QCD Phase Diagram and Astrophysical Implications

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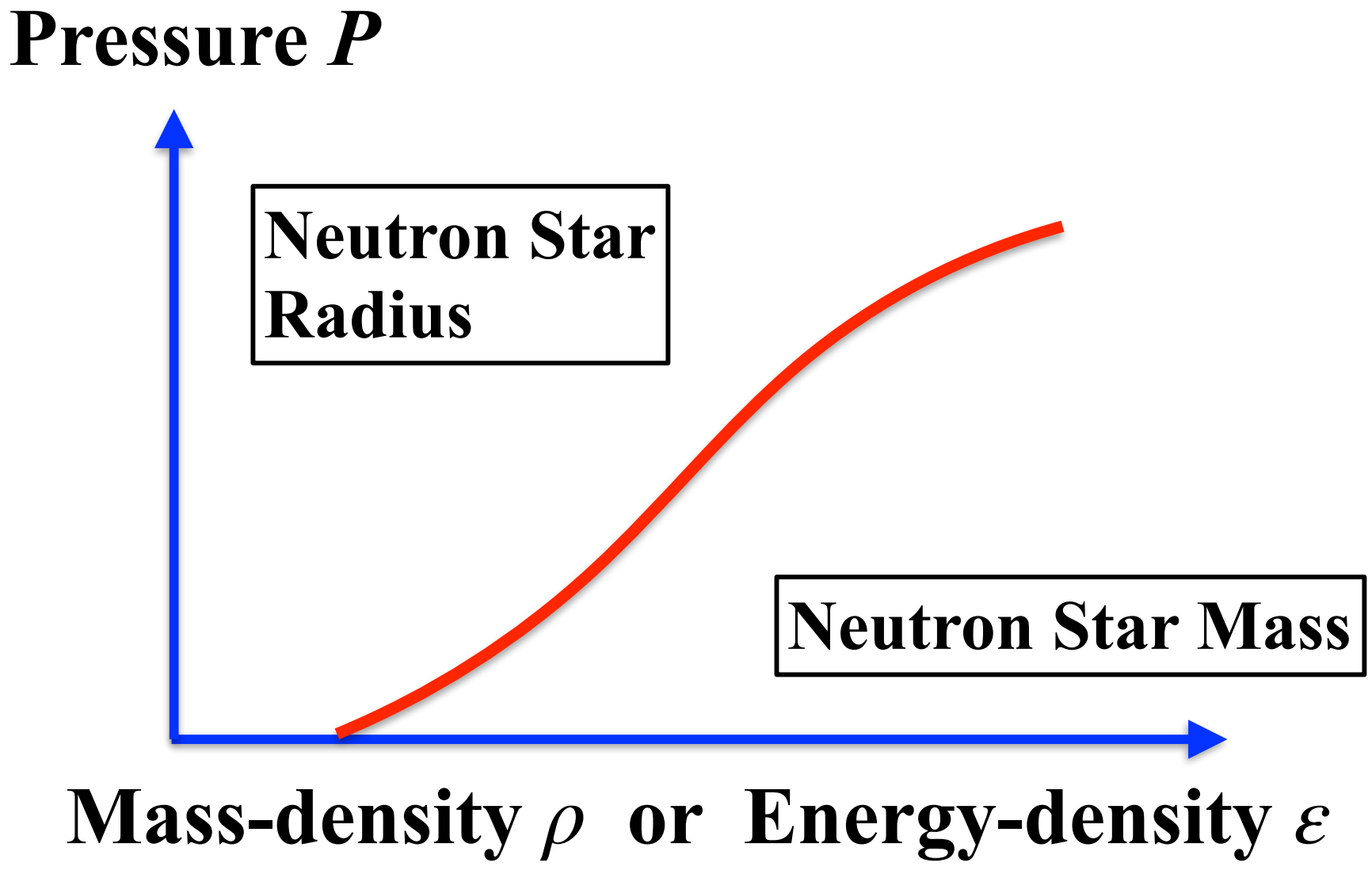
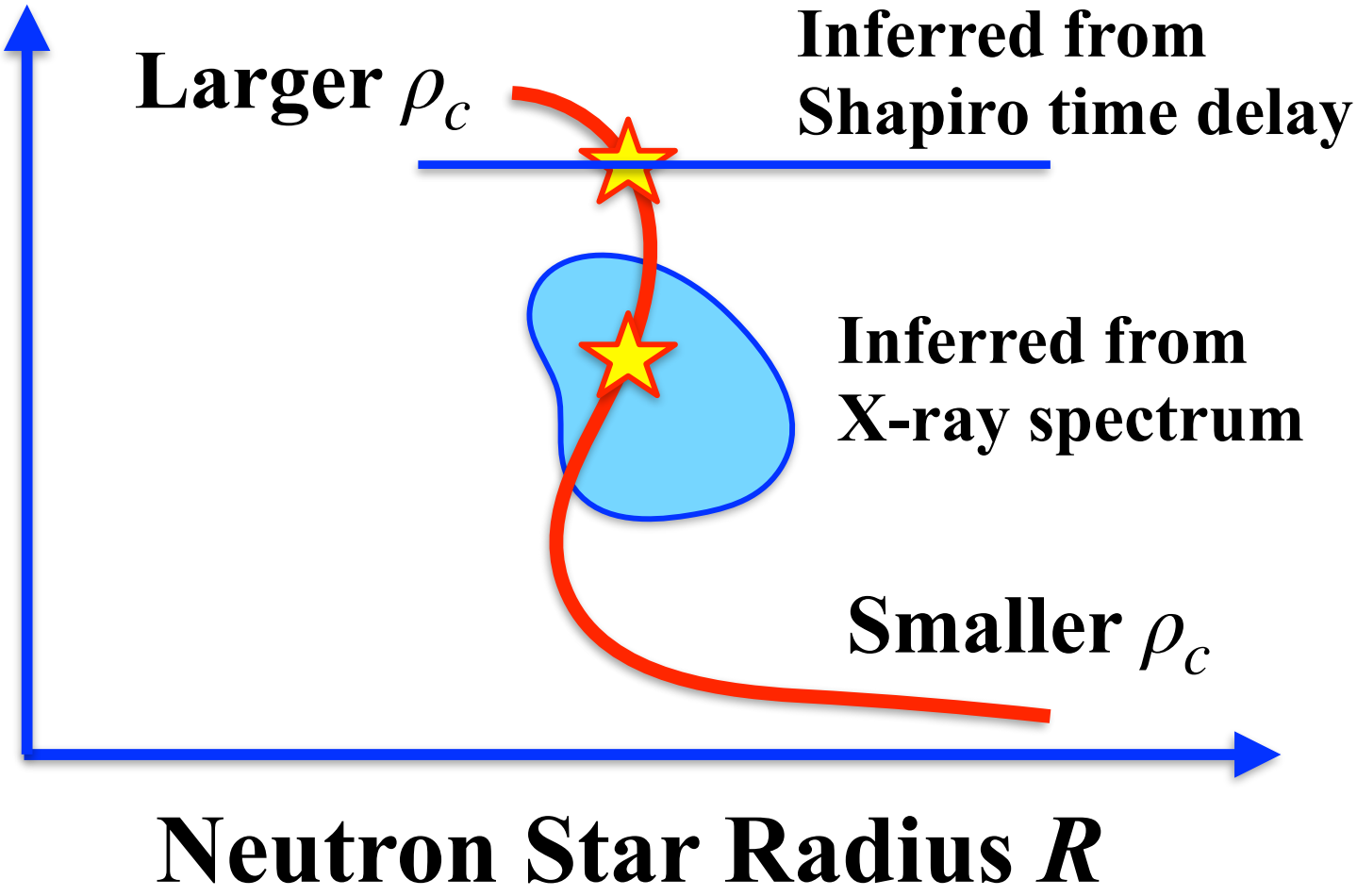
— Aspects of Criticality II —

