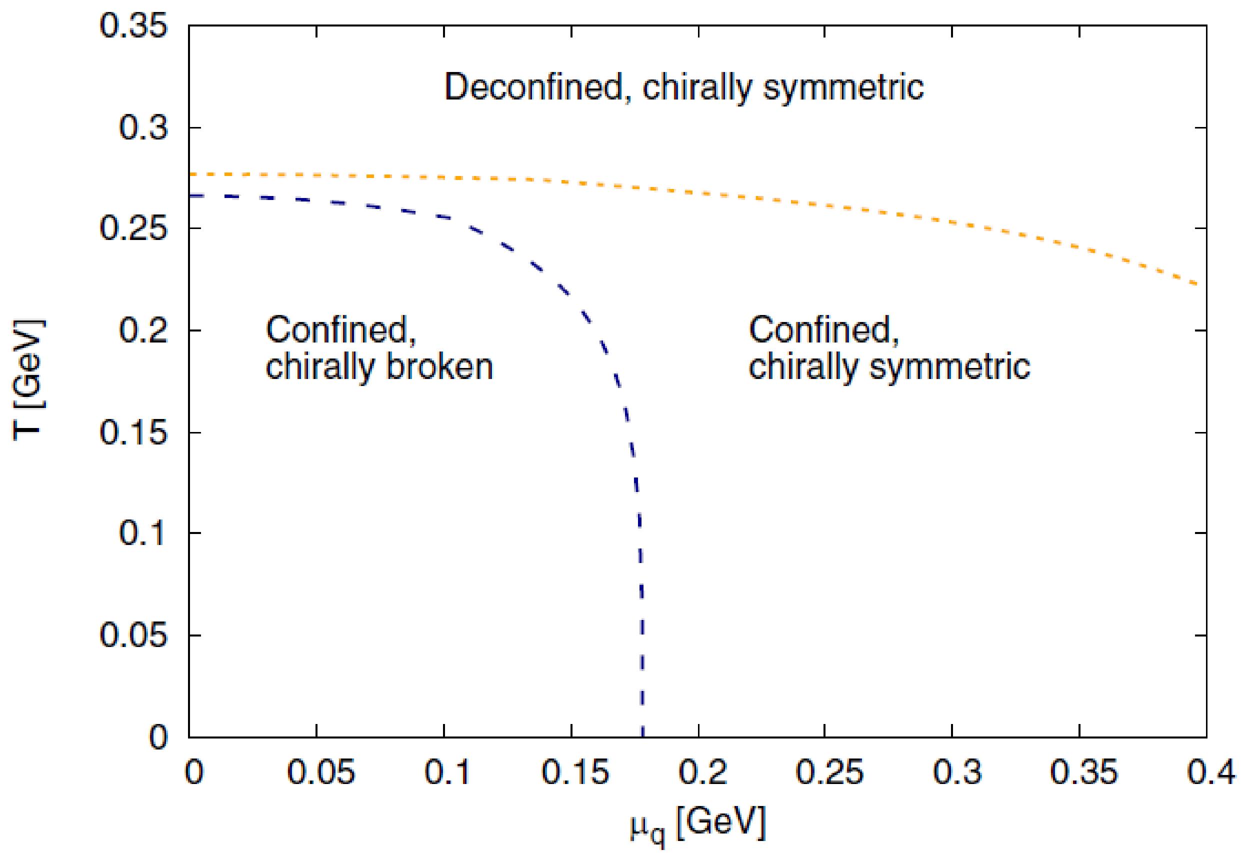


FIG. 3. The μ_q quark chemical potential dependence of the ϕ_N condensate at different N_c values. $N_c = 3.00$ corresponds to the rightmost curve, while $N_c = 3.45$ corresponds to the leftmost curve. The top figure is obtained with set A, while the bottom figure with set B of Table I.

