Baryon number fluctuations at finite temperature and magnetic field

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Various Faces of QCD,
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Why study QCD in large magnetic fields?

May be important for phenomenology:

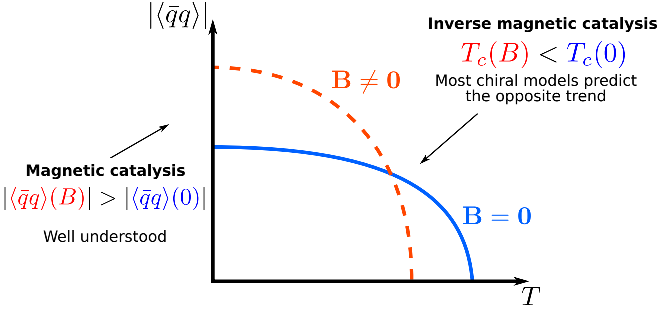
- Non-central heavy-ion collisions
- Magnetars
- Early Universe

Additional parameter to study QCD under extreme conditions

- Can be probed directly in LQCD simulations
  - Quark condensate
  - Fluctuations of conserved charges

This talk $\rightarrow$ Effective model with screend interactions

Schematic behavior of the quark condensate from first-principle numerical simulations



Opposite $T_C(B)$ for models and LQCD $\rightarrow$ Possible missing interactions

Our work: We explore the effect of screening of four-quark interactions in an effective chiral quark model[1]

[1] 2107.05521, 2109.04439

Starting point → Chiral model inspired by Coulomb gauge QCD[2]

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m_0)\psi(x) + \int d^4y \rho^a(x) V^{ab}(x-y)\rho^b(y)$$

with

- $\rho^a(x) = \bar{\psi}(x)\gamma^0 T^a \psi(x)$ → color quark current
- $V^{ab}(x-y)$ → Interaction potential

This work → Contact interaction, gap equation:

$$M = m_0 + C_F V_0 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(1 - N_{th}(E,\mu) - \bar{N}_{th}(E,\mu)\right)$$

The same form as the NJL model if $C_F V_0 \to 4 N_c N_f (2 G_{NJL})$

- NJL → Scalar-scalar interaction

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + G_{NJL}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2\right]$$

- Current model → Vector-vector interaction
  - Systematic improvements possible → dressing by polarization

[2] See e.g. P. M. Lo, E. S. Swanson Phys. Rev. D **81** 034030 (2010)

Dressing by polarization, ring diagram approximation

$$\tilde{V}_0^{-1} = V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p}) \quad \Rightarrow \quad \tilde{V}_0 = \frac{1}{V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p})}$$

Static limit

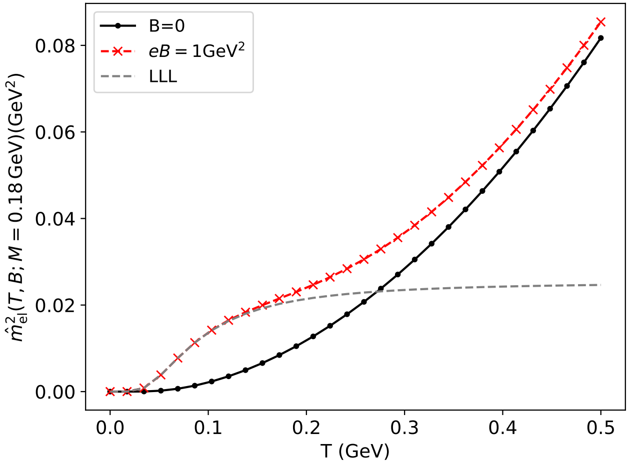
$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0)$$

Screening → Medium-dependent coupling

Contact interaction → No confinement

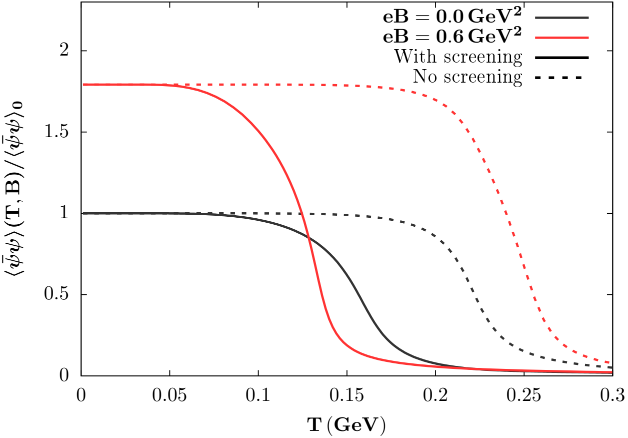
- Polyakov loop → Statistical confinement
- $\Pi_{00}(M) \to \Pi_{00}(M, \ell, \bar{\ell})$
- Regulates the screening strength