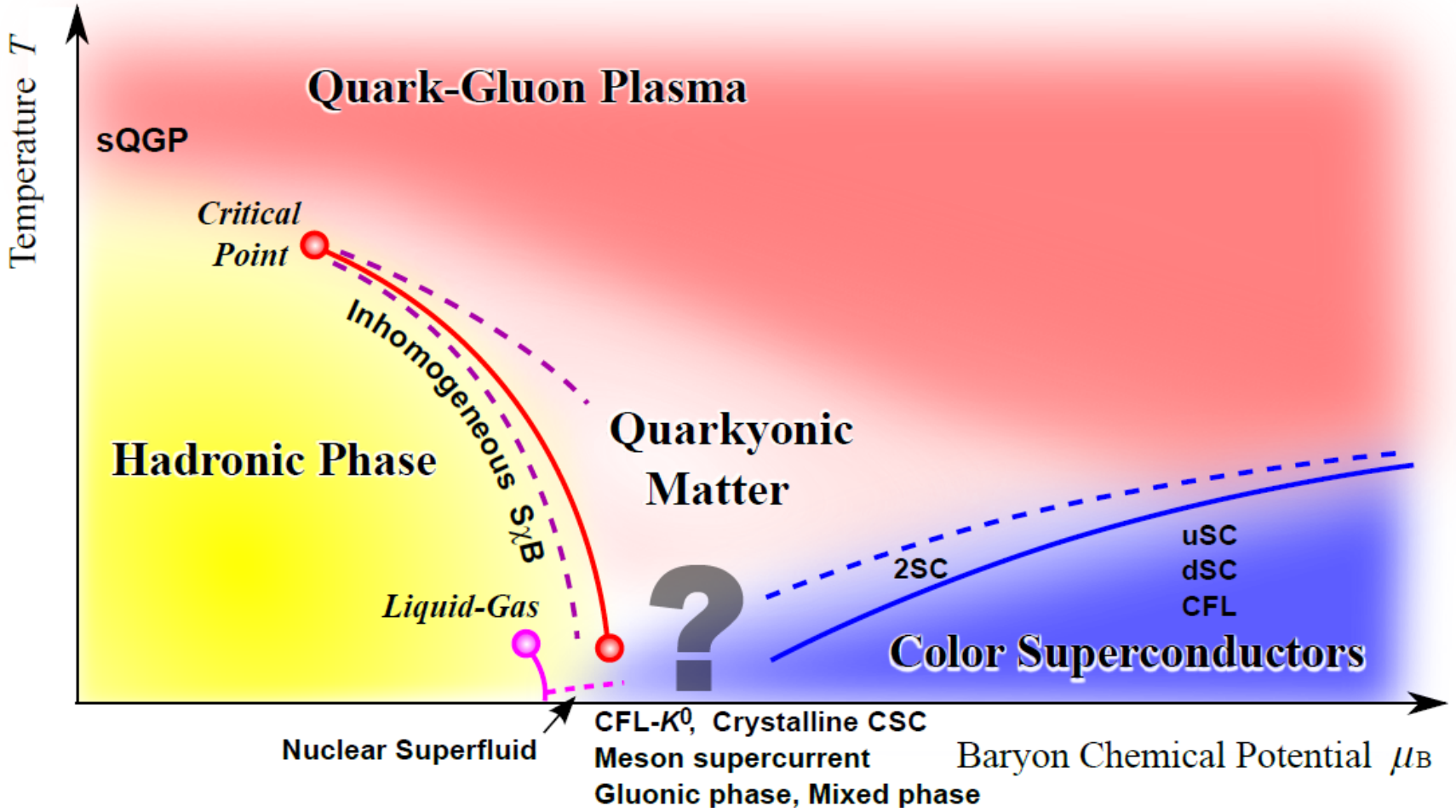
Equation of State



Equation of State

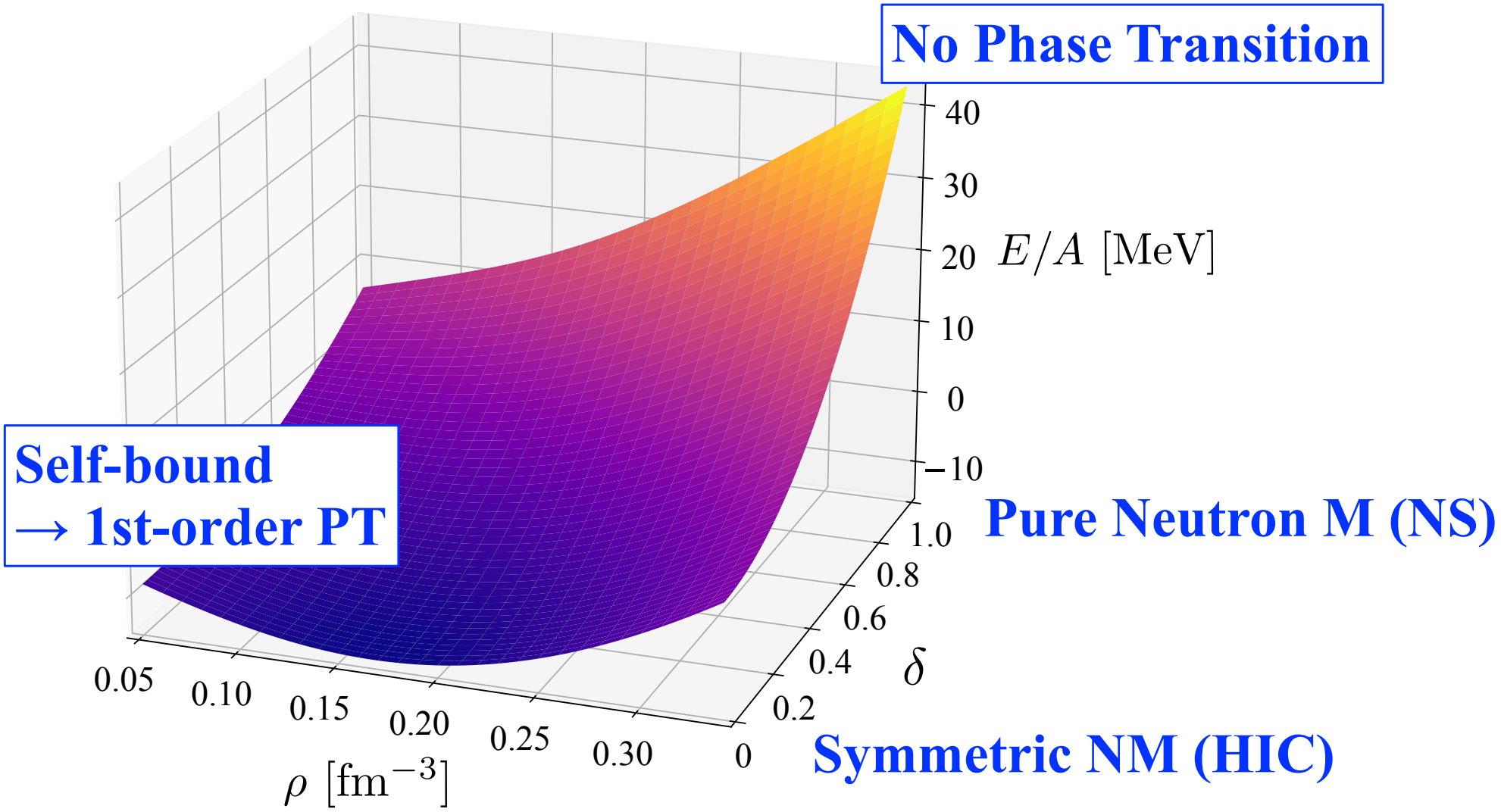


