IDEAS OF CONFINEMENT FROM COULOMB GAUGE QCD

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EMMI WORKSHOP:
ASPECT OF CRITICALITY II
02-04.07.2024 WROCLAW

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QED & QCD IN COULOMB GAUGE

COULOMB GAUGE QED

$$\mathcal{H}_{\text{Coulomb}} = \bar{\psi}\left(-i\vec{\gamma}\cdot\nabla + m\right)\psi - g\bar{\psi}\vec{\gamma}\psi\cdot\vec{A}_{\perp} + \frac{1}{2}\vec{\Pi}_{\perp}^{2} + \frac{1}{2}\vec{B}^{2}$$

$$+\frac{1}{2}g^{2}\rho\,\frac{-1}{\nabla^{2}}\,\rho$$

where $\rho = \bar{\psi}\gamma^{0}\psi$

only 2 DoFs

$$\nabla\cdot\vec{A} = 0 \rightarrow \vec{A} = \vec{A}_{\perp}$$

$$(A^{0}, \vec{A})$$

*A0 is **NOT** dynamical*

Trade it w Gauss Law!

$$-\nabla^{2}A^{0} = \rho$$

=> $A^{0} = -\frac{1}{\nabla^{2}}\rho$

Potential $\frac{-1}{\nabla^{2}} \rightarrow \frac{1}{4\pi r}$

N. H. Christ and T. D. Lee
Phys. Rev. D 22, 939 (1980)

COULOMB GAUGE QCD

$$\mathcal{H} = -i\bar{\psi}\vec{\gamma}\cdot\nabla\psi + m\bar{\psi}\psi + \frac{1}{2}(\vec{E}^2 + \vec{B}^2) - g\bar{\psi}\,\vec{\gamma}\,T^a\cdot\vec{A}^a$$

$$+ \frac{1}{2}\,\rho\,[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]\,\rho$$

$$\rho^a = \bar{\psi}\gamma^0 T^a\psi + f^{\mathrm{abc}}\,A_i^b\,E_c^i$$

$$\vec{D}^{ab} = \delta^{ab}\vec{\nabla} + igT^c_{ab}\vec{A}^c$$

both quarks and gluons are color charged!

Potential $V_{ab}(x,y;\vec{A}_\perp) = \langle x,a|[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]|y,b\rangle$

[22] N. H. Christ and T. D. Lee, "Operator ordering and feynman rules in gauge theories," *Phys. Rev. D*, vol. 22, pp. 939–958, Aug 1980.

$$
\begin{aligned}
H = \int \{ & \tfrac{1}{2} \mathcal{J} \;\; \Pi_i \cdot \mathcal{J}\, \Pi_i + \tfrac{1}{2} \mathcal{B}_i \\
& + \psi_c^\dagger \gamma_4 [\gamma_i (\nabla_i - i\tfrac{1}{2} g_0 \vec{\tau} \cdot \vec{A}_i) + m] \psi_c \} \, d^3r \\
& + \tfrac{1}{2} g_0^2 \int \mathcal{J}^{-1} \sigma^{\ell}(\vec{r}) \\
& \cdot \langle \ell, \vec{r} \,|\, (\nabla_i \mathcal{D}_i)^{-1} (-\nabla^2) (\nabla_j \mathcal{D}_j)^{-1} \,|\, \ell', \vec{r}\,' \rangle \\
& \cdot \mathcal{J} \sigma^{\ell'}(\vec{r}\,') \, d^3r \, d^3r'
\end{aligned}
\tag{18.97}
$$

where, for notational clarity, we replace the coupling g in the classical Hamiltonian by g_0, which denotes the unrenormalized coupling constant in the quantum theory. Except for this change, all other notations are the same as before; e.g., we have

$$\vec{\Pi}_i^{tr} = -\vec{\mathcal{E}}_i^{tr} \;, \qquad \mathcal{D}_i = \nabla_i + g_0 \vec{A}_i \times \;,$$

$$\vec{\sigma} = \vec{A}_i \times \vec{\Pi}_i^{tr} + \psi_c^\dagger \tfrac{1}{2} \vec{\tau} \psi_c = -\vec{\Pi}_i^{tr} \times \vec{A}_i + \psi_c^\dagger \tfrac{1}{2} \vec{\tau} \psi_c = \sigma^\dagger$$

and

$$\vec{\mathcal{B}}_i = \nabla_j \vec{A}_k - \nabla_k \vec{A}_j + g_0 \vec{A}_j \times \vec{A}_k \tag{18.98}$$

with i, j, k cyclic. As in (18.48), $\mathcal{J}$ is the Jacobian. [For the non-Abelian gauge-field theory, it is often referred to in the literature as the Faddeev-Popov determinant.] The evaluation of the Jacobian is straightforward but somewhat tedious; the result * is that in (18.97) we may write

$$\mathcal{J} = \det \nabla_i \mathcal{D}_i \;, \tag{18.99}$$

where $\nabla_i \mathcal{D}_i$ stands for the matrix whose elements are

$$\langle \ell, \vec{r} \,|\, \nabla_i \mathcal{D}_i \,|\, \ell', \vec{r}\,' \rangle \;.$$

It is important to keep the operator ordering given in the above Hamiltonian. The generalization from SU_2 to SU_N of an arbitrary

* For details see Phys.Rev. D22, 939 (1980).

-abelian nature of the theory introduces several complications. Firstly, the Faddeev-Popov determinant is no longer field independent. In other gauge, this is usually handled by the introduction of auxiliary ghost fields. As we shall see, being a physical gauge, there is no need to introduce the ghost degrees of freedom in Coulomb gauge. Again, one integrates away A^0 and obtains a generalized Coulomb potential in the Lagrangian. We will also discuss the Gribov problem, which originates from the inability of the gauge condition to fix the gauge uniquely.

The Lagrangian density of QCD reads

$$\mathcal{L}_{QCD} = \bar{\psi}(i\partial\!\!\!/ - M - gA\!\!\!/)\psi - \frac{1}{4}G^2 \tag{B.59}$$

with

$$
\begin{aligned}
[T^a, T^b] &= i f^{abc} T^c \\
A &= T^a A^a \\
G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu .
\end{aligned}
\tag{B.60}
$$

The Lagrangian is invariant under the gauge transformation:

$$
\begin{aligned}
\psi &\longrightarrow e^{-ig\alpha^a T^a} \psi \\
A^a_\mu &\longrightarrow A^{a\prime}_\mu \approx A^a_\mu + \partial_\mu \alpha^a - g f^{abc} A^b_\mu \alpha^c
\end{aligned}
\tag{B.61}
$$

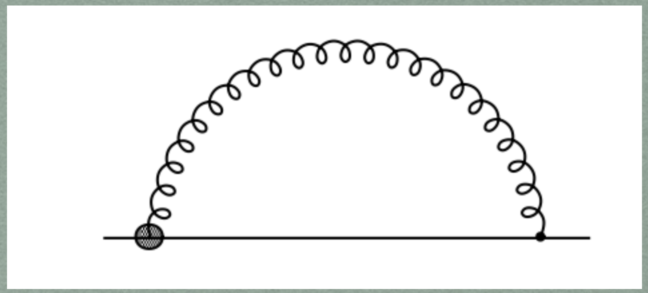
It is useful to define the operator

$$\mathcal{D}^{ac}_\mu = \partial_\mu \delta^{ac} - g f^{abc} A^b_\mu, \tag{B.62}$$

so that the gauge transformation for photon fields can be conveniently written as

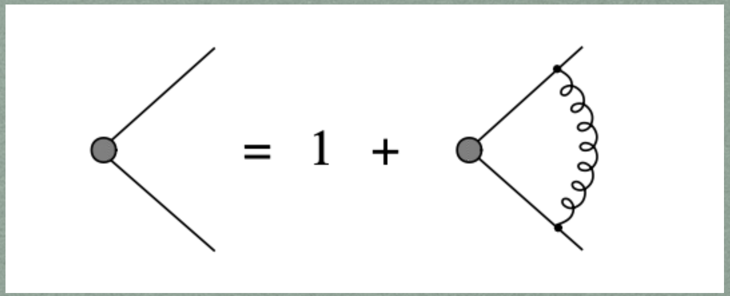
INGREDIENTS OF CONFINEMENT

2 form factors *Infrared enhancement*



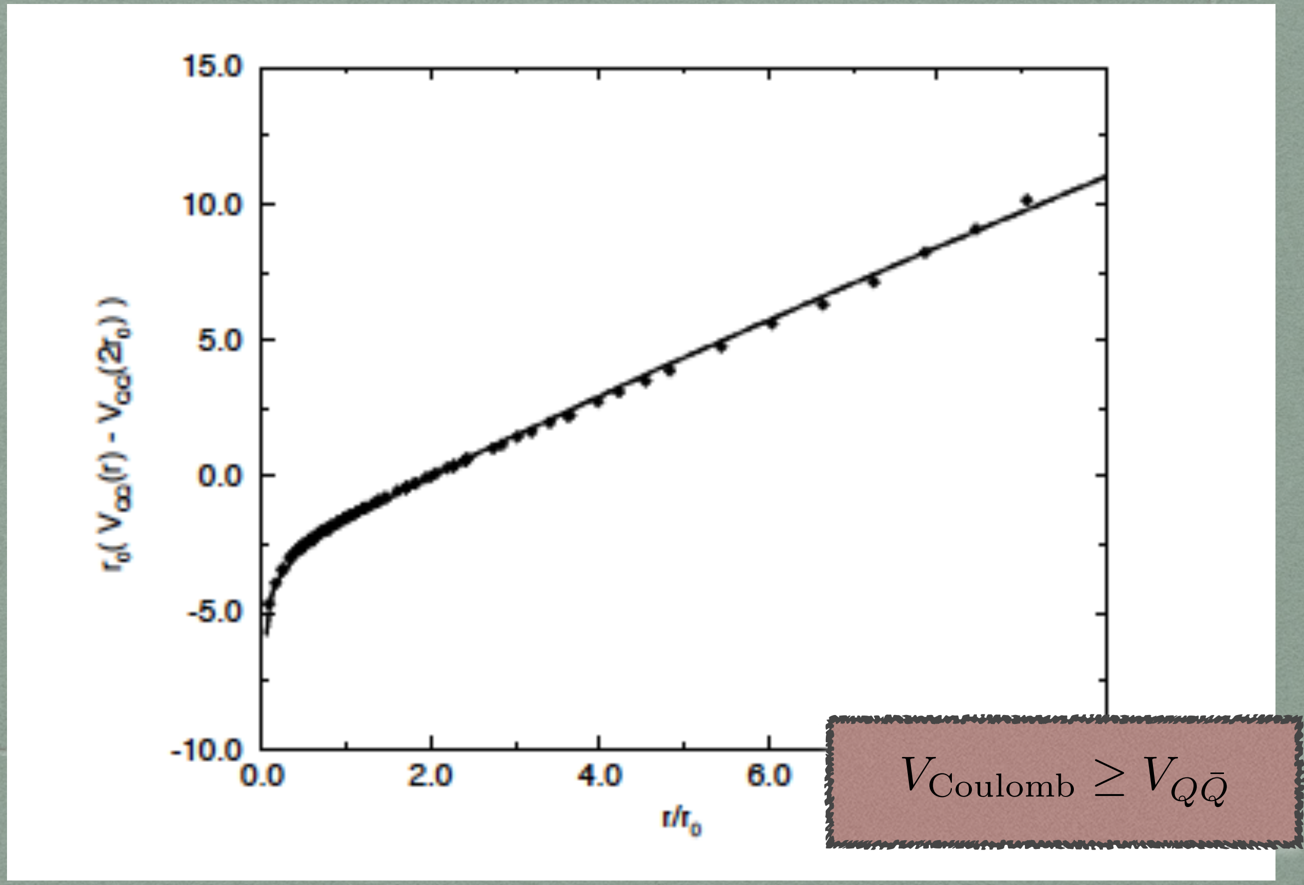
$$\langle \frac{-1}{\nabla \cdot \vec{D}[\vec{A}_\perp]} \rangle \to d \times \frac{-1}{\nabla^2}$$

$$\langle \frac{-1}{\nabla \cdot \vec{D}[\vec{A}_\perp]} \nabla^2 \frac{-1}{\nabla \cdot \vec{D}[\vec{A}_\perp]} \rangle \to f \times d^2 \times \frac{-1}{\nabla^2}$$