Vacuum term → Proper-time regularization

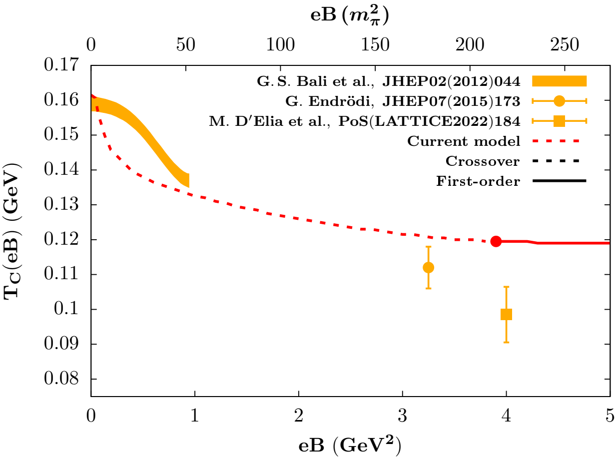


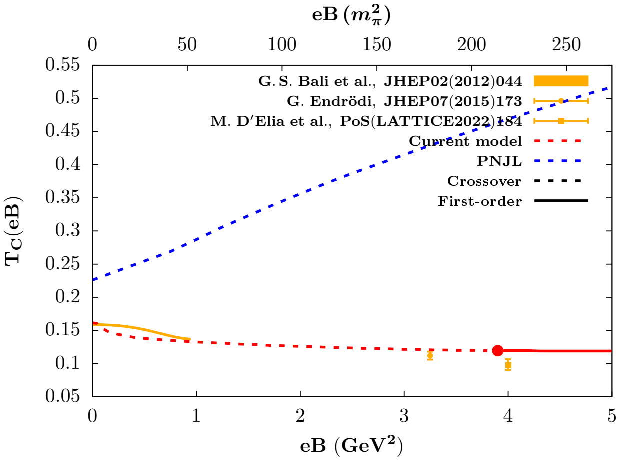
- LQCD $\rightarrow$ First-order transition for $4\,\text{GeV}^2 < eB^{LQCD}_{crit.} < 9\,\text{GeV}^2$ [3]
- Current model $\rightarrow$ $eB_{crit.} \approx 0.5\,\text{GeV}^2 \ll eB^{LQCD}_{crit.}$
- Approximation: $\Pi(M, \ell, \bar{\ell}) \rightarrow \xi \times \Pi(\bar{M} \approx 0.130\,\text{MeV}, \ell, \bar{\ell})$

[3] M. D'Elia et al., PoS(LATTICE2022)184



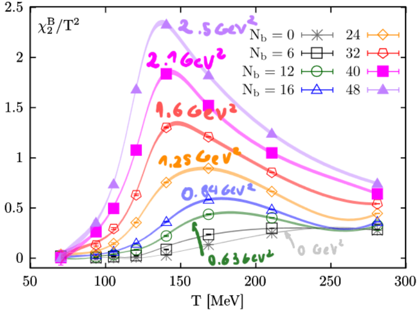
Screening $\rightarrow$ MC and IMC captured

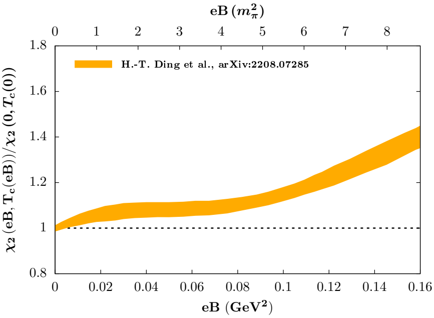

eB (m_π^2)
0
50
100
150
200
250
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
G. S. Bali et al., JHEP02(2012)044
G. Endrödi, JHEP07(2015)173
M. D′Elia et al., PoS(LATTICE2022)184
Current model
Crossover
First-order
$T_C(eB)$ (GeV)
0
1
2
3
4
5
eB (GeV^2)


eB (m_π^2)
0
50
100
150
200
250
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
T_C(eB)
G. S. Bali et al., JHEP02(2012)044
G. Endrödi, JHEP07(2015)173
M. D′Elia et al., PoS(LATTICE2022)184
Current model
PNJL
Crossover
First-order
0
1
2
3
4
5
eB (GeV^2)