QCD Phase Diagram



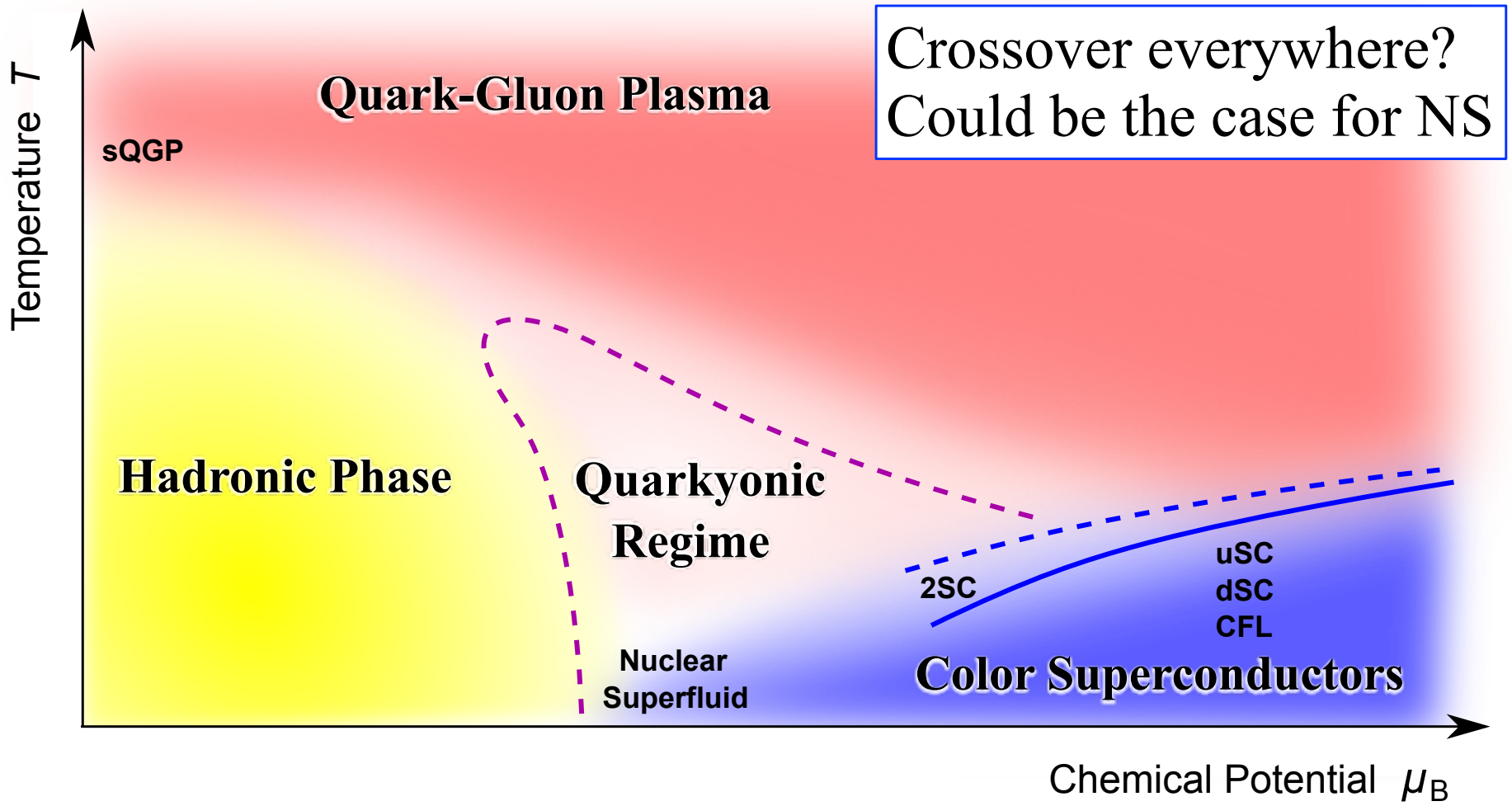
Fukushima-Hatsuda (2010); see also 50 Years of QCD Chap.7 (2023)