+ non-trivial vacuum with $\vec{A}_\perp$

Hartree-Fock Bogoliubov



A. Szczepaniak and E. Swanson
Phys. Rev. D 65, 025012 (2002)

A. Szczepaniak and E. Swanson

HEAVY QUARK FREE ENERGY

$$|\langle L \rangle|^2 = e^{-\beta F_{Q\bar{Q}}[r\to\infty, T]}$$

Kaczmarek *et. al.*

$T < T_c$

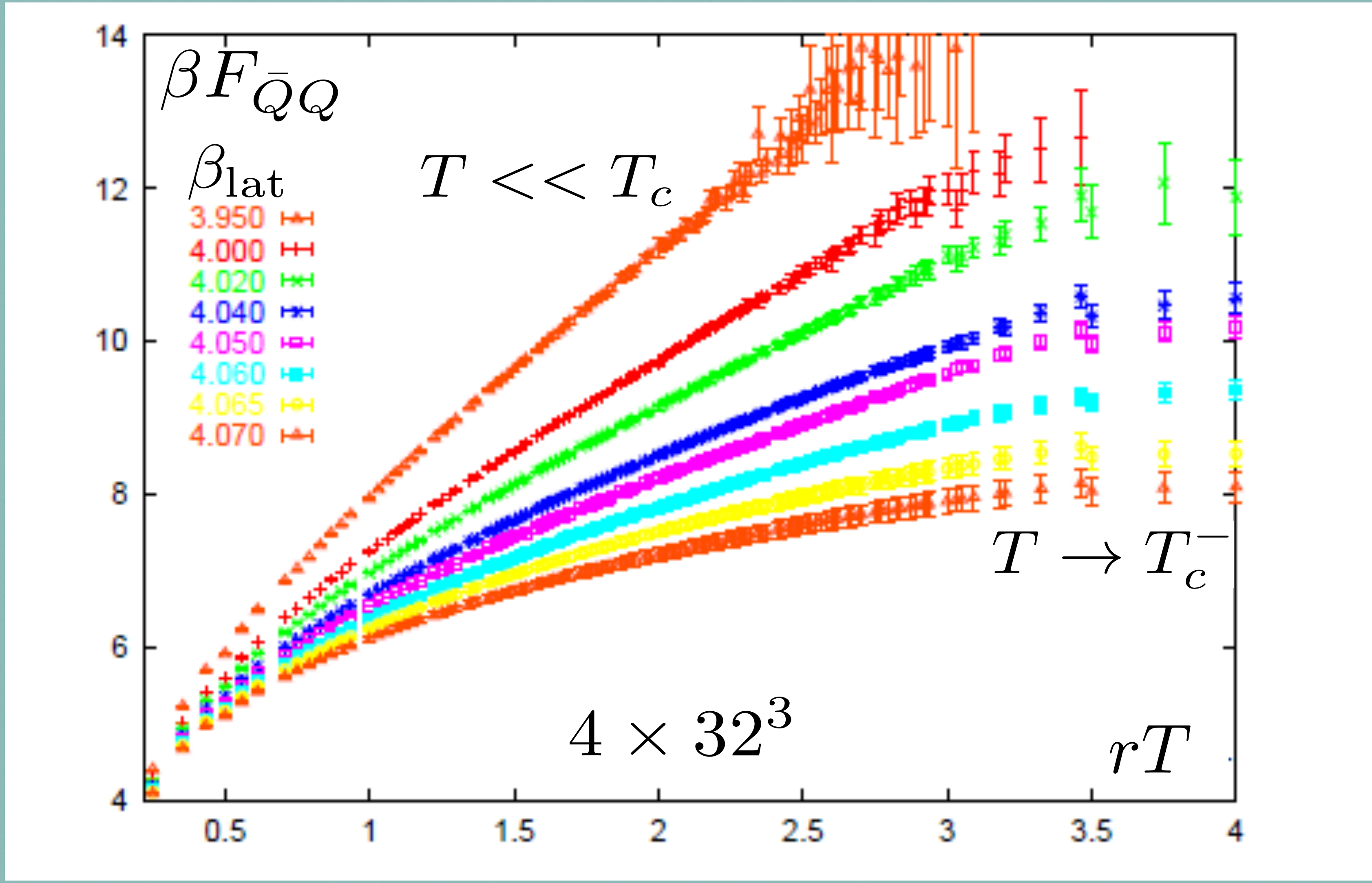
$\langle L \rangle = 0$

confined

$T > T_c$

$\langle L \rangle \neq 0$

deconfined



HEAVY QUARK FREE ENERGY

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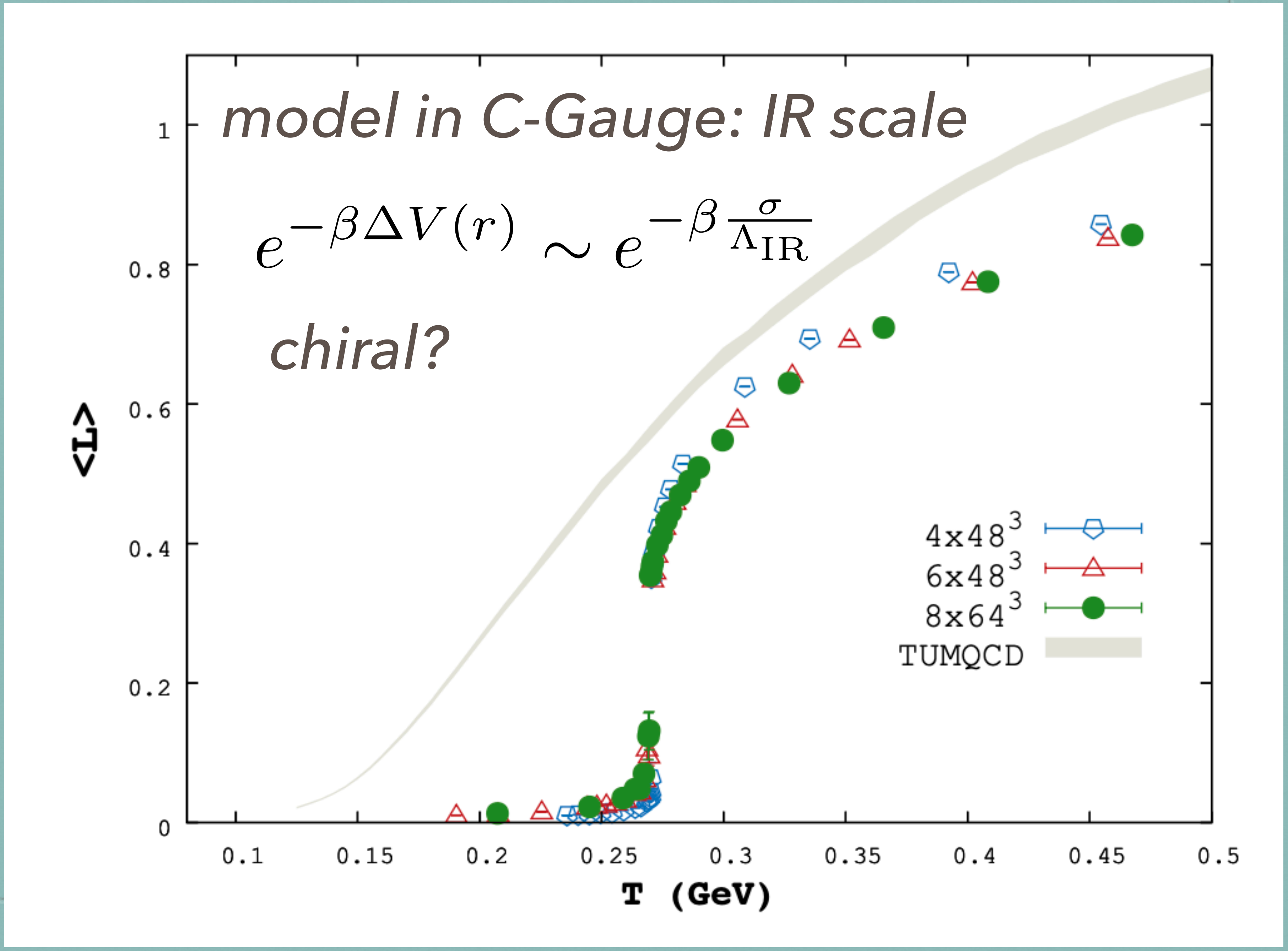


QCD IN COULOMB GAUGE

- An instantaneous potential obtained from QCD
- All degree of freedom are physical
  ghost-free!
- Confining and momentum dependent
  VS NJL

CONFINEMENT OF QUARKS

$$S^{-1}(p) = A_0(p)\, p^0\gamma^0 - A(p)\,\vec{p}\cdot\vec{\gamma} - B(p)$$

$$\Sigma(p) \approx C_F \int \frac{d^4q}{(2\pi)^4}\, V(\vec{p}-\vec{q})\, i\,\gamma^0 S(q)\gamma^0.$$

$$V_{ab}(x,y;\vec{A}_\perp) = \langle x,a|[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]|y,b\rangle$$

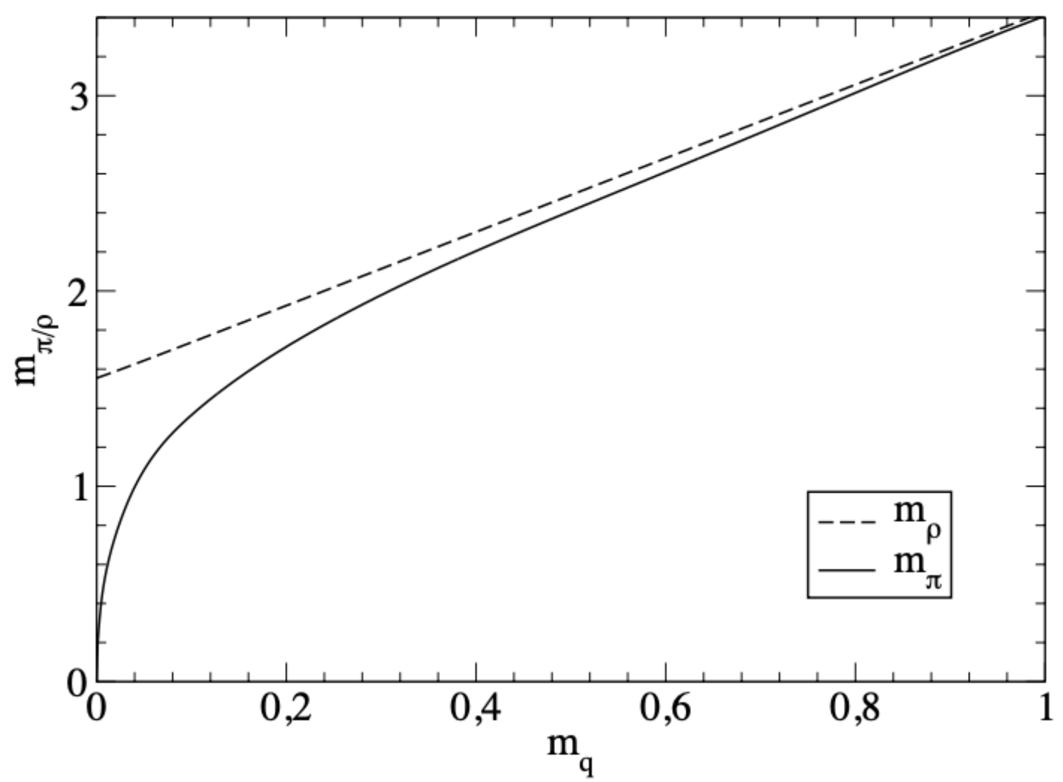
$A(p), B(p)$ *are IR div! But*

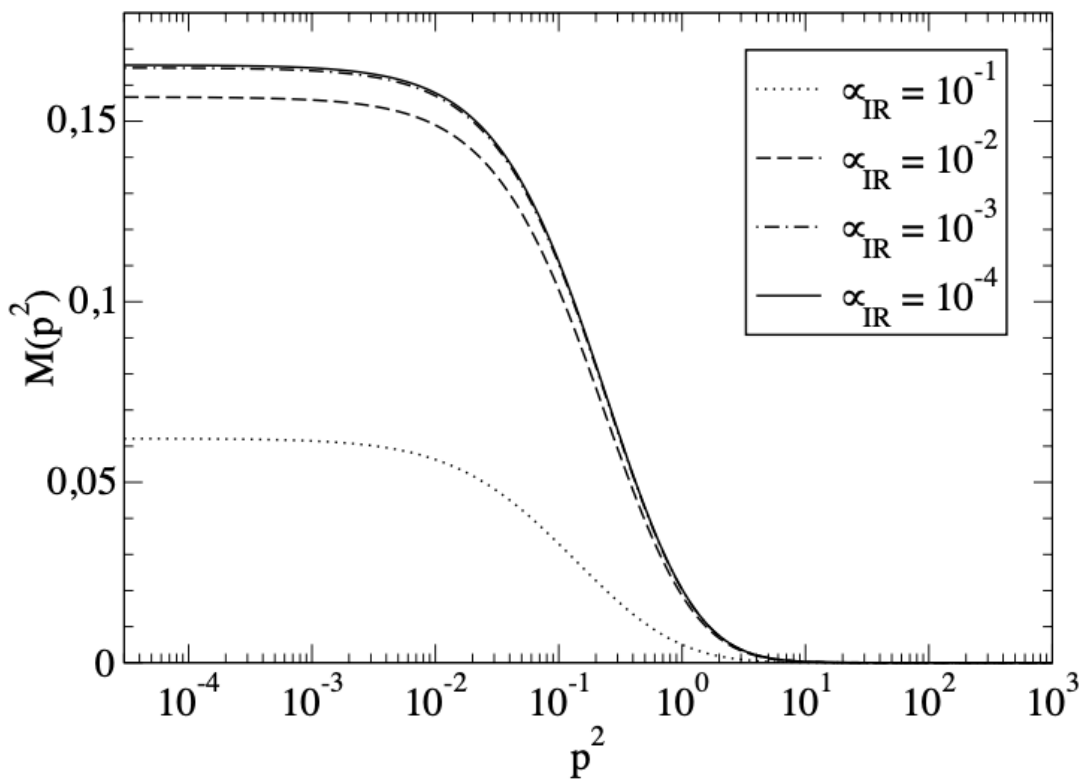
$$M(p) = \frac{B(p)}{A(p)}$$ *is finite!*

$$\langle\bar{\psi}\psi\rangle = N_c \int \frac{d^3q}{(2\pi)^3}\frac{-4\,B(q)}{2\sqrt{A(q)^2q^2+B(q)^2}}$$