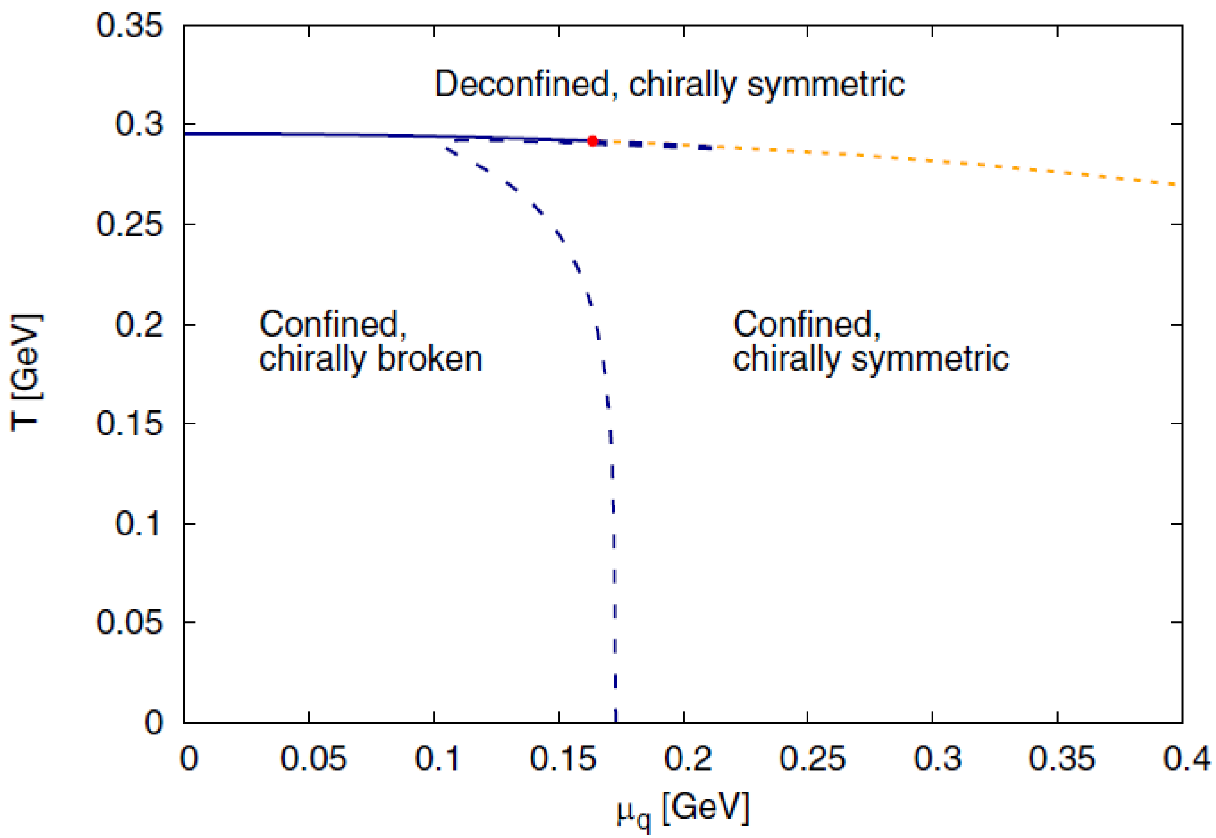
Phase diagram: Nc =3

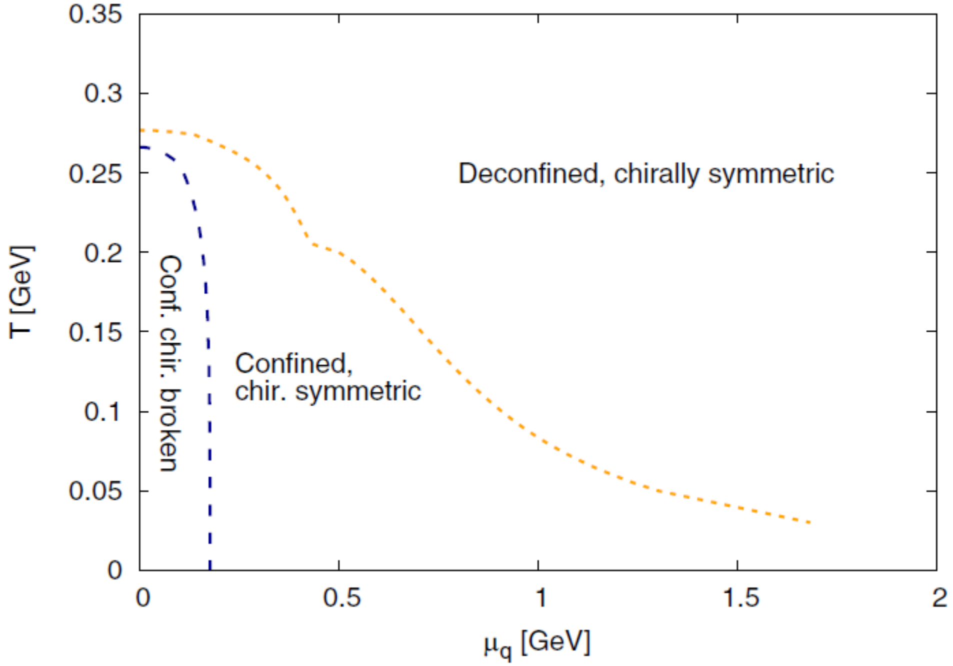


Phase diagram: Nc = 33 (only cross-over, no CP)



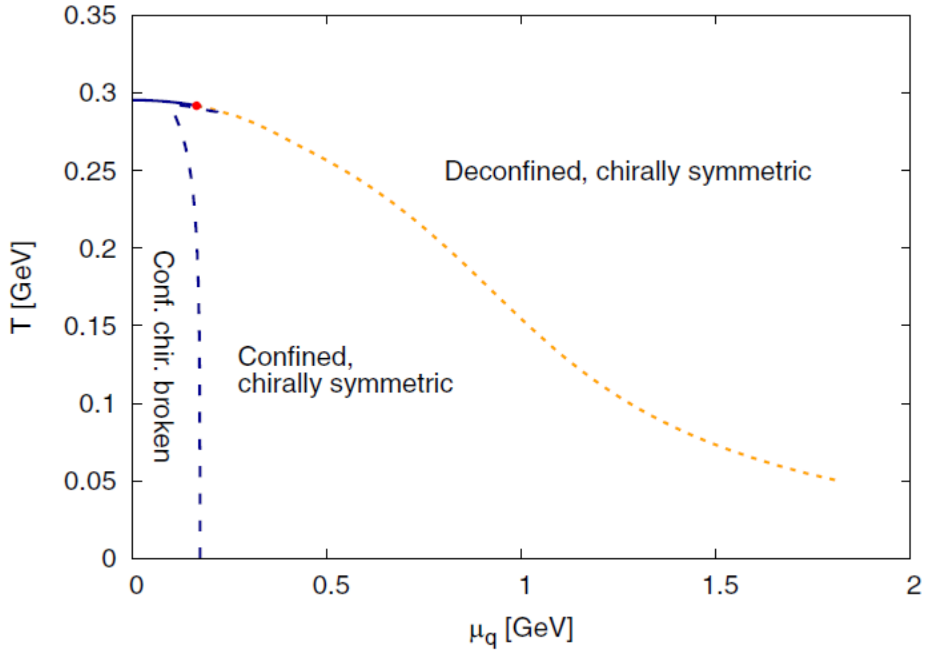
Phase diagram: Nc = 63





0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
T [GeV]
0
0.5
1
1.5
2
μq [GeV]
Deconfined, chirally symmetric
Conf. chir. broken
Confined,
chir. symmetric

Nc = 33



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
T [GeV]
0
0.5
1
1.5
2
μq [GeV]
Deconfined, chirally symmetric
Conf. chir. broken
Confined,
chirally symmetric

Nc = 63

Schematic phase diagram at large Nc

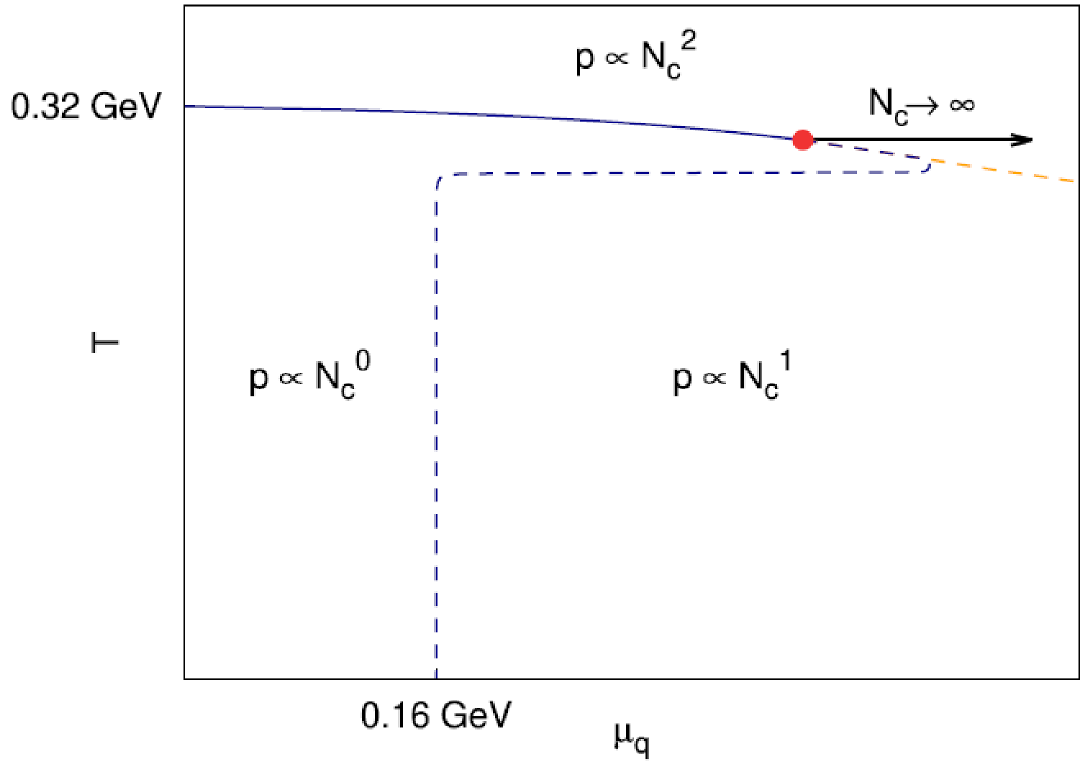
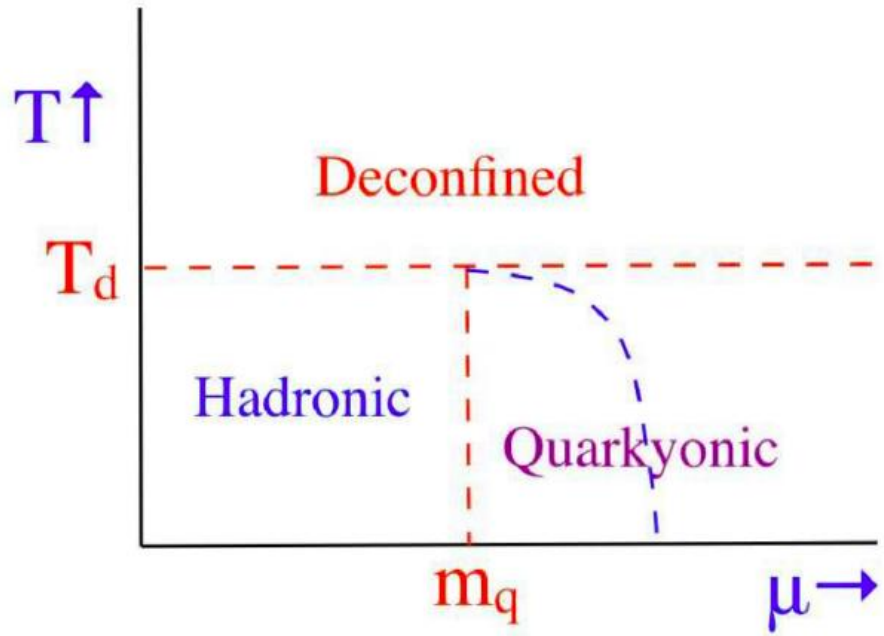