QCD Phase Diagram



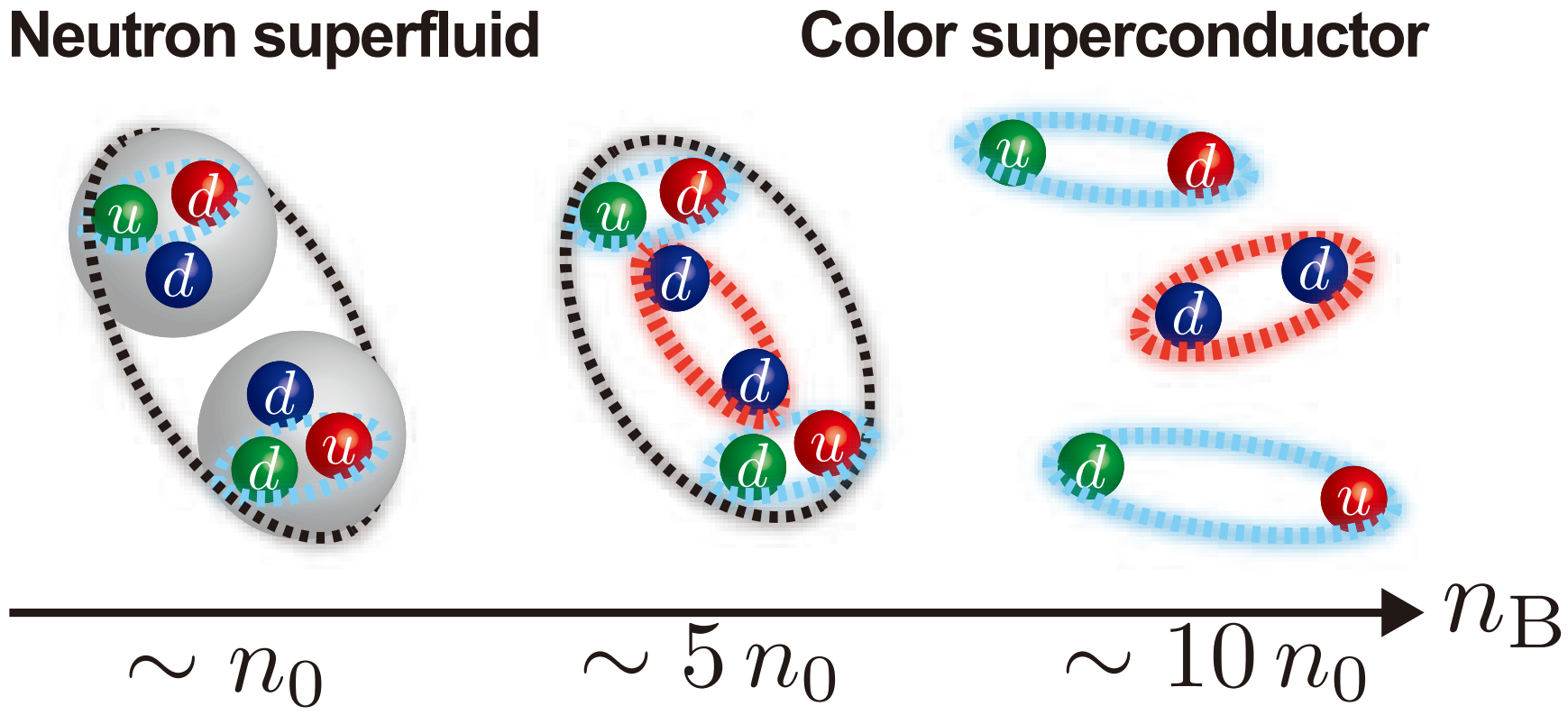
QCD Phase Diagram

Fukushima-Sasaki (2013)



Crossover to Quark Matter

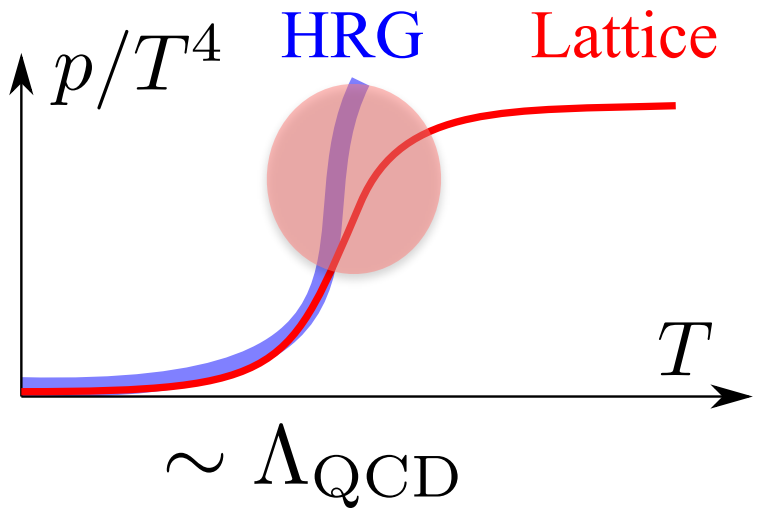
Fujimoto-Fukushima-Weise (2019)