VS string-flip model:
M -> infinity
(too large sigma mass)

DYNAMICAL MASS GENERATION



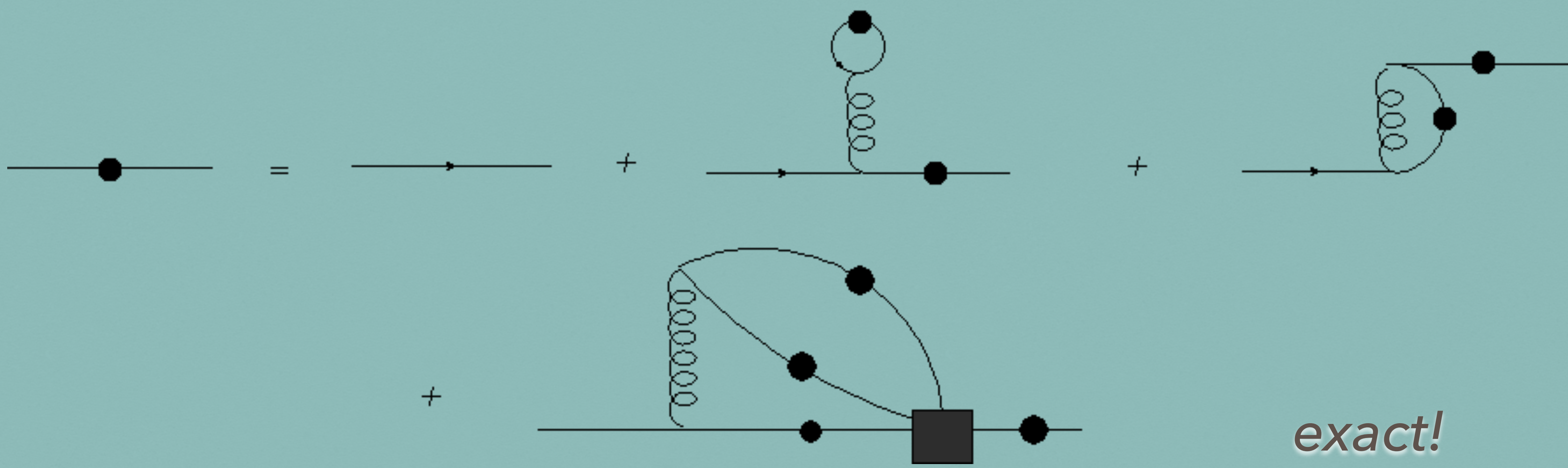


Alkofer et. al.

Phys. Rev. Lett. 96, 022001

DYNAMICAL MASS GENERATION

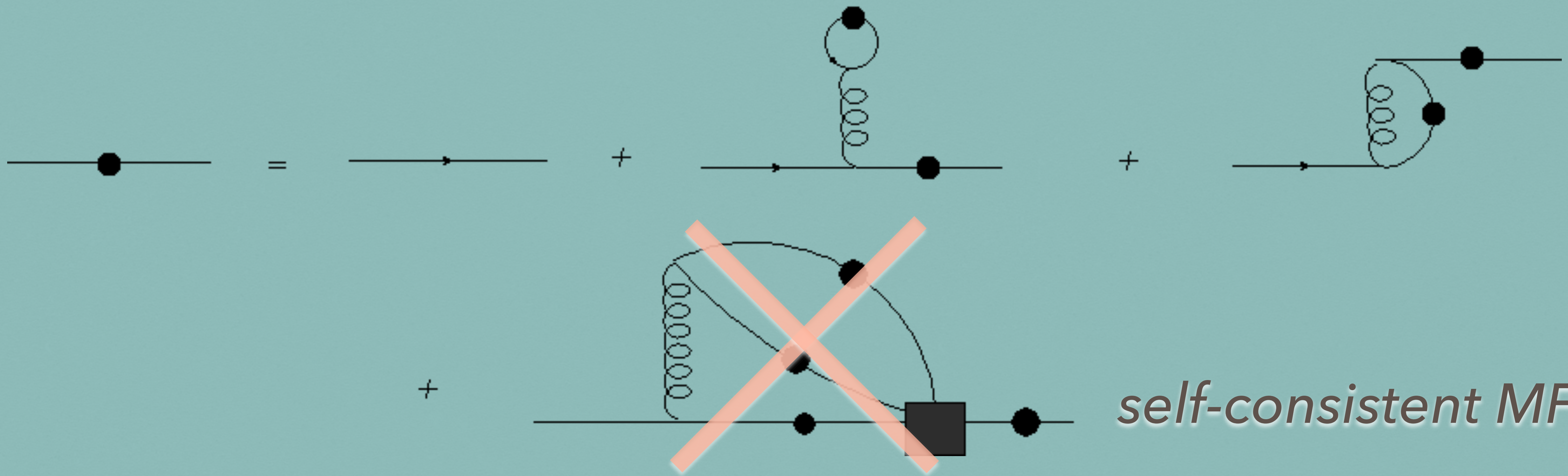
Dyson-Schwinger Equations



exact!

DYNAMICAL MASS GENERATION

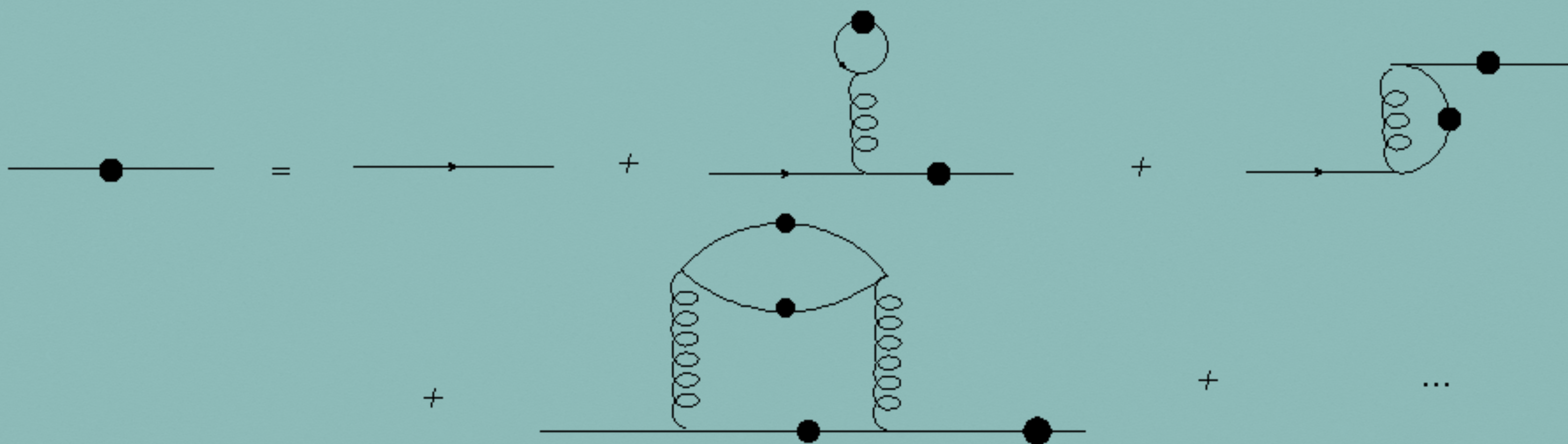
Dyson-Schwinger Equations



self-consistent MF

DYNAMICAL MASS GENERATION

Dyson-Schwinger Equations



RPA approx

P. M. Lo and E. Swanson
Phys. Rev. D 81, 034030 (2010)

Quark SDE

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{A(q)\, \hat{p} \cdot \hat{q}}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{B(q)}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$\tilde{E}(p) = \sqrt{A(p)^2\, p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q) - \mu'(q))} + 1}.$$

Quark SDE

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{1}{2}\left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

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$$\tilde{E}(p) = \sqrt{A(p)^2\,p^2 + B(p)^2}$$

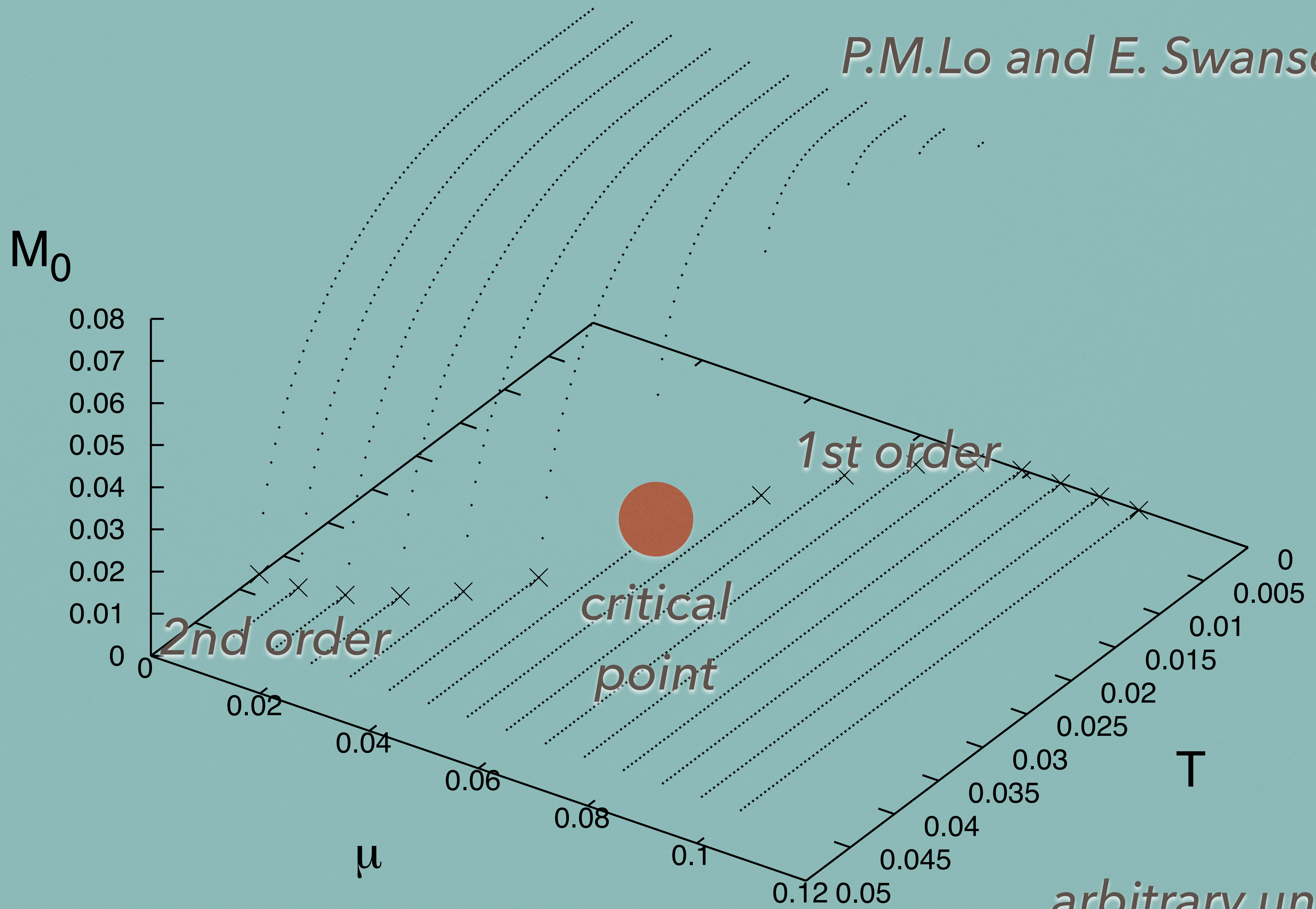
$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q)-\mu'(q))} + 1}.$$

Alkofer et al. A->1 in thermal

Conf. via:
A -> Infinity
thermal weights -> 0;
non-sense!