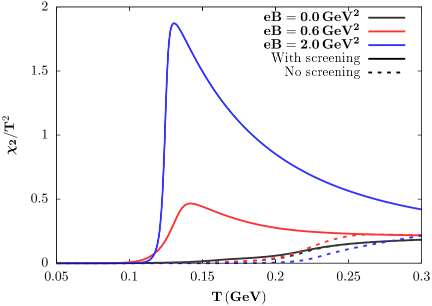
This talk → Baryon number fluctuations

$$\chi_n = \frac{\partial^n P}{\partial \mu_B^n}$$

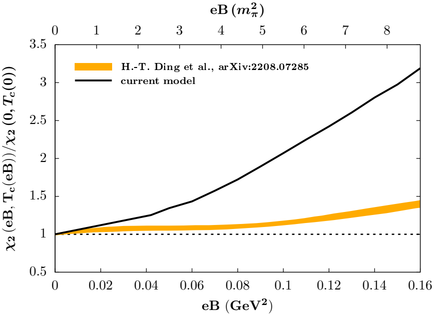




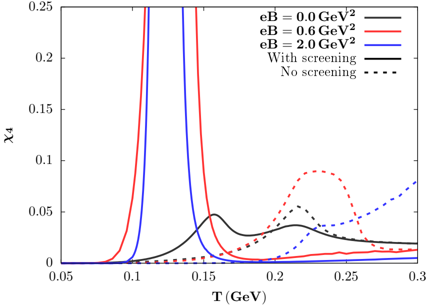
Left: H. T. Ding et al. Eur. Phys. J. A **57**, no.6, 202 (2021)
Right: H. T. Ding et al. Acta Phys. Polon. Supp. **16**, no.1, 1-A134 (2023)

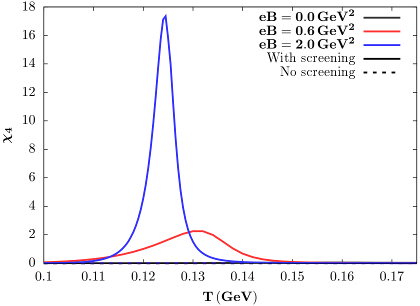


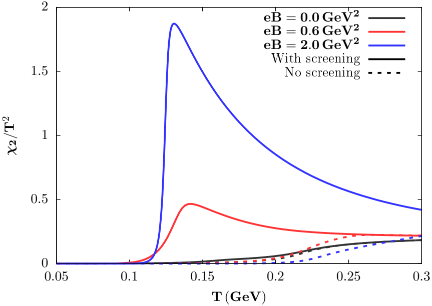
χ_2/T^2
$T\,(GeV)$
$eB = 0.0\,GeV^2$
$eB = 0.6\,GeV^2$
$eB = 2.0\,GeV^2$
With screening
No screening



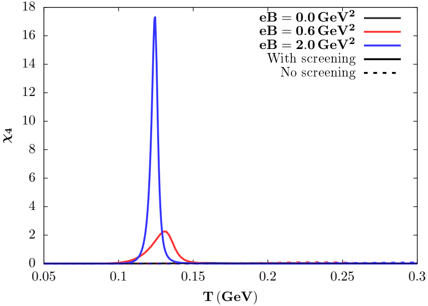
$eB\,(m_\pi^2)$
$\chi_2\,(eB, T_c(eB))/\chi_2\,(0, T_c(0))$
$eB\,(GeV^2)$
H.-T. Ding et al., arXiv:2208.07285
current model


eB = 0.0 GeV^2
eB = 0.6 GeV^2
eB = 2.0 GeV^2
With screening
No screening
χ_4
T (GeV)
0.25
0.2
0.15
0.1
0.05
0
0.05
0.1
0.15
0.2
0.25
0.3

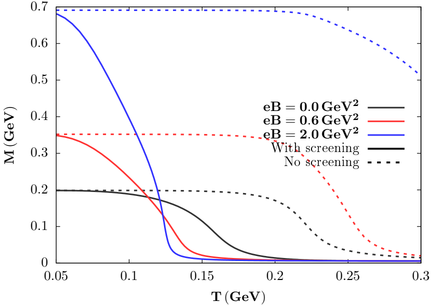

eB = 0.0 GeV^2
eB = 0.6 GeV^2
eB = 2.0 GeV^2
With screening
No screening
χ_4
T (GeV)
18
16
14
12
10
8
6
4
2
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17