FIG. 13. The schematic phase diagram for large N_c and the N_c scaling of the pressure in the different phases.

Then, for the QCD diagram: 3 is not a large number!!!!

…agrees well with quarkyonic…



- Confined, quarkyonic phase may appears for large density

McLerran, Pisarski: **Nucl. Phys. A 796, 83-100 (2007)**

McLerran, Redlich, Sasaki: **Nucl. Phys. A 824, 86-100 (2009)**

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Uniwersytet Jana Kochanowskiego w Kielcach

Does nuclear matter bind at large N_c?

Luca Bonanno *, Francesco Giacosa

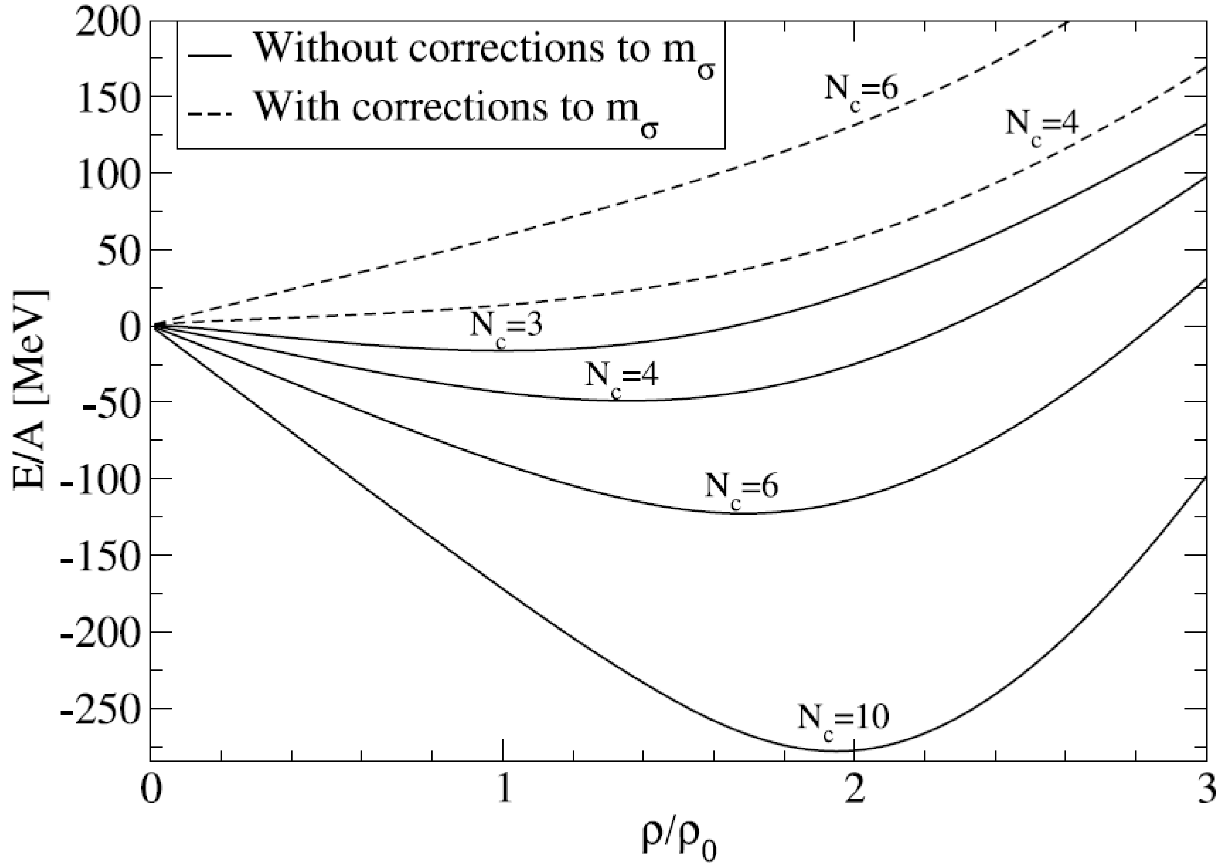
The Lagrangian of the Walecka model reads [9]:

$$\mathcal{L} = \bar{\psi}\left[\gamma^{\mu}(i\partial_{\mu} - g_{\omega}\omega_{\mu}) - (m_N - g_{\sigma}\sigma)\right]\psi + \frac{1}{2}\partial^{\mu}\sigma\,\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu} - V_{\sigma}(\sigma),$$

$$m_{\sigma} \longrightarrow m_{\sigma};$$

$$m_{\omega} \longrightarrow m_{\omega}, \qquad m_N \longrightarrow m_N\frac{N_c}{3};$$