CHIRAL PHASE STRUCTURE

P.M.Lo and E. Swanson



arbitrary units

Quark SDE

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{A(q)\, \hat{p} \cdot \hat{q}}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{B(q)}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

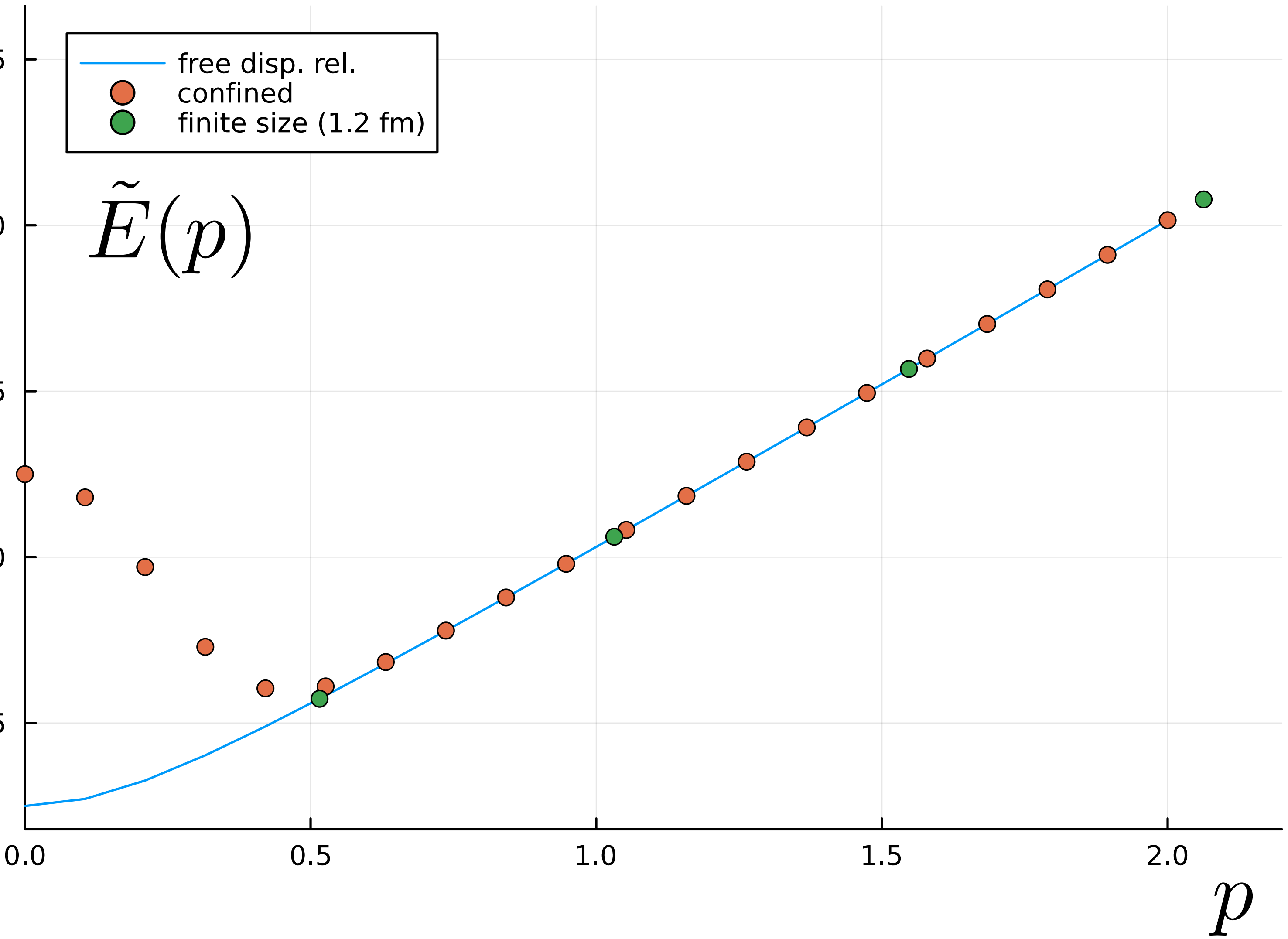
$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q) - \mu'(q))} + 1}.$$

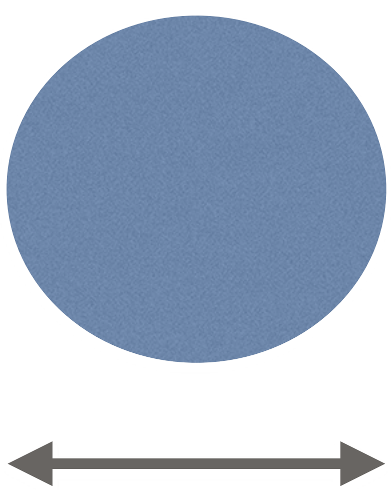
Conf. via:
A -> Infinity
thermal weights -> 0;
non-sense??
Quark Suppression

A-conf

$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$



Bag Model



as a finite size effect

$$k \to k_n = \frac{n\pi}{L}$$

$$\Lambda_{\mathrm{IR}}$$

proton wavefunction

G. Krein EPJA 18 (2003)

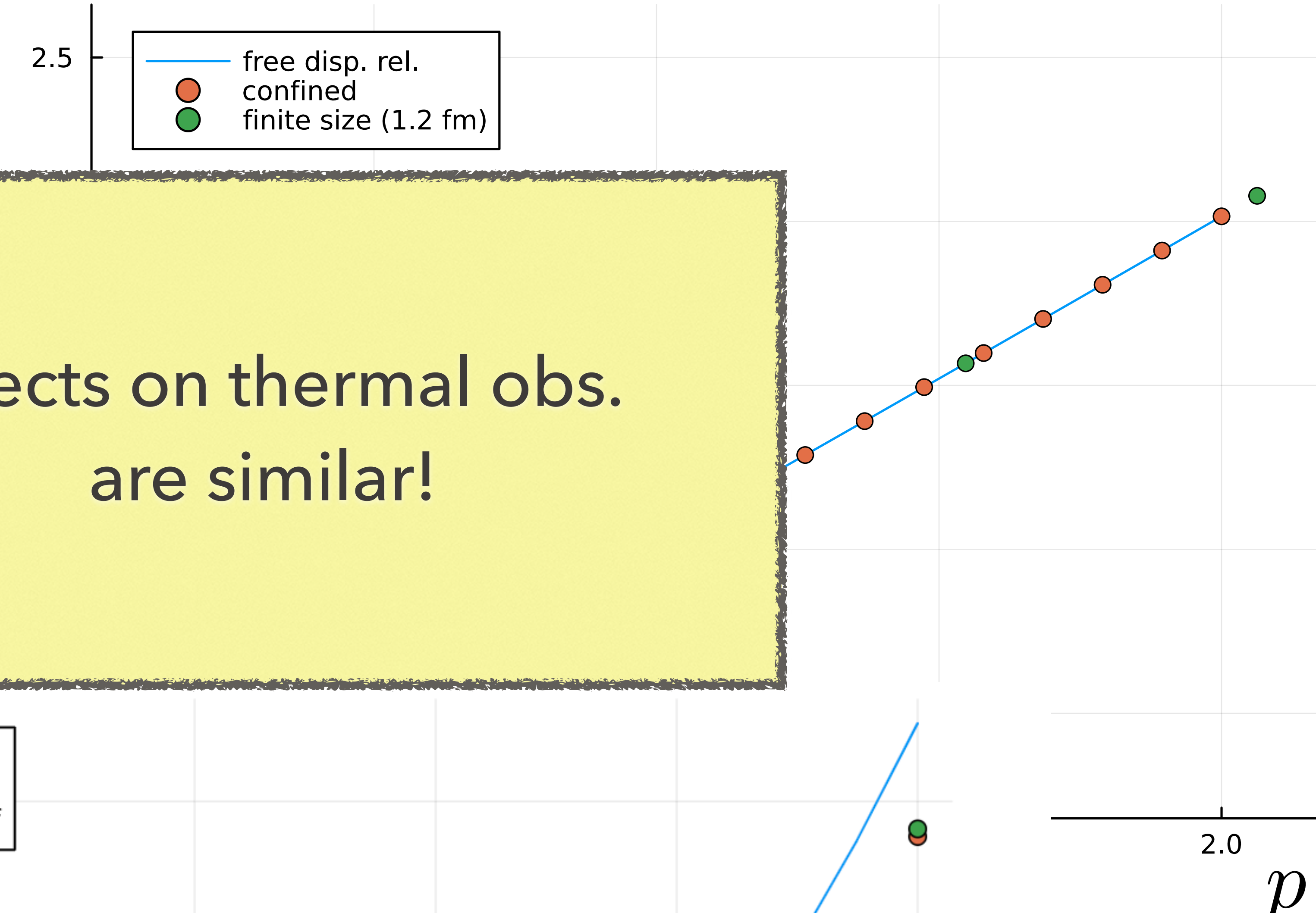
Bicudo et. al. PRD 45 5 (1992)

also recently

K. Fukushima, T. Kojo, W. Weise
Phys. Rev. D 102, 096017 (2020)

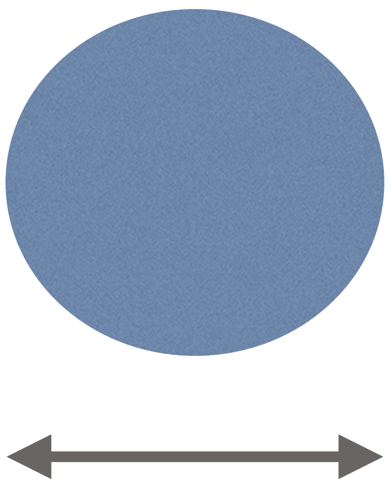
A-conf

$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 +$$

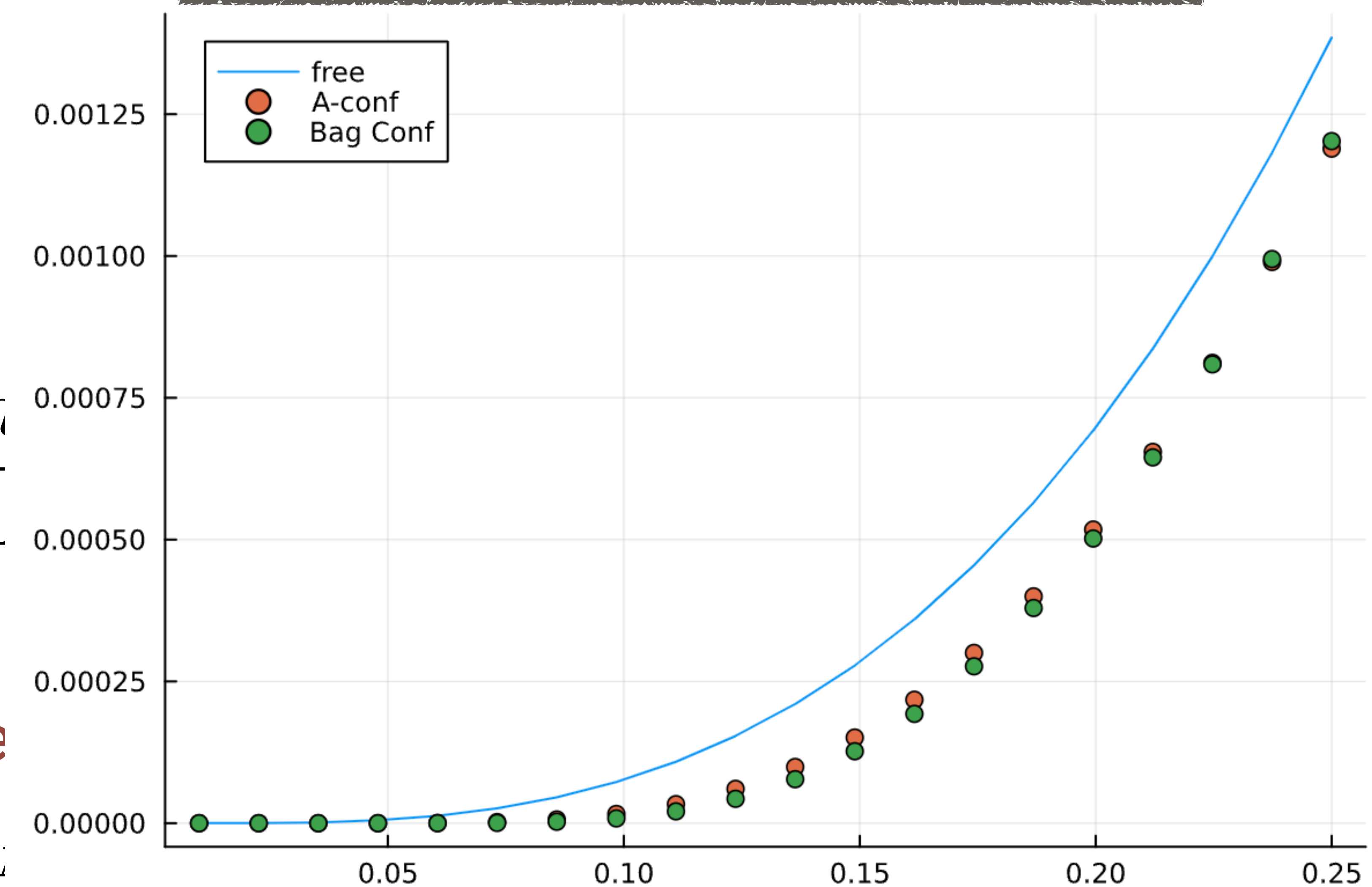


effects on thermal obs.
are similar!