χ_2/T^2
2
1.5
1
0.5
0
0.05
0.1
0.15
0.2
0.25
0.3
T (GeV)
eB = 0.0 GeV^2
eB = 0.6 GeV^2
eB = 2.0 GeV^2
With screening
No screening



χ_4
18
16
14
12
10
8
6
4
2
0
0.05
0.1
0.15
0.2
0.25
0.3
T (GeV)
eB = 0.0 GeV^2
eB = 0.6 GeV^2
eB = 2.0 GeV^2
With screening
No screening

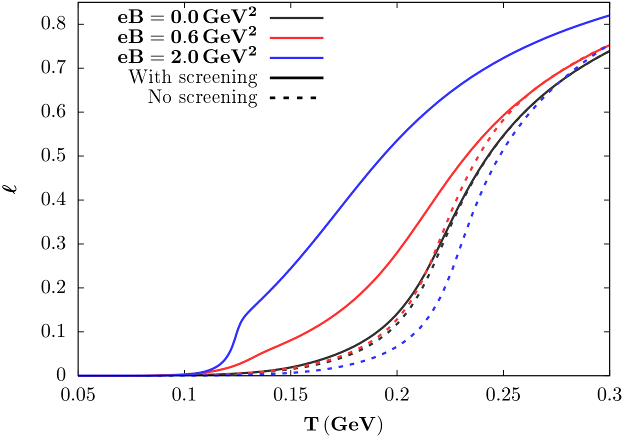


M (GeV)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
T (GeV)
eB = 0.0 GeV^2
eB = 0.6 GeV^2
eB = 2.0 GeV^2
With screening
No screening

Conclusions and outlook

- Effect of the screening of the 4-quark interaction
  - $B = 0$: Reduction of T_C
  - $B \neq 0$: MC and IMC captured
- Baryon number fluctuations $\rightarrow$ strongly enhanced
- Future plans:
  - Higher-order fluctuations
  - Charge fluctuations
  - Going beyond the contact interaction

For more details see 2107.05521, 2109.04439, 2309.03124



Polykaov loop increases with $B \to$ Consistent with LQCD observations[4]

[4] See eg. F. Bruckmann et al. JHEP04(2013)112, M. D'Elia et al. PRD **98**, 054509 (2018)

Final set of gap equations:

$$M = m_0 + C_F \tilde{V}_0(M, \ell) \left[I_{vac} - \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(N_{th}(E, \ell, \bar{\ell}, \mu) + \bar{N}_{th}(E, \ell, \bar{\ell}, \mu) \right) \right]$$

$$\tilde{V}_0(M, \ell, \bar{\ell}) = \frac{1}{V_0^{-1} + m_{el}^2(T, M, \ell, \bar{\ell})}$$

$$\frac{\partial}{\partial \ell} (\mathcal{U}_G + \mathcal{U}_Q) = 0$$

$$\frac{\partial}{\partial \bar{\ell}} (\mathcal{U}_G + \mathcal{U}_Q) = 0$$

Non-consistent approximation:

$$m_{el}^2(T, M, \ell, \bar{\ell}) \to \xi m_{el}^2(T, \bar{M}, \ell, \bar{\ell})$$

ξ – controls the ring strenght, $\bar{M} \approx 0.14\,\mathrm{GeV}$

Regularization:

$$I_{vac} = \int \frac{d^3q}{(2\pi^3)} \frac{M}{2E} \to \int_{1/\Lambda^2}^{\infty} \frac{ds}{16\pi^2} \frac{1}{s^2} e^{-M^2 s}$$

Coupling to the Polyakov loop → Statistical confinement

- Pure gluon system → Deconfinement order parameter
- Effective models → Accounts for non-preturbative gluon dynamics