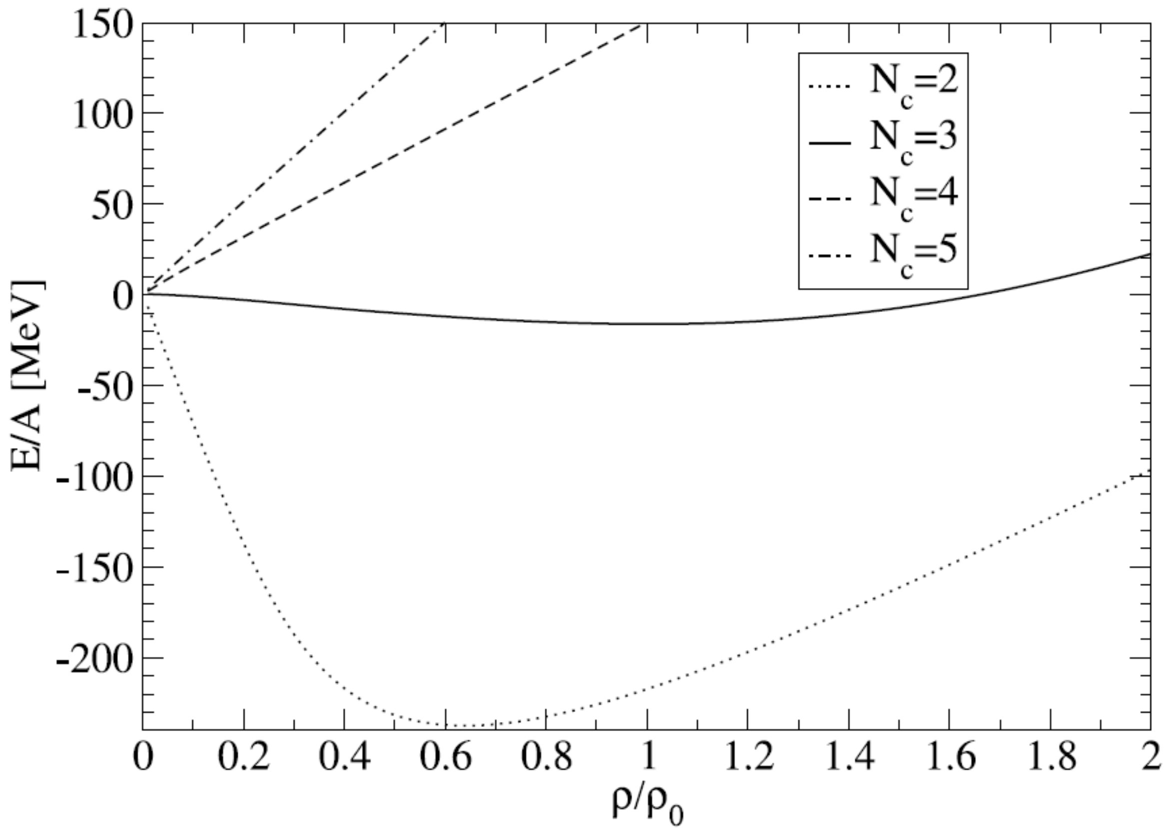
$$g_{\sigma} \longrightarrow g_{\sigma}\sqrt{\frac{N_c}{3}}, \qquad g_{\omega} \longrightarrow g_{\omega}\sqrt{\frac{N_c}{3}}.$$



$$m_\sigma^2(N_c) = m_\sigma^2 + b_\sigma^2\left(\frac{1}{3} - \frac{1}{N_c}\right).$$

Minimal variation of the scaling…
quark model places this state higher.
Enough to unbind nuclear matter

If the lightest scalar is not a quarkonium



Summary: for nuclear matter, 3 is not a large number!!!!

Other scenarios

- Two scalar fields: tetraquark+quarkonium, no nuclear matter.

- f0(500) as pion-pion molecular states, dissolves at large Nc, no nuclear matter.

- One-pion-exchange: what does eventually happen at very large Nc? (not taken into account here because beyond MFE)

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Uniwersytet Jana Kochanowskiego w Kielcach

Neutron stars in the large-N_c limit

Francesco Giacosa [a,b], Giuseppe Pagliara [c,*]

$$p_q = b_1 N_c \mu_q^4 - N_c^2 B$$

$$b_1 = \frac{N_f}{12\pi^2}.$$

Quark matter athigh density: free gas plus bag

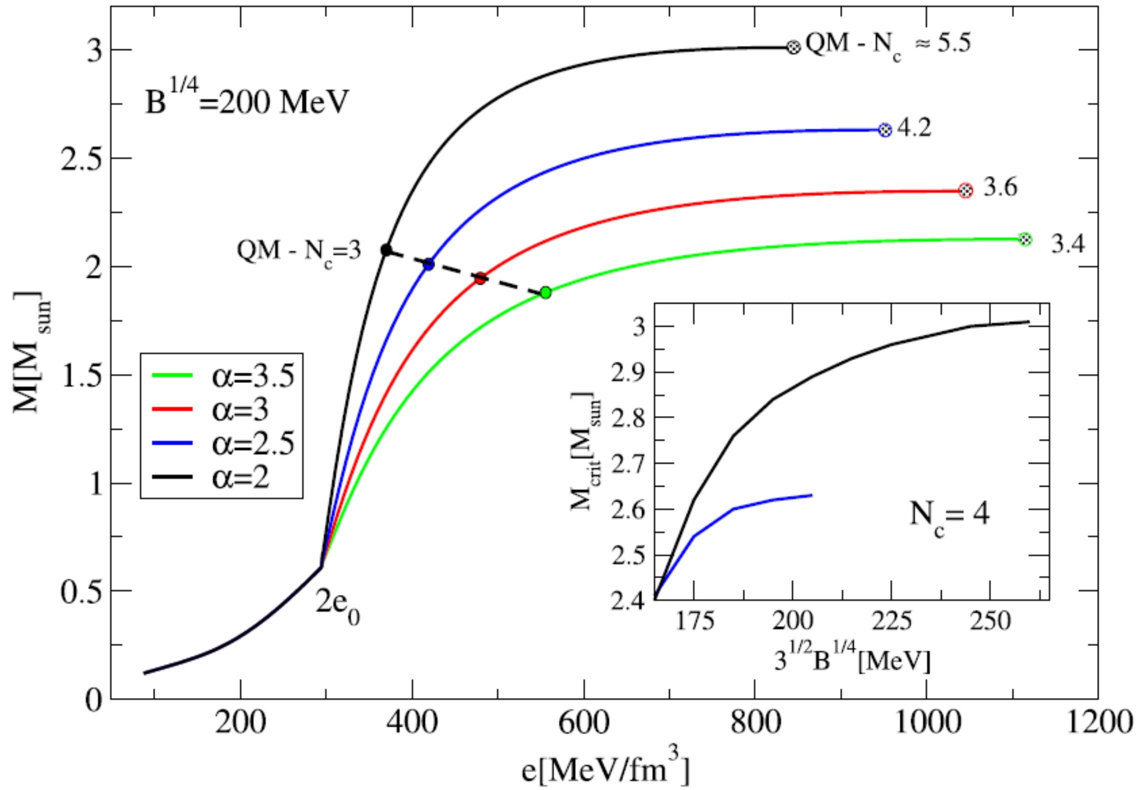
$$p_b = \tilde{a}_1 N_c \mu_q^2 - \tilde{K} N_c$$

Baryonic matter at high density (starting from 2ρ0):

The speed of sound is 1 in this case

The stiffest equation of state corresponds, in agreement with causality, to $\alpha = 2$.

Results



Moreover one should not observe stars with masses larger than about $2.1 M_\odot$

Also for neutron stars: Nc = 3 is not large!