Bag Model



$$k \to k_n = \frac{n}{-}$$



proton wave ...ntly

G. Krein EPJ... , W. Weise

Bicudo et. al. PRD 45 5 (1992)

Phys. Rev. D 102, 096017 (2020)

SOME APPLICATIONS

ASYMPTOTIC FREEDOM

Mean fields

$$\mu' = \mu - 2G_V \int \frac{d^3q}{(2\pi)^3}\;(n_F - \bar{n}_F)$$

$$\propto {\mu'}^3$$

$$\mu' \propto \mu^{\frac{1}{3}} \longrightarrow c_S^2 \to 1$$

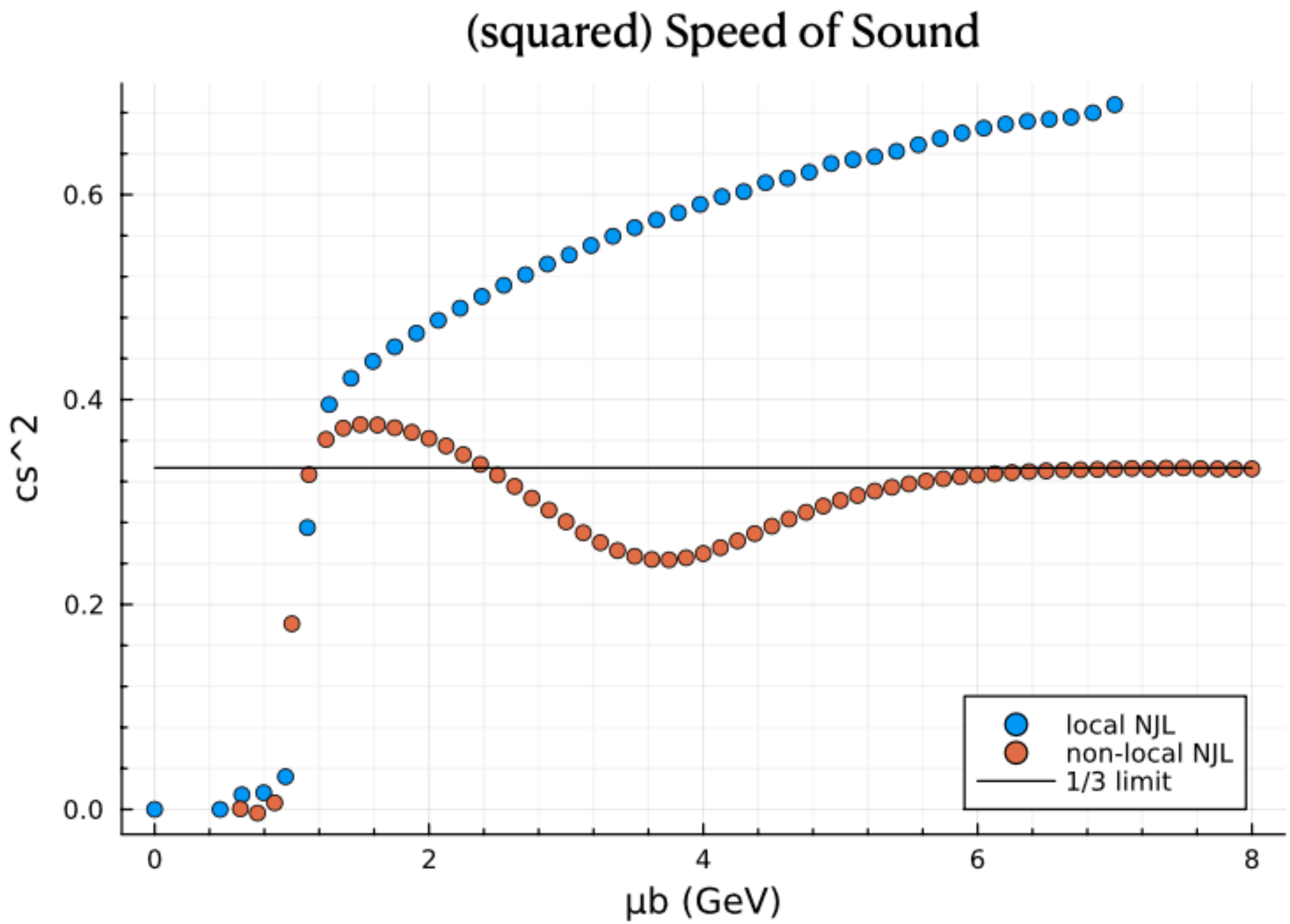
VS

Dynamical model

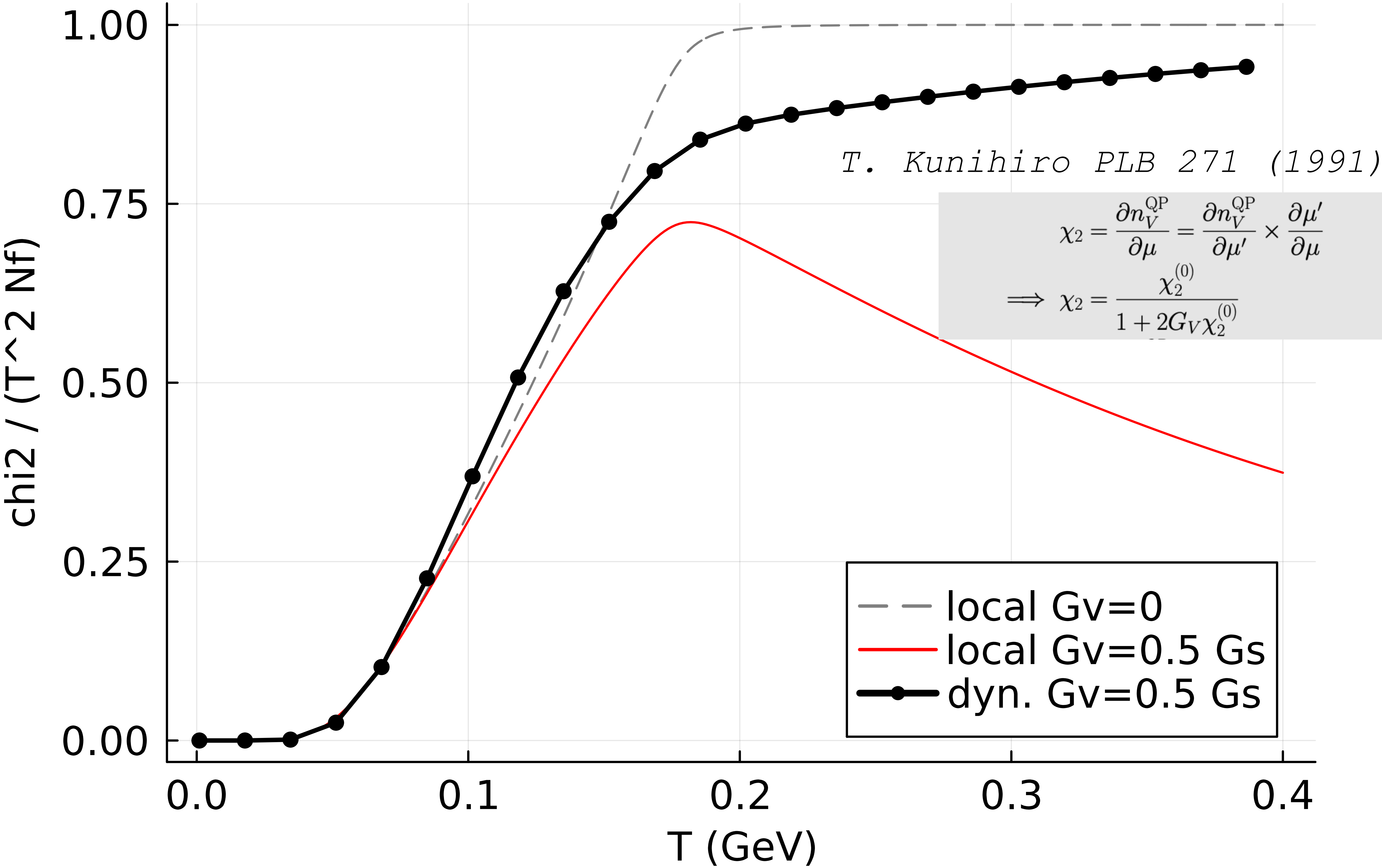
$$\mu'(p) = \mu + \int \frac{d^3q}{(2\pi)^3}\,\frac{1}{2}\,V(\vec{p} - \vec{q})\;(n_F - \bar{n}_F)$$

If V->0 as p-> Inf: $\mu' \to \mu \longrightarrow c_s^2 \to \frac{1}{3}$

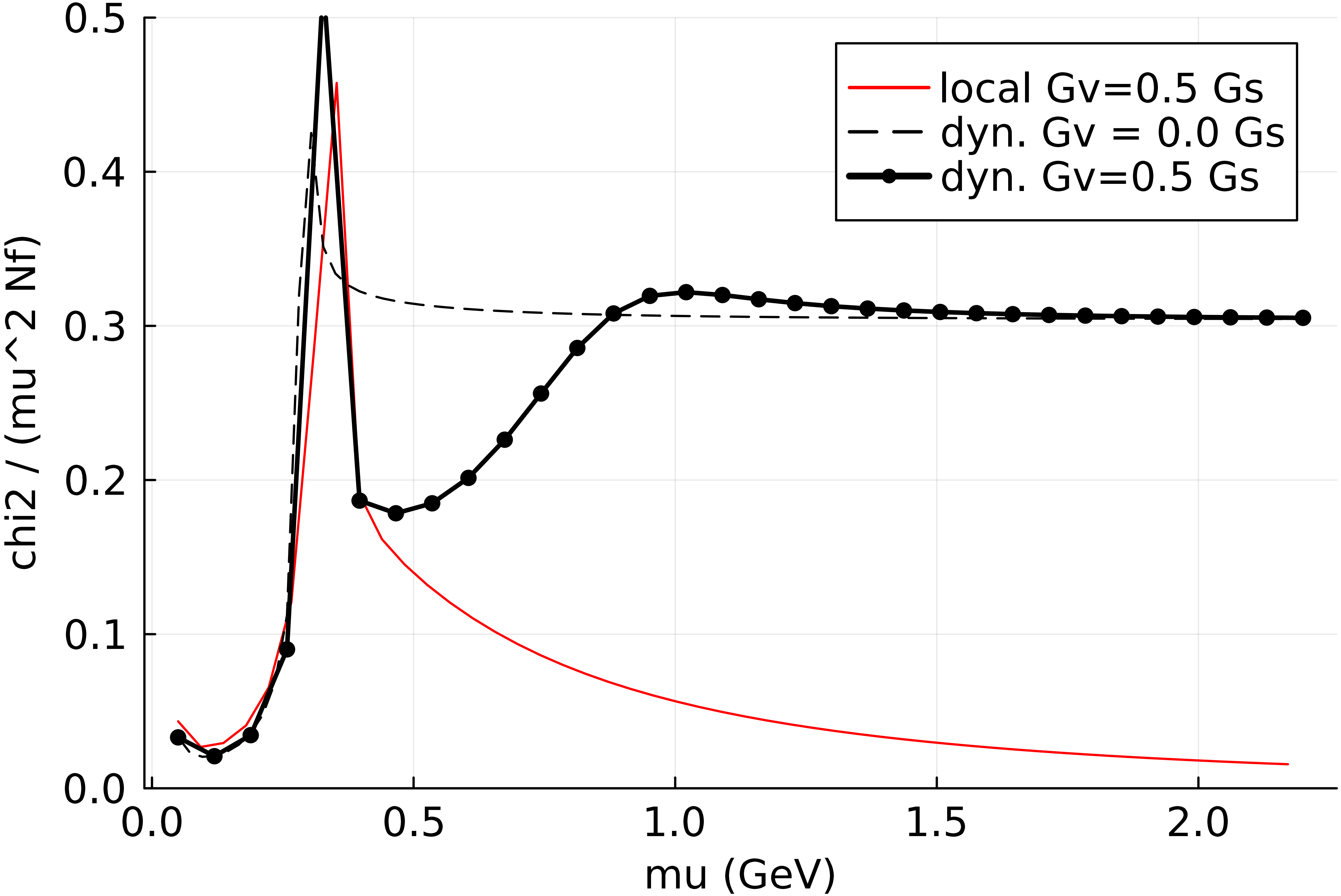
e.g. $V(p,q) \approx G_0\, e^{-p^2} e^{-q^2}$ *separable model*



(squared) Speed of Sound
cs^2
μb (GeV)
0.0
0.2
0.4
0.6
0
2
4
6
8
local NJL
non-local NJL
1/3 limit