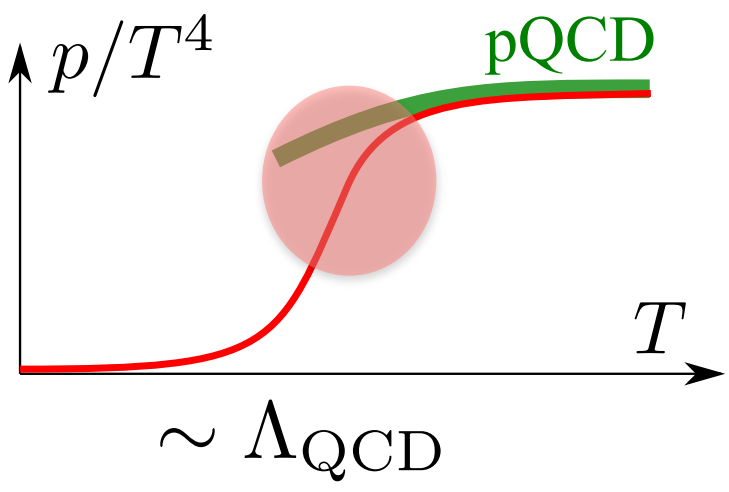


Lesson from High-T

Crossover = Duality Point

Rising *p* from small *T* is understood by a free gas of (thousands of) mesons (Hadron Resonance Gas).

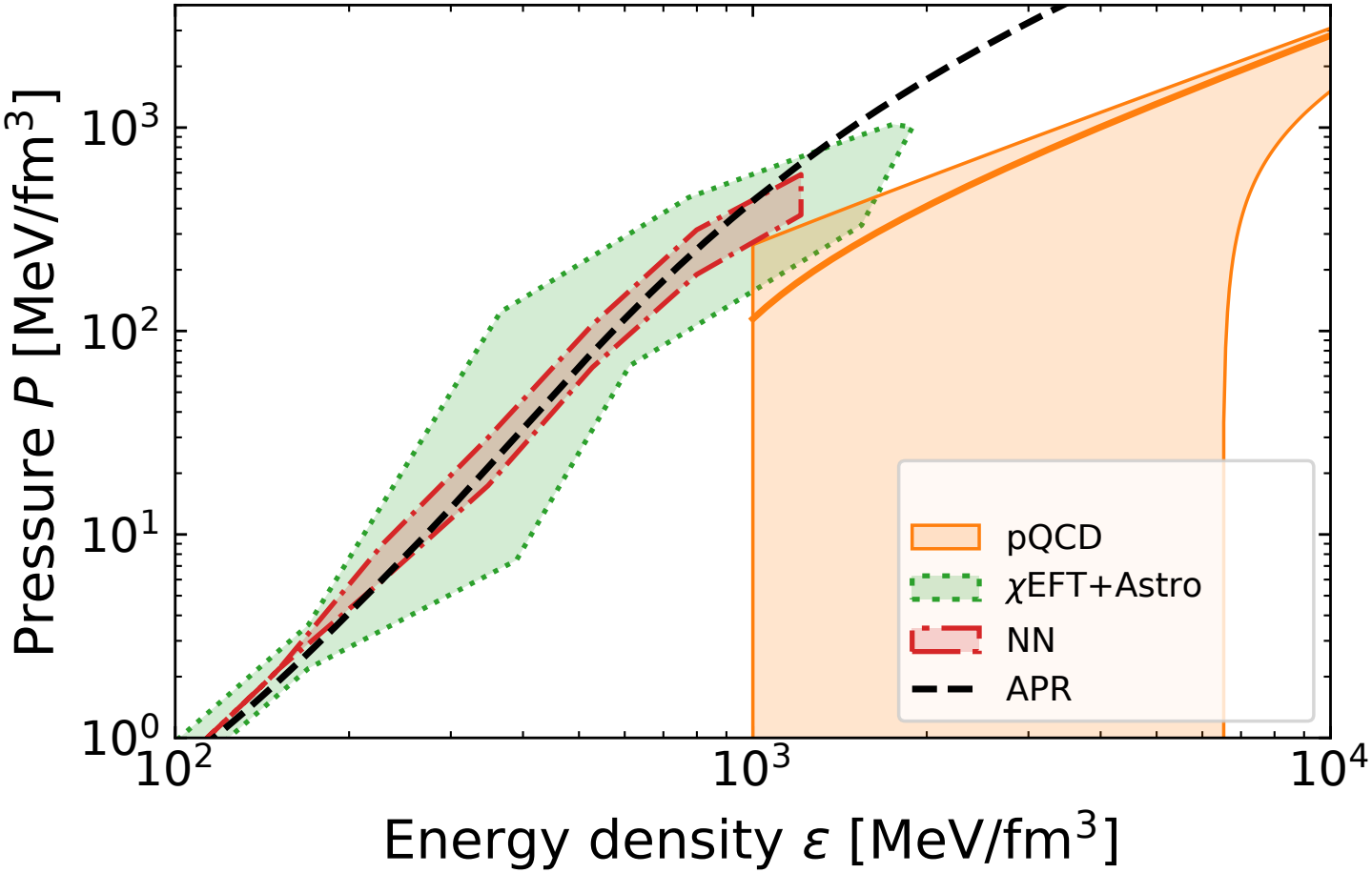




Pressure is "saturated" by the pQCD degrees of freedom.