Conclusions

Large-Nc

• useful tool for QCD (as well as for a variety of models/theories)

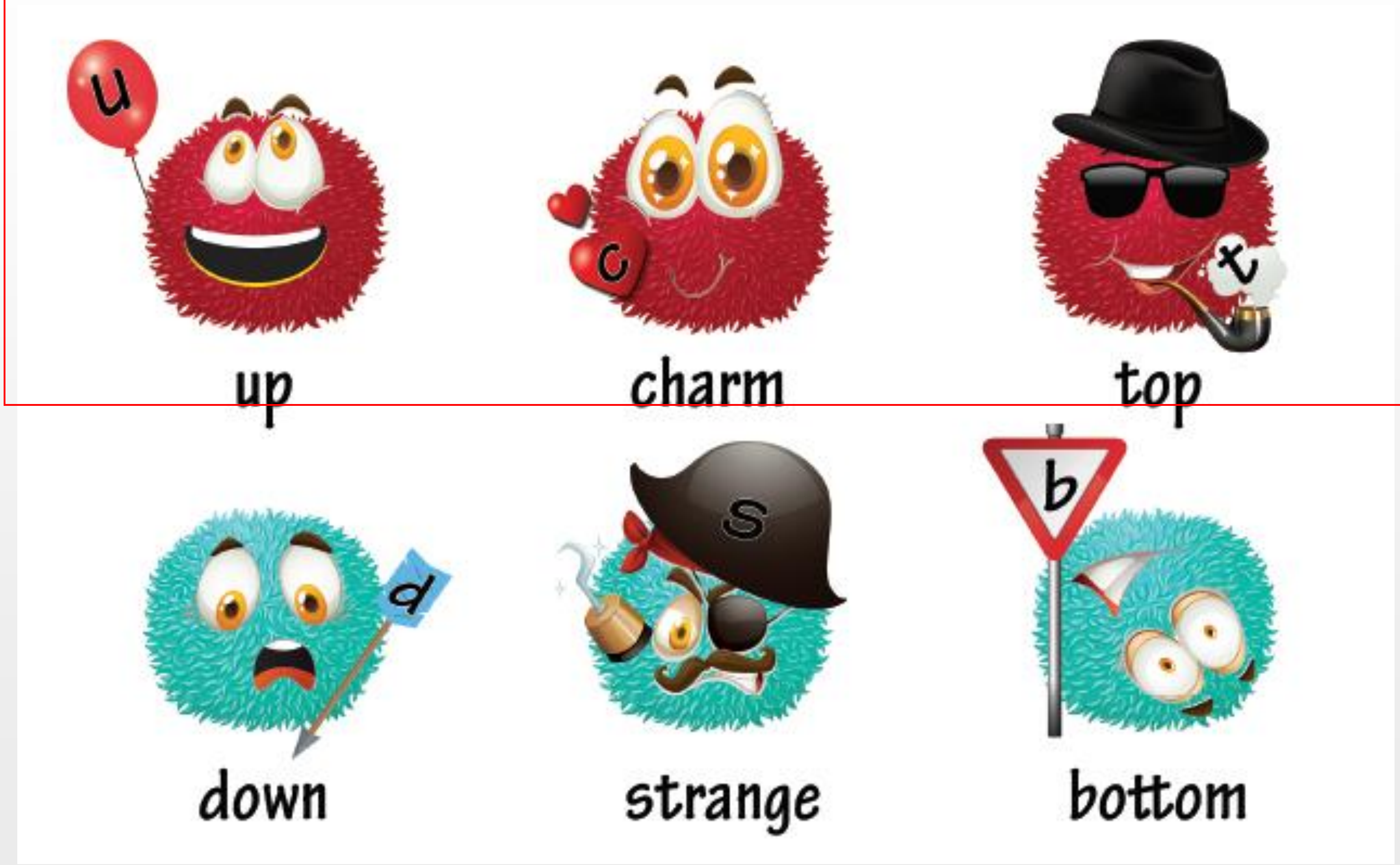
• Phenomenology in the vacuum can be better understood (i.e. OZI), certains terms appear as dominant, other are suppressed… 3 is a large number.

• Exception: chiral anomaly, relevant for the η' and the pseudoscalar glueball

•Applications at nonzero temperature and density, in various cases 3 is not a large number.

Thanks!

Confinement: quarks never ‘seen’ directly.
How they might look like ☺



Picture by Pawel Piotrowski

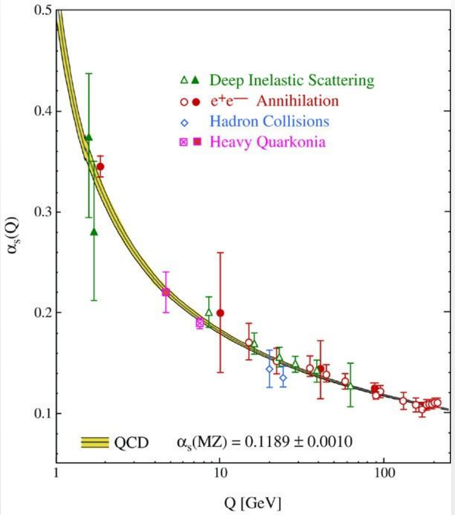
Trace anomaly:
the emergence of a dimension

Chiral limit: $m_i = 0$

$x^\mu \to x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250 \text{ MeV}$

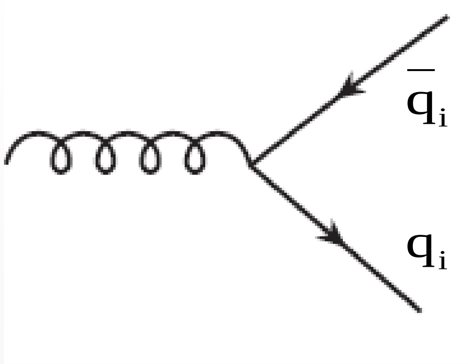
$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \to m^*_{gluon} \approx 500 - 800 \, \text{MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Flavor symmetry



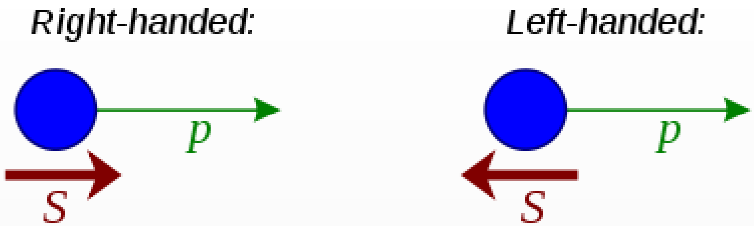
Gluon-quark-antiquark vertex

It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$\mathrm{U} \in \mathrm{U}(3)_{\mathrm{V}} \rightarrow \mathrm{U}^{+}\mathrm{U} = 1$$

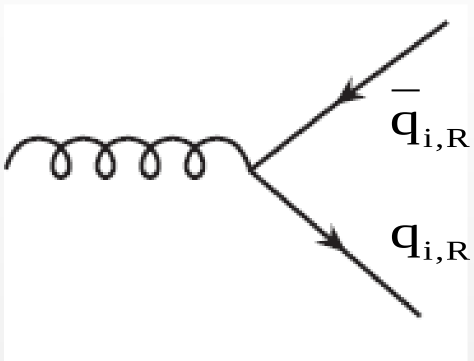
Chiral symmetry

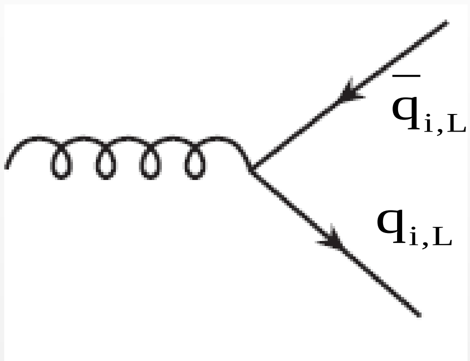


$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$





$$q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\partial^\mu(\bar{q}^i \gamma_\mu \gamma_5 q^i) = \frac{3g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}(G_{\mu\nu} G_{\rho\sigma})$$