T. Kunihiro PLB 271 (1991)
$$\chi_2 = \frac{\partial n_V^{\mathrm{QP}}}{\partial \mu} = \frac{\partial n_V^{\mathrm{QP}}}{\partial \mu'} \times \frac{\partial \mu'}{\partial \mu}$$
$$\Longrightarrow \chi_2 = \frac{\chi_2^{(0)}}{1 + 2G_V \chi_2^{(0)}}$$
local Gv=0
local Gv=0.5 Gs
dyn. Gv=0.5 Gs
chi2 / (T^2 Nf)
T (GeV)
1.00
0.75
0.50
0.25
0.00
0.0
0.1
0.2
0.3
0.4

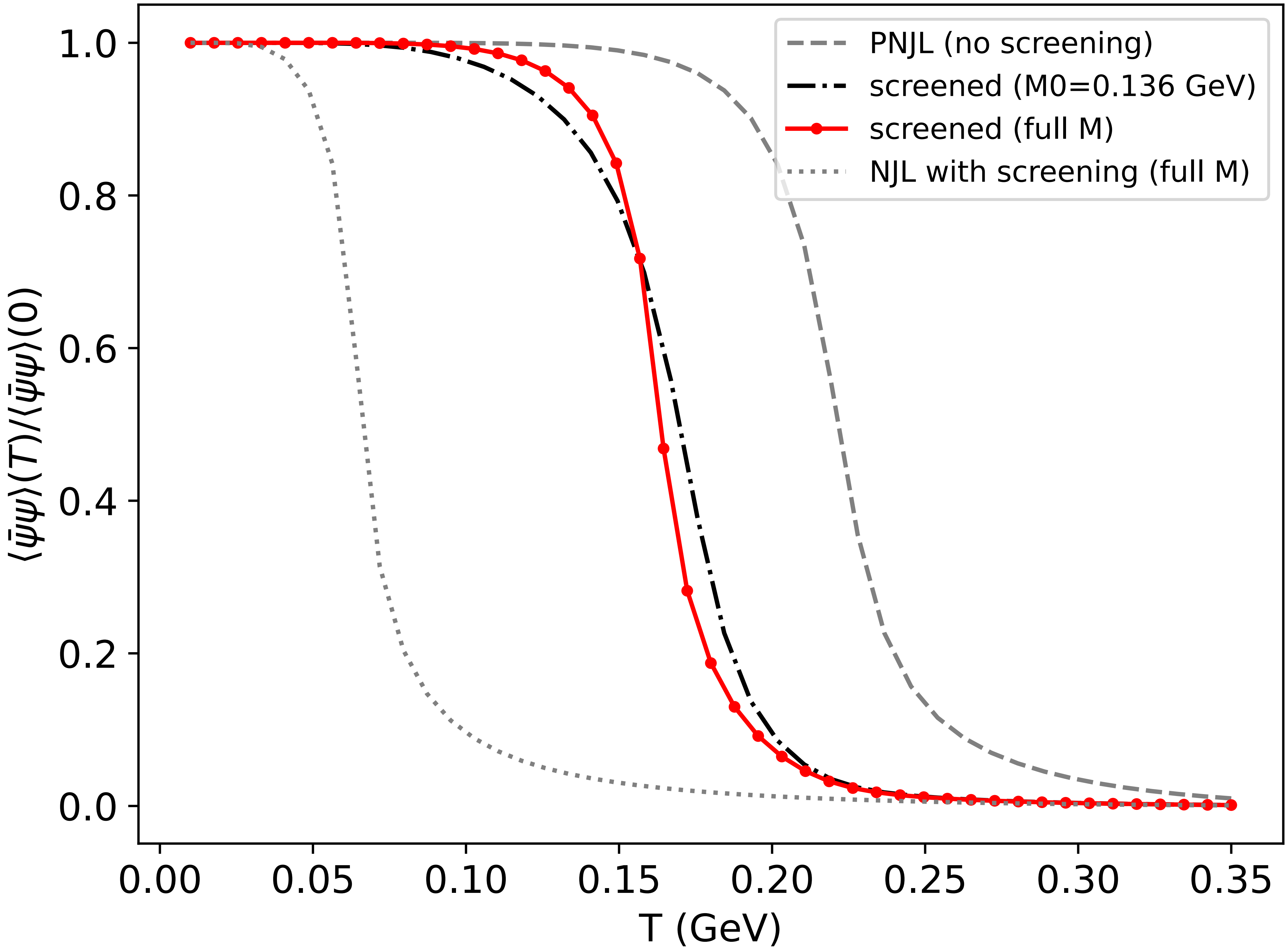


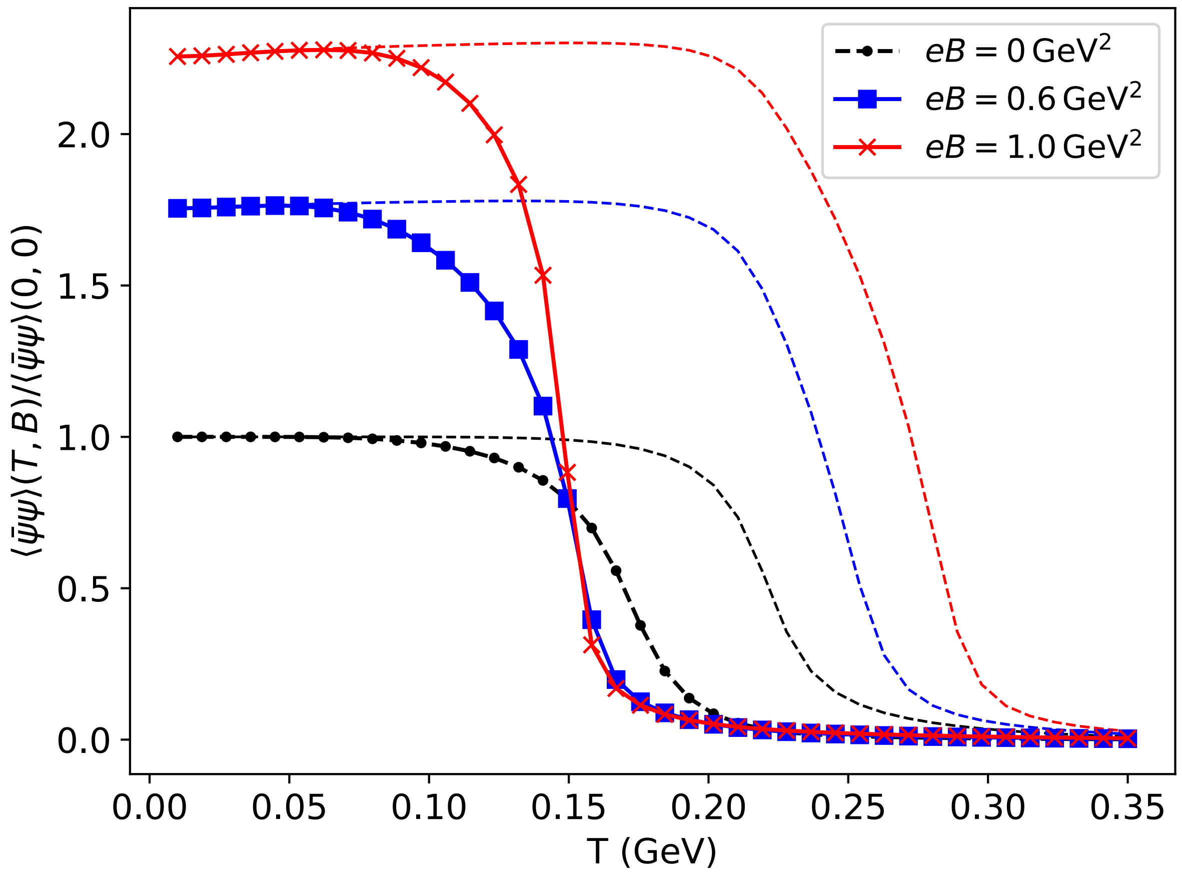
local Gv=0.5 Gs
dyn. Gv = 0.0 Gs
dyn. Gv=0.5 Gs
chi2 / (mu^2 Nf)
mu (GeV)
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.5
1.0
1.5
2.0

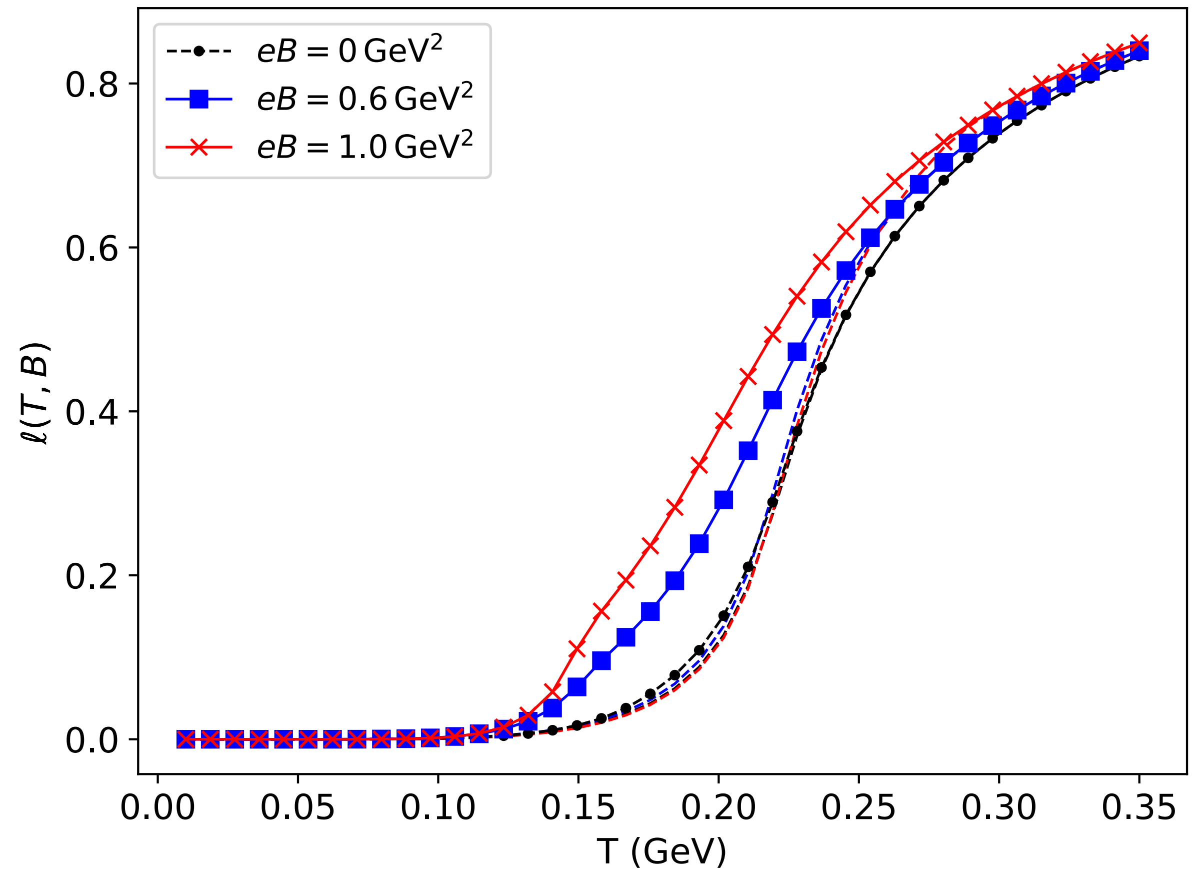
IR STRUCTURE AND CHIRAL SYMMETRY BREAKING

FIG. 3. Gap Equations: summing polarization insertions in the truncated Schwinger-Dyson equations.

$$\begin{aligned} A(\vec{p}) &= 1 + \frac{C_F}{2}\int \frac{d^3q}{(2\pi)^3} V_{\rm ring}(\vec{p}-\vec{q})\frac{A_q}{E_q}\frac{\vec{p}\cdot\vec{q}}{p^2}\Theta(q) \\ B(\vec{p}) &= m + \frac{C_F}{2}\int \frac{d^3q}{(2\pi)^3} V_{\rm ring}(\vec{p}-\vec{q})\frac{B_q}{E_q}\Theta(q) \\ \tilde{\mu}(\vec{p}) &= \mu + \frac{C_F}{2}\int \frac{d^3q}{(2\pi)^3} V_{\rm ring}(\vec{p}-\vec{q})[n(q)-\bar{n}(q)] \\ E_p^2 &= A_p^2 p^2 + B_p^2. \end{aligned} \tag{13}$$


PNJL (no screening)
screened (M0=0.136 GeV)
screened (full M)
NJL with screening (full M)
$\langle\bar{\psi}\psi\rangle(T)/\langle\bar{\psi}\psi\rangle(0)$
T (GeV)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35