EoS Construction

Nuclear EoS vs. pQCD EoS



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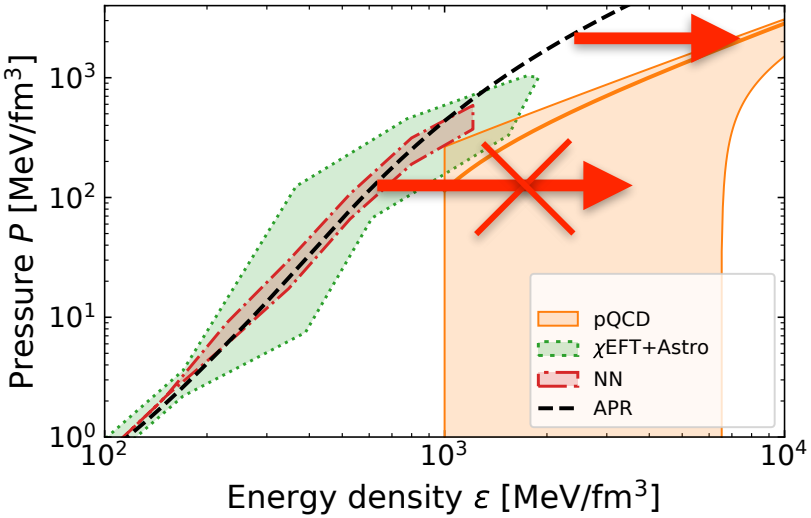
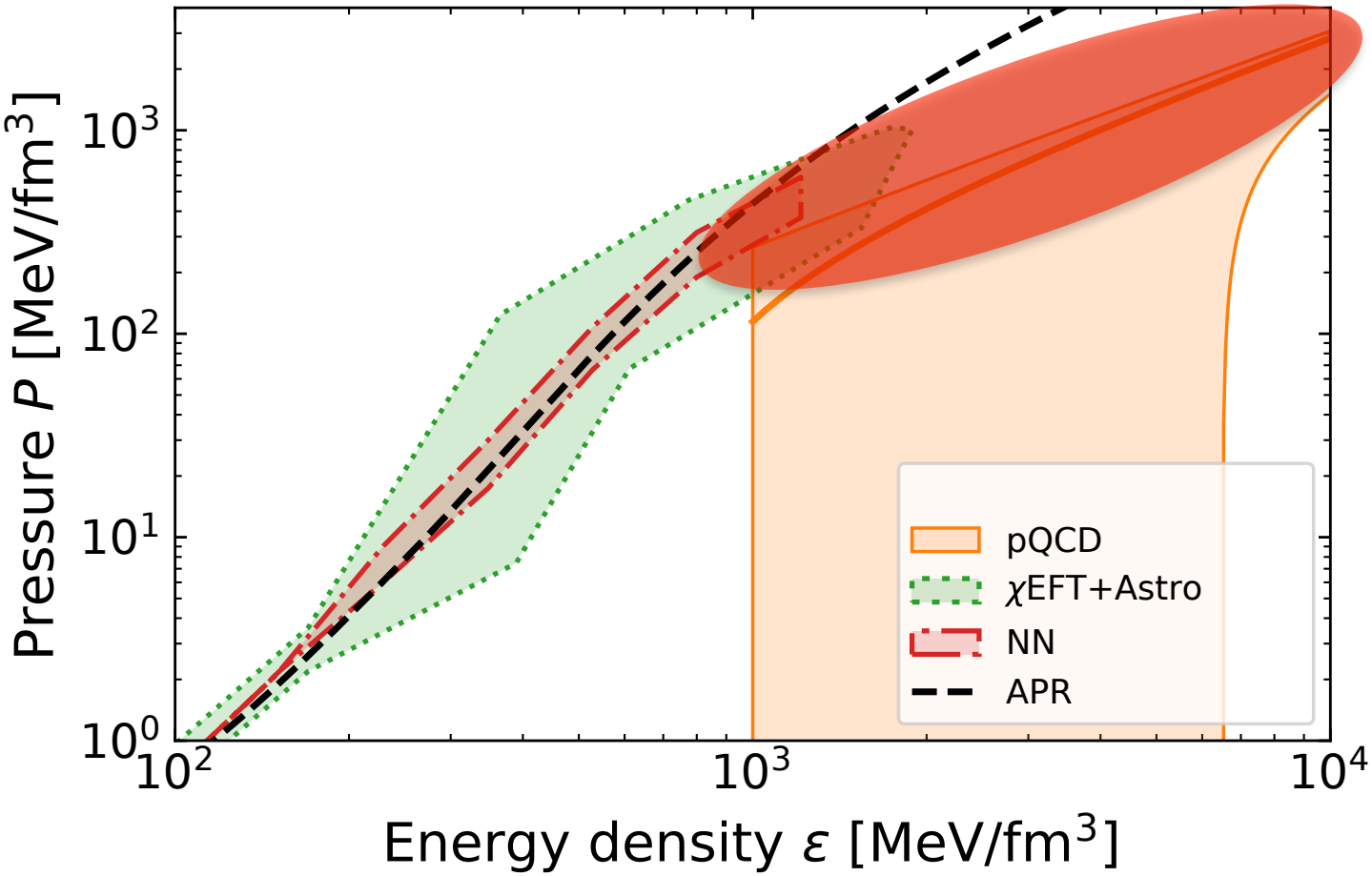