Spontaneous breaking of chiral symmetry: chiral condensate and constituent mass

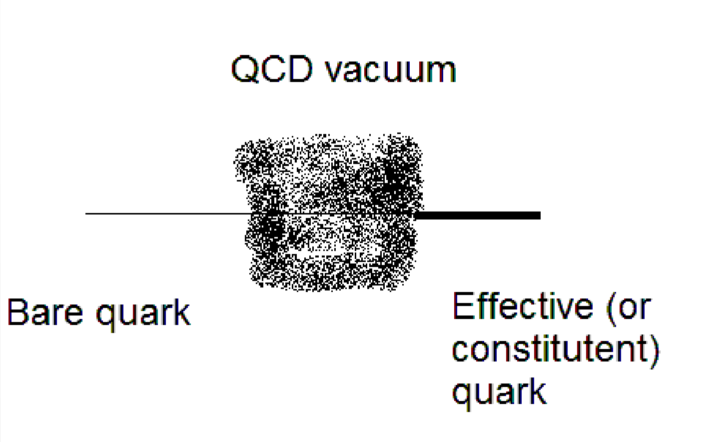
$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

$$\mathrm{SSB:}\ \mathrm{SU(3)_R \times SU(3)_L \rightarrow SU(3)_{V=R+L}}$$

Chiral symmetry → Flavor symmetry

$$\left\langle \bar{q}_i q_i \right\rangle = \left\langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \right\rangle \neq 0$$

$$\mathrm{m \approx m_u \approx m_d \approx 5\ MeV \rightarrow m^* \approx 300\ MeV}$$



$$\mathrm{m_{\rho-meson} \approx 2m^*}$$

$$\mathrm{m_{proton} \approx 3m^*}$$

At nonzero T the chiral condensate decreases

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	θ_{quad} [°]	θ_{lin} [°]
$1\,^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'(958)}$	-11.3	-24.5
$1\,^3S_1$	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$	39.2	36.5
$1\,^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^{\dagger}$	$h_1(1380)$	$\boldsymbol{h_1(1170)}$		
$1\,^3P_0$	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$		
$1\,^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^{\dagger}$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$		
$1\,^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^{\dagger}$	$\eta_2(1870)$	$\boldsymbol{\eta_2(1645)}$		
$1\,^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}$		$\boldsymbol{\omega(1650)}$		
$1\,^3D_2$	2^{--}		$\boldsymbol{K_2(1820)}$				
$1\,^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$	31.8	30.8
$1\,^3F_4$	4^{++}	$\boldsymbol{a_4(2040)}$	$\boldsymbol{K_4^*(2045)}$		$\boldsymbol{f_4(2050)}$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$		
$2\,^3S_1$	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}$	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$		
$3\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1800)}$			$\eta(1760)$		

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Tensor and (axial-)tensors

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

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From well-known tensor mesons to yet unknown axial-tensor mesons

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While the ground-state tensor ($J^{PC} = 2^{++}$) mesons $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, and $f_2'(1525)$ are well known experimentally and form an almost ideal nonet of quark-antiquark states, their chiral partners, the ground-states axial-tensor ($J^{PC} = 2^{--}$) mesons are poorly settled: only the kaonic member $K_2(1820)$ of the nonet has been experimentally found, whereas the isovector state ρ_2 and two isoscalar states ω_2 and ϕ_2 are still missing. Here, we study masses, strong, and radiative decays of tensor and axial-tensor mesons within a chiral model that links them: the established tensor mesons are used to test the model and to determine its parameters, and subsequently various predictions for their chiral partners, the axial-tensor mesons, are obtained. The results are compared to current lattice QCD outcomes as well as to other theoretical approaches and show that the ground-state axial-tensor mesons are expected to be quite broad, the vector-pseudoscalar mode being the most prominent decay mode followed by the tensor-pseudoscalar one. Nonetheless, their experimental finding seems to be possible in ongoing and/or future experiments.

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Large-Nc: basics/1

$SU_c(3)$

$|q\rangle = \begin{pmatrix} R \\ G \\ B \end{pmatrix}$; $|q\rangle \mapsto U|q\rangle$, $U \in SU_c(3)$

$q_a \quad a = 1, 2, 3$

$|MESON\rangle = \sqrt{\frac{1}{3}}\,(\bar{R}R + \bar{G}G + \bar{B}B)$

invariant under $SU_c(3)$ = 'white'

$|BARYON\rangle = N \cdot \varepsilon_{abc}\, q^a q^b q^c$

$= N\,(RGB + BRG + GBR - GRB - BGR - RBG)$

invariant under $SU(3)$

$SU_c(N_c)$

$|q\rangle = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_{N_c} \end{pmatrix}$; $|q\rangle \mapsto U|q\rangle$, $U \in SU(N_c)$

$q_a \quad a = 1, 2, \ldots, N_c$

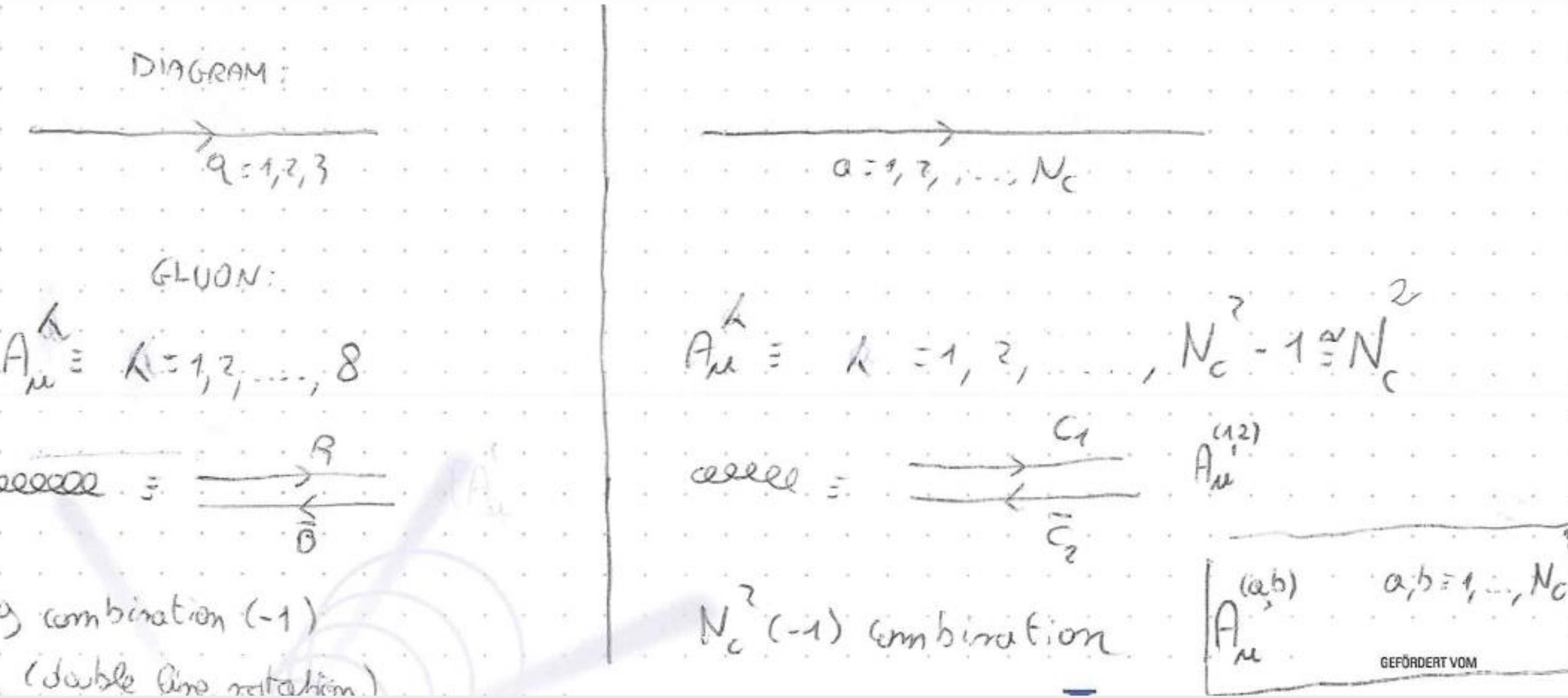
$|MESON\rangle = \sqrt{\frac{1}{N_c}}\,(\bar{C}_1 C_1 + \cdots + \bar{C}_{N_c} C_{N_c})$