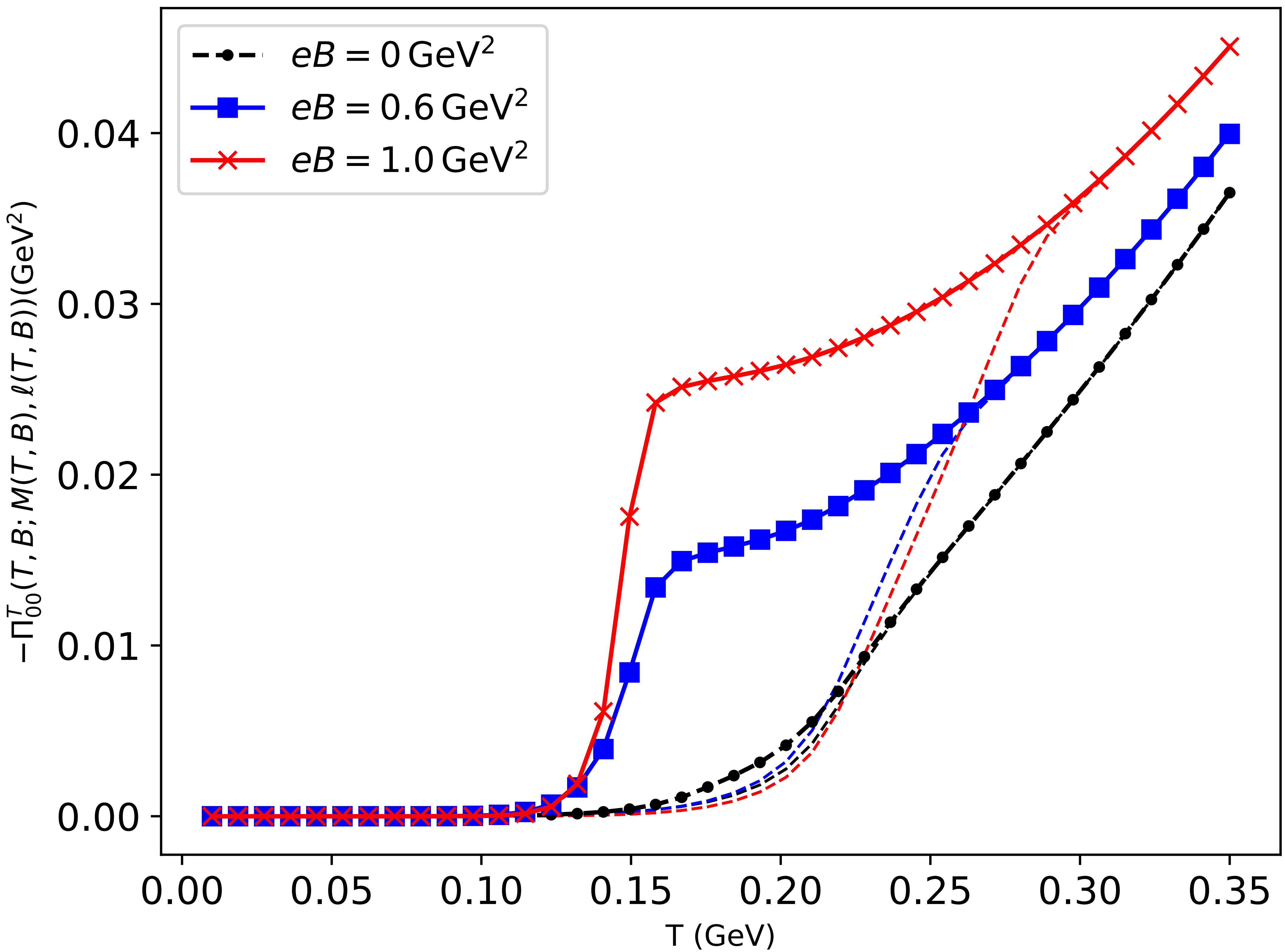


$$M = m + C_F\,\tilde{V_0}\int \frac{d^3q}{(2\pi)^3}\,\frac{M}{2E} \times (1 - 2\,N_{\rm th}(E))$$

$$\tilde{V_0}^{-1} = V_0{}^{-1} - \frac{1}{2}\,N_f\,\Pi_{00}$$

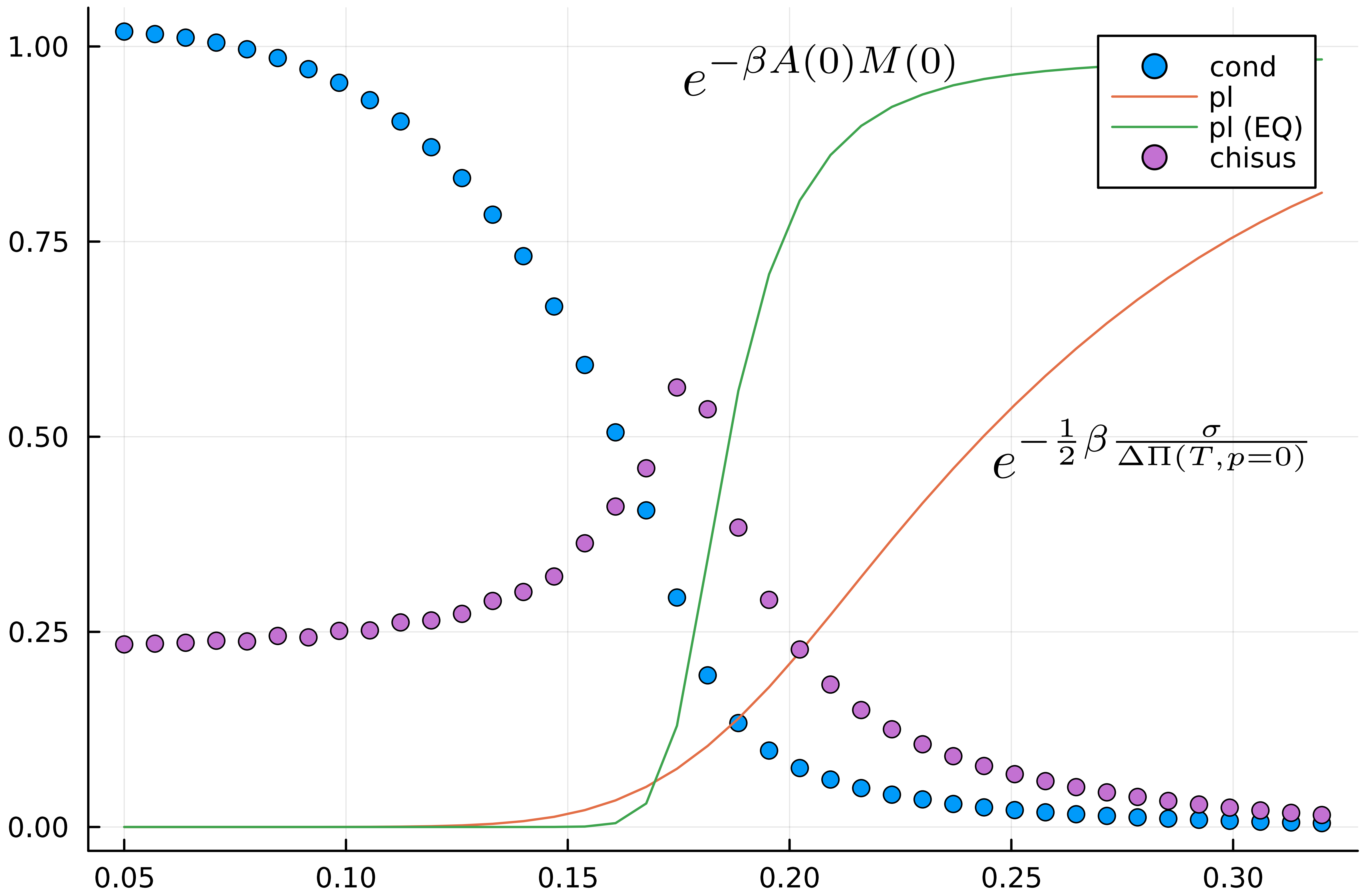
$$\begin{aligned}\hat{m}_{el}^2 &= -\frac{1}{2}\,N_f\,\Pi_{00}(p^0 = 0, \vec{p} \to \vec{0}) \\ &= \frac{1}{2}\,N_f \times \int \frac{d^3q}{(2\pi)^3}\,4\beta N_{\rm th}(1 - N_{\rm th}).\end{aligned}$$

$$\begin{aligned}N_{\rm th}(E,\ell) &\to \frac{1}{3}\sum_{j=1}^{3}\frac{\hat{\ell}_F^{(j)}}{e^{\beta E} + \hat{\ell}_F^{(j)}} \\ &= \frac{1}{3}\,\frac{3\ell\,e^{-\beta E} + 6\ell\,e^{-2\beta E} + 3e^{-3\beta E}}{1 + 3\ell\,e^{-\beta E} + 3\ell\,e^{-2\beta E} + e^{-3\beta E}},\end{aligned}$$



$eB = 0\,\mathrm{GeV}^2$
$eB = 0.6\,\mathrm{GeV}^2$
$eB = 1.0\,\mathrm{GeV}^2$
0.00
0.01
0.02
0.03
0.04
$-\Pi^T_{00}(T, B; M(T, B), \ell(T, B))(\mathrm{GeV}^2)$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
T (GeV)

new model *illustration*



TO DO LIST

- Gluons Should Not be Neglected! (x3)
- Explore Dense Matter EoS
- Deriving pions
- Vs Quark Meson Model (on-going!)
- Understand quark hadron duality & look more seriously to the deconfinement transition.