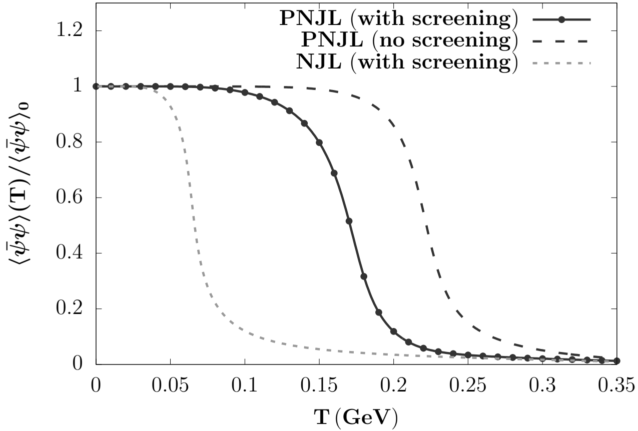
$$N_{th}(E,\mu) \to N_{th}(E,\ell,\bar{\ell},\mu) = \frac{\ell e^{-\beta(E-\mu)} + 2\bar{\ell}e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}{1 + 3\ell e^{-\beta(E-\mu)} + 3\bar{\ell}e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}$$
$$= \begin{cases} \dfrac{1}{1+e^{3\beta(E-\mu)}}\,, & \ell = \bar{\ell} = 0\,, \quad \text{baryon-like} \\ \dfrac{1}{1+e^{\beta(E-\mu)}}\,, & \ell = \bar{\ell} = 1\,, \quad \text{quark-like} \end{cases}$$

Two additional gap equations

$$\frac{\partial}{\partial \ell}(\mathcal{U}_G + \mathcal{U}_Q) = 0 \qquad \frac{\partial}{\partial \bar{\ell}}(\mathcal{U}_G + \mathcal{U}_Q) = 0$$

- $\mathcal{U}_G$ – pure gauge potential[5]
- $\mathcal{U}_Q$ – quark-gluon interaction

[5] P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 074502 (2013)



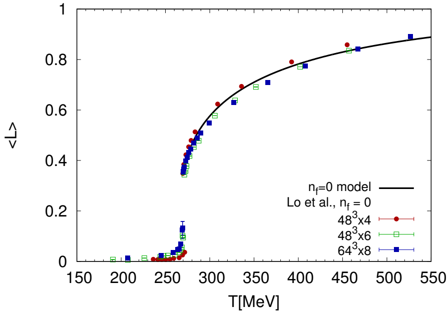
Screening

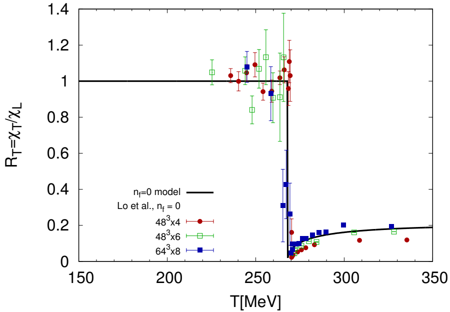
- Reduction of pseudocritical temperature
- No additional modification of model parameters necessary
- Role of Polyakov loop → suppression of the screening strength

Pure gauge part → Polyakov loop potential[1]

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\ln M_H(\ell,\bar{\ell}) + \frac{1}{2}c(T)(\ell^3+\bar{\ell}^3) + d(T)(\ell\bar{\ell})^2$$

- Polyakov loop & fluctuations determined from LQCD





Quark-gluon interaction

$$\mathcal{U}_Q = -2T\int\frac{d^3q}{(2\pi)^3}2\ln\left(1+3\ell e^{-\beta E}+3\ell e^{-2\beta E}+e^{-3\beta E}\right)$$

[1] *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88**, *074502 (2013)*

Electric mass

$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0) = \frac{1}{2} N_f \times \int \frac{d^3 q}{(2\pi)^3} 4\beta N_{th}(1 - N_{th})$$

Constant, uniform magnetic field $\rightarrow$ Landau quatization

$$E^2 = m^2 + \vec{p}^2 \quad \rightarrow \quad E_{k,s}^{\,2} = m^2 + p_z^2 + 2|q_f B|(k + \frac{1}{2} - s)\,,$$

$$2N_f \int \frac{d^3 p}{(2\pi)^3} \quad \rightarrow \quad \sum_f \frac{|q_f B|}{2\pi} \sum_{s=\pm 1/2}^{\infty} \sum_{k=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

Electric mass (per flavor)

$$m_{el}^2 = \frac{1}{2} \frac{|q_f B|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int \frac{dq_z}{2\pi} 4\beta N_{th}(E_k)(1 - N_{th}(E_k))$$

$$\approx \frac{1}{2} \frac{|q_f B|}{4\pi} \int \frac{dq_z}{2\pi} \frac{4\beta e^{\beta\sqrt{(q_z)^2 + m^2}}}{(e^{\beta\sqrt{(q_z)^2+m^2}} + 1)^2}\,, \qquad |q_f B| \gg T^2$$