$|BARYON\rangle = N\, \varepsilon_{a_1 \ldots a_{N_c}}\, q^{a_1} q^{a_2} \cdots q^{a_{N_c}}$

$a_1, \ldots, a_{N_c} = 1, 2, \ldots, N_c$

invariant under $SU_c(N_c)$

Large-Nc: basics/2



Glueball production and decays: gluon-rich processes

Glueballs should be find in gluon-rich processes
(such as J/ψ decays, proton-antiproton fusion, …)

Glueball should have suppressed decay into flavor breaking channels (eg eta-etaprime)

Moreover, glueballs should have a suppressed (but nonzero!) decay into photons.

In the $N_c \to \infty$ limit eg.:

- Stable, noninteracting mesons and glue-balls (infinite number with fixed qn.) in the hadronic phase with $m \propto N_c^0$ masses.
- Baryon masses diverges as $m_B \propto N_c^1$.
- Hadronic phase built from noninteracting mesons and glueballs, energy density scales as $\propto N_c^0$
- Phase boundary to quark-gluon plasma at a temperature $\propto N_c^0$
- Energy density of quark-gluon phase N_c^2.
  $\Rightarrow$ First or second order phase transition expected.
- Quark loops are suppressed: the thermodynamics expected to became similar to Yang-Mills.
- Confined, quarkyonic phase may appears for large density

McLerran, Pisarski: **Nucl. Phys. A 796, 83-100 (2007)**
McLerran, Redlich, Sasaki: **Nucl. Phys. A 824, 86-100 (2009)**

FInite T, sigma model

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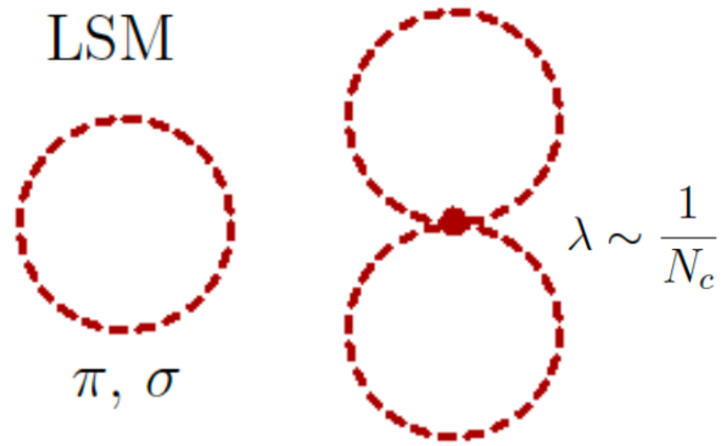
$$\mathcal{L}_\sigma(N_c) = \frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4,$$

$$\Phi^t = (\sigma, \vec{\pi})$$

$$0 = \varphi(T)^2 - \frac{N_c}{3\lambda}\mu^2 + 3\int (G_\sigma + G_\pi).$$

$$\int G_i = \int_0^\infty \frac{dkk^2}{2\pi^2\sqrt{k^2 + M_i^2}}\left[\exp\left(\frac{\sqrt{k^2 + M_i^2}}{T}\right.\right.$$

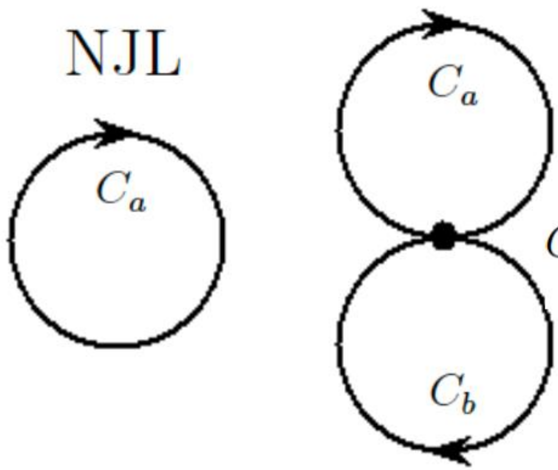


$$P \sim N_c^0 \qquad P \sim \lambda \sim \frac{1}{N_c}$$

$$T_c(N_c) = \sqrt{2} f_\pi \sqrt{\frac{N_c}{3}} \propto N_c^{1/2}.$$

Finite T, NJL model

$$\mathcal{L}_{\rm NJL}(N_c) = \bar{\psi}(\imath\gamma^{\mu}\partial_{\mu} - m_q)\psi + \frac{3G}{N_c}[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\psi)]^2$$



$$G_{\rm NJL} \sim \frac{1}{N_c}$$

$$P \sim N_c \qquad P \sim G_{\rm NJL}\, N_c^2, \sim N_c$$

$$T_c(N_c) \simeq \Lambda\sqrt{\frac{3}{\pi^2}}\sqrt{1 - \frac{\pi^2}{6\Lambda^2 G}} \propto N_c^0.$$

What if the lightest scalar is a tetraquark?

$$\mathcal{L} = \bar{\psi}\left[\gamma^{\mu}(i\partial_{\mu} - g_{\omega}\omega_{\mu}) - (m_N - g_{\chi}\chi)\right]\psi + \frac{1}{2}\partial^{\mu}\chi\partial_{\mu}\chi - \frac{1}{2}m_{\chi}^2\chi^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega^{\mu}\omega_{\mu},$$

$$\chi = [\bar{R}, \bar{B}][R, B] + [\bar{G}, \bar{B}][G, B] + [\bar{R}, \bar{G}][R, G]$$

for Nc = 3

$$d_{a_1} = \varepsilon_{a_1 a_2 a_3 \cdots a_{N_c}} q^{a_2} q^{a_3} \cdots q^{a_{N_c}} \quad \text{with} \quad a_2, \ldots, a_{N_c} = 1, \ldots, N_c.$$

for Nc> 3

$$\chi = \sum_{a_1=1}^{N_c} d_{a_1}^{\dagger} d_{a_1}.$$

Extended ‘tetraquark’ version!
(indeed, a well-defined one)