THANK YOU

POLYAKOV LOOP

CENTER SYMMETRY AND VACUUM STRUCTURE

- Order parameter **characterizes** the state of the system

$$\langle L \rangle$$

*Operator does **not** respect the symmetry*

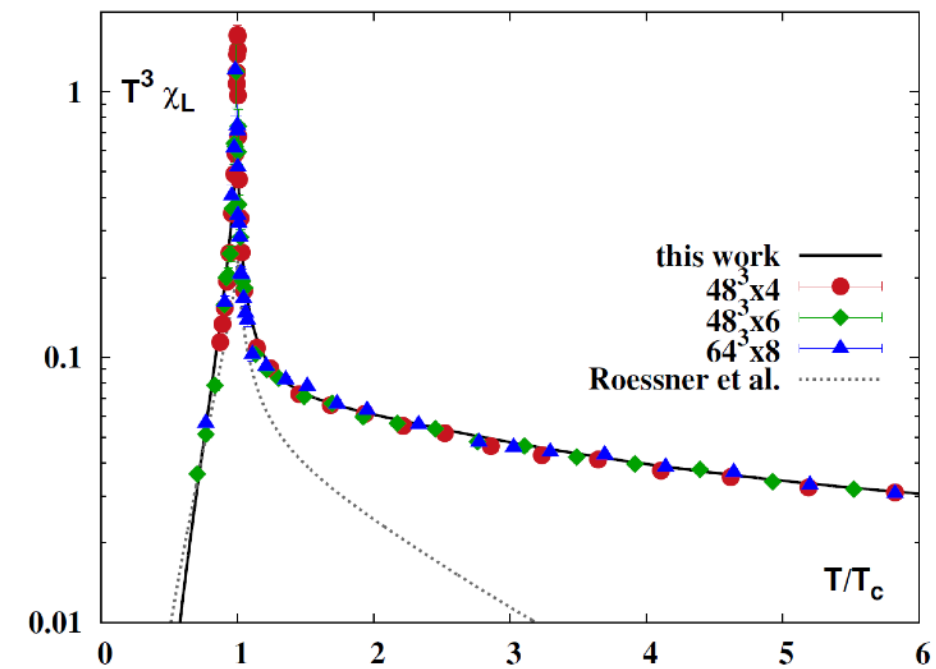
$$L \to L\, e^{2ni\pi/3}$$

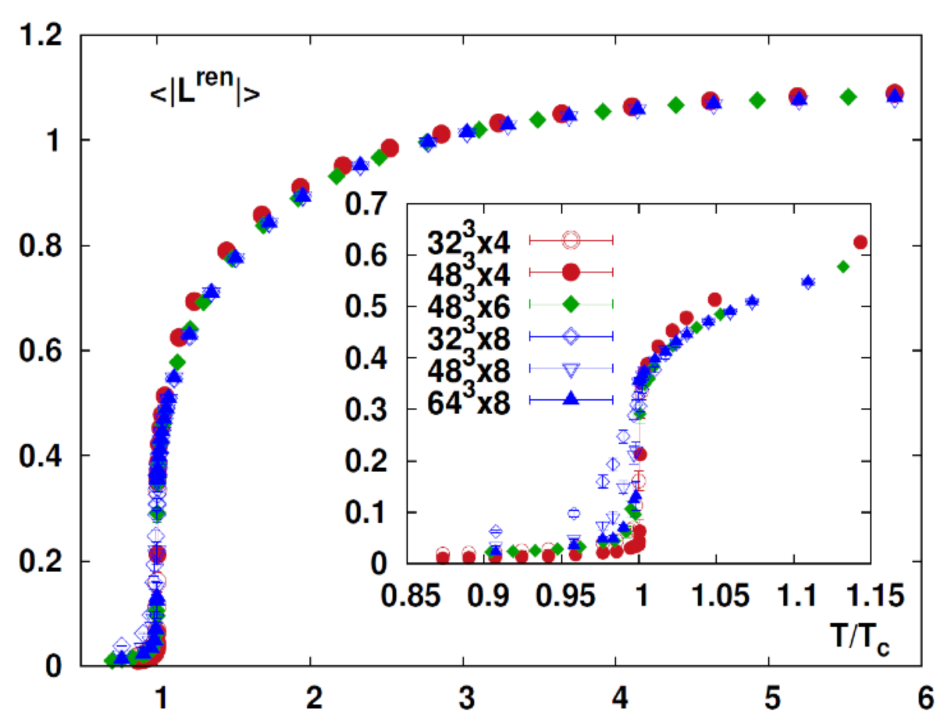
confined phase: $\langle L \rangle = 0$

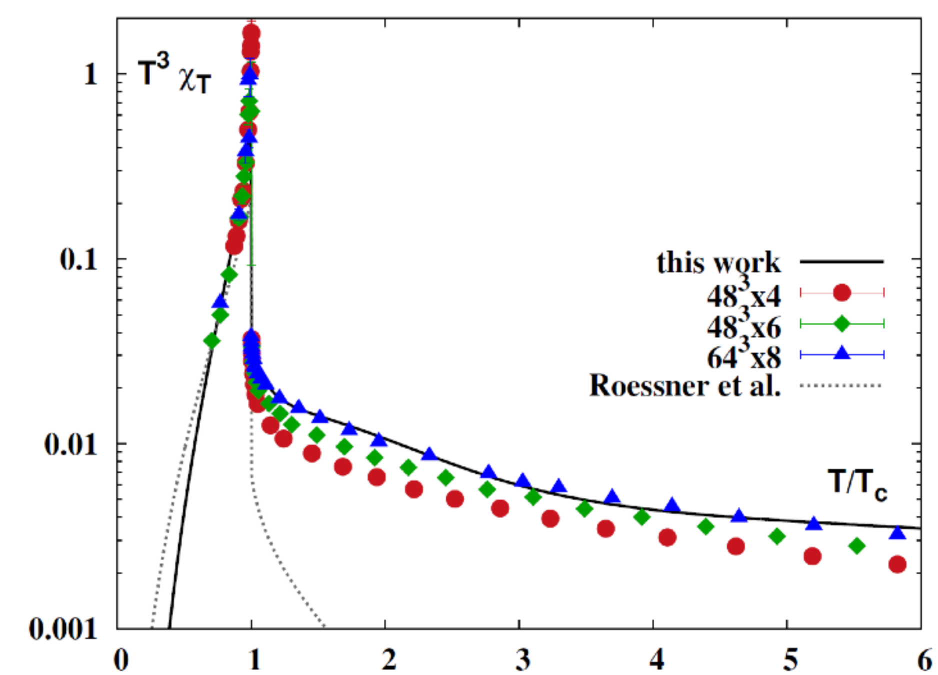
deconfined phase: $\langle L \rangle \neq 0$

order parameter and fluctuations

$$L = L_L + iL_T \longrightarrow \begin{aligned} \chi_L &= V(\langle L_L L_L \rangle - \langle L_L \rangle^2) \\ \chi_T &= V(\langle L_T L_T \rangle - \langle L_T \rangle^2) \end{aligned}$$







2 DoFs

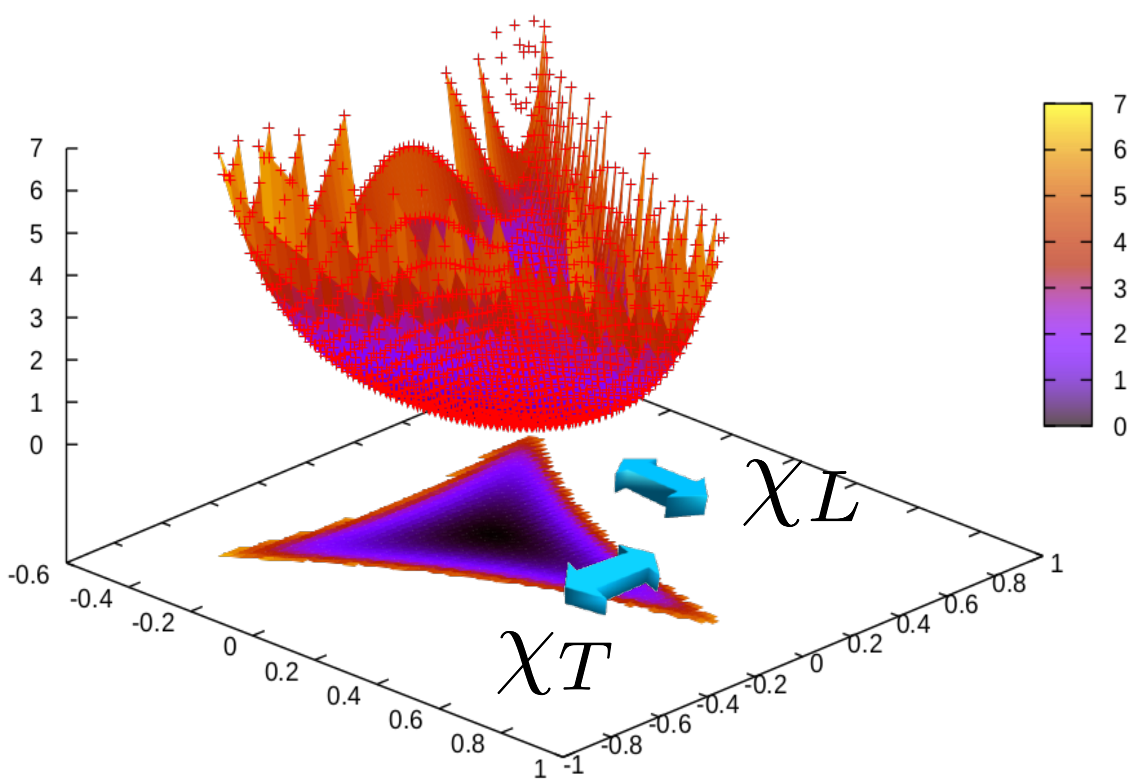
or w 8 adjoint angles depend on

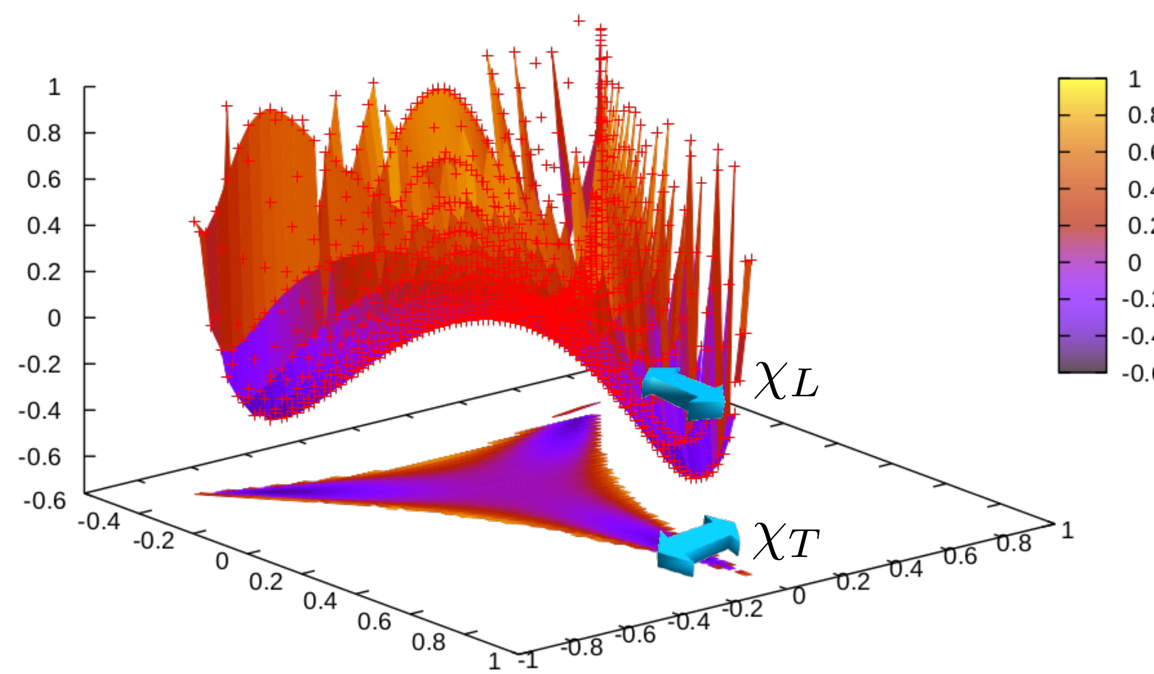
$$\{\gamma_3, \gamma_8\}$$

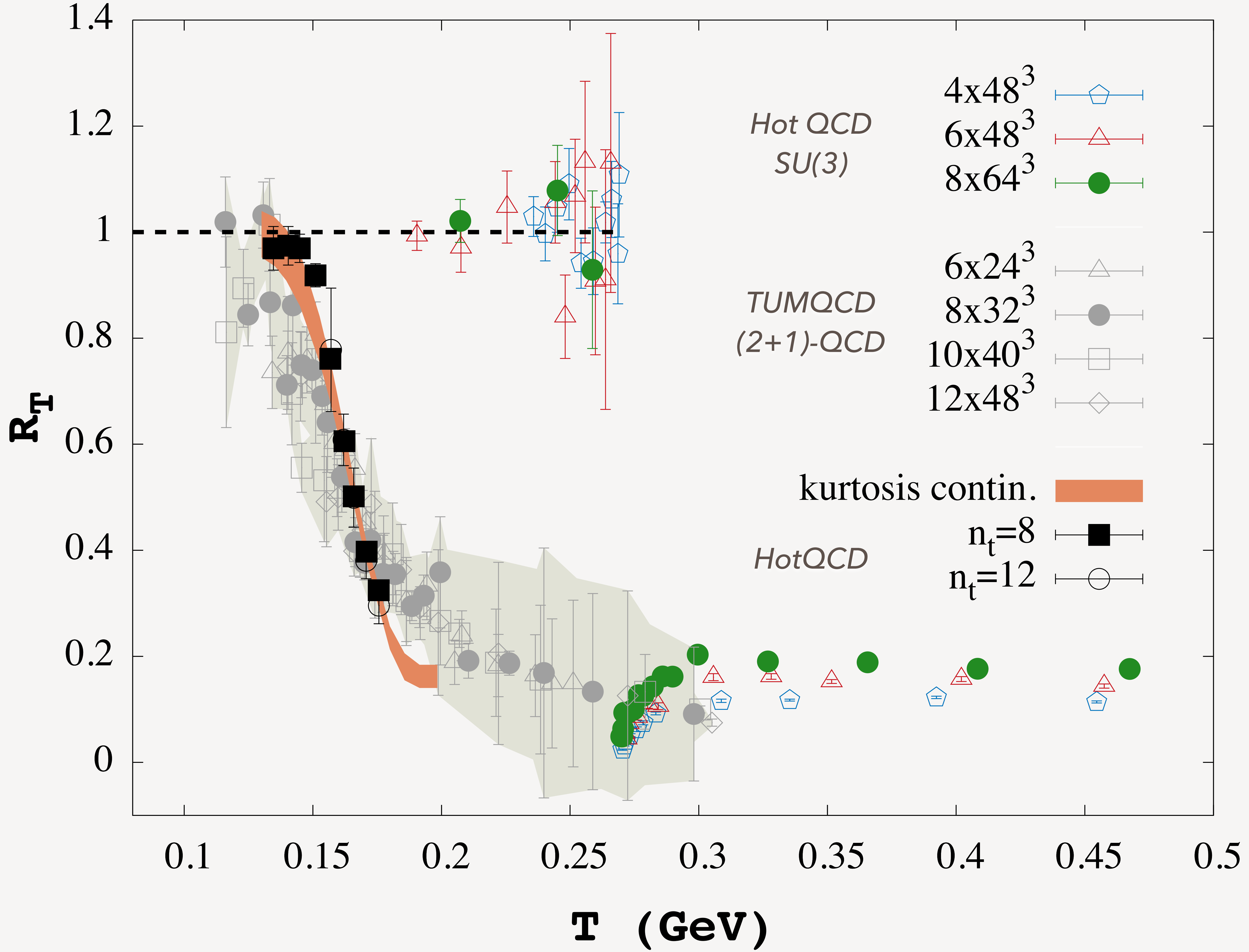
$$\ell = X + i\,Y$$

$$X = \frac{1}{3}\left(\cos q_1 + \cos q_2 + \cos(q_1 + q_2)\right) \qquad \sim 1 - \mathcal{O}(q_1^2, q_2^2, q_1 q_2)$$

$$Y = \frac{1}{3}\left(\sin q_1 + \sin q_2 - \sin(q_1 + q_2)\right). \qquad \sim \mathcal{O}(q_1 q_2^2, q_1^2 q_2)$$







1.4
1.2
1
0.8
0.6
0.4
0.2
0
R_T
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
T (GeV)
Hot QCD
SU(3)
$4x48^3$
$6x48^3$
$8x64^3$
TUMQCD
(2+1)-QCD
$6x24^3$
$8x32^3$
$10x40^3$
$12x48^3$
kurtosis contin.
HotQCD
n_t=8
n_t=12