$$m_{\chi} \to m_{\chi}\frac{2N_c - 2}{4}, \qquad g_{\chi} \to g_{\chi}.$$

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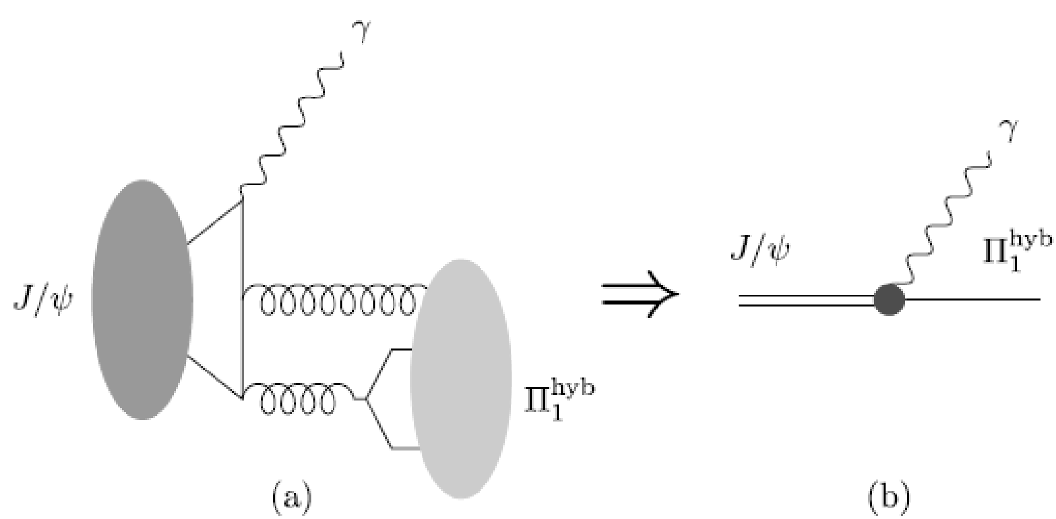
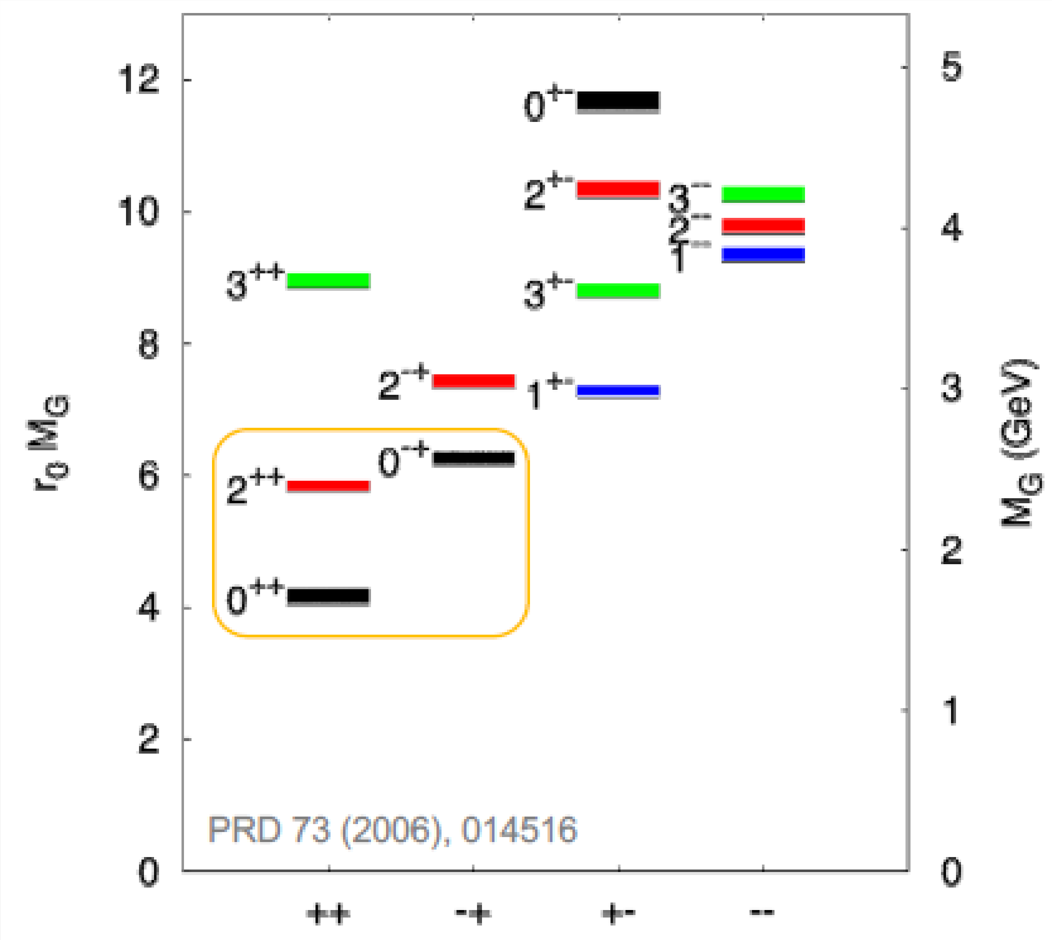
Radiative production and decays of the exotic $\eta_1'(1855)$ and its siblings

Vanamali Shastry [a,*], Francesco Giacosa [a,b]



Glueball spectrum from lattice QCD



$$T_c \sim T_{dec} \sim \Lambda_{QCD} \sim N_c^0$$

$$T_c \sim f_\pi \sim N_c^{1/2}$$

$$T_c \sim N_c^0$$

The stiffest equation of state corresponds, in agreement with causality, to $\alpha = 2$.

$$p_b = \tilde{a}_1 N_c \mu_q^2 - \tilde{K} N_c$$

The speed of sound is 1 in this case

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Neutron stars in the large-N_c limit

Francesco Giacosa [a,b], Giuseppe Pagliara [c,*]

$$p_q = b_1 N_c \mu_q^4 - N_c^2 B$$

$$b_1 = \frac{N_f}{12\pi^2}.$$

Quark matter athigh density:
free gas plus bag

$$p_b = a_1 \mu_b^\alpha - K$$

$$a_1(N_c) \propto \left(\frac{g_V^2}{m_V^2}\right)^{\frac{\alpha-4}{2}} \propto N_c^{\frac{\alpha-4}{2}}$$

$$K = \tilde{K} N_c^{(3\alpha-4)/2}$$

Baryonic matter at high density
(starting from 2ρ0):
parameter α unknown

$$v_b = \sqrt{\frac{dp_b}{d\varepsilon_b}} = \frac{1}{\sqrt{\alpha - 1}}.$$

$$\alpha \geq 2$$

Stiffest equation and transition

The stiffest equation of state corresponds, in agreement with causality, to $\alpha = 2$.

$$p_b = \tilde{a}_1 N_c \mu_q^2 - \tilde{K} N_c$$

The speed of sound is 1 in this case
The speed of sound is 1 in this case

$$p_b = \tilde{a}_1 N_c^{\frac{3\alpha-4}{2}} \mu_q^{\alpha} - \tilde{K} N_c^{\frac{3\alpha-4}{2}} = b_1 N_c \mu_q^4 - N_c^2 B = p_q$$

$$\mu_q^{\mathrm{crit}} = \left(\frac{B N_c}{b_1}\right)^{1/4} [1 + \ldots] \text{ for } 2 \leq \alpha \leq \frac{16}{7}$$

Neutron stars, main outcome

From the $N_c = 3$ analysis, one further reduces the range (20), namely α must be *smaller* than about $\alpha_{\max} \simeq 2.5$ (the blue line) in order to explain the existence of $2M_\odot$ stars:

$$2 \leq \alpha \lesssim 2.5 \text{ for } N_c = 3.$$

Moreover one should not observe stars with masses larger than about $2.1 M_\odot$

Also for neutron stars: Nc = 3 is not large!

Also for neutron stars: Nc = 3 is not large!