How the EoS jumps to the pQCD branch ?

1st-order PT cannot be excluded… but it must be weak at low density or strong at irrelevantly high density…

EoS Construction

Nuclear EoS vs. pQCD EoS



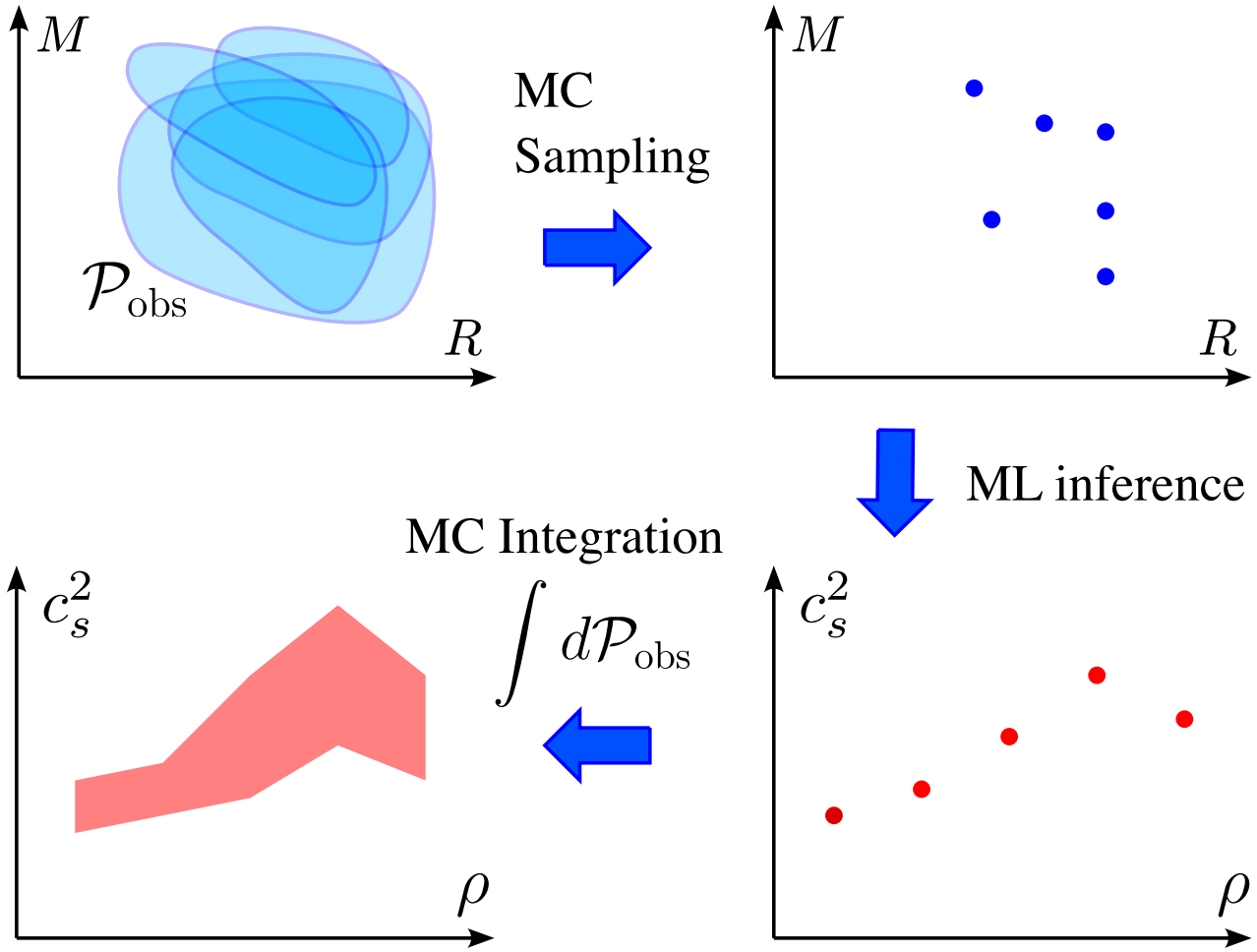
Or… smooth crossover?

pQCD branch is almost conformal.

$$c_s^2 \simeq 1/3$$

EoS Construction

Fujimoto-Fukushima-Kamata-Murase (2018-2024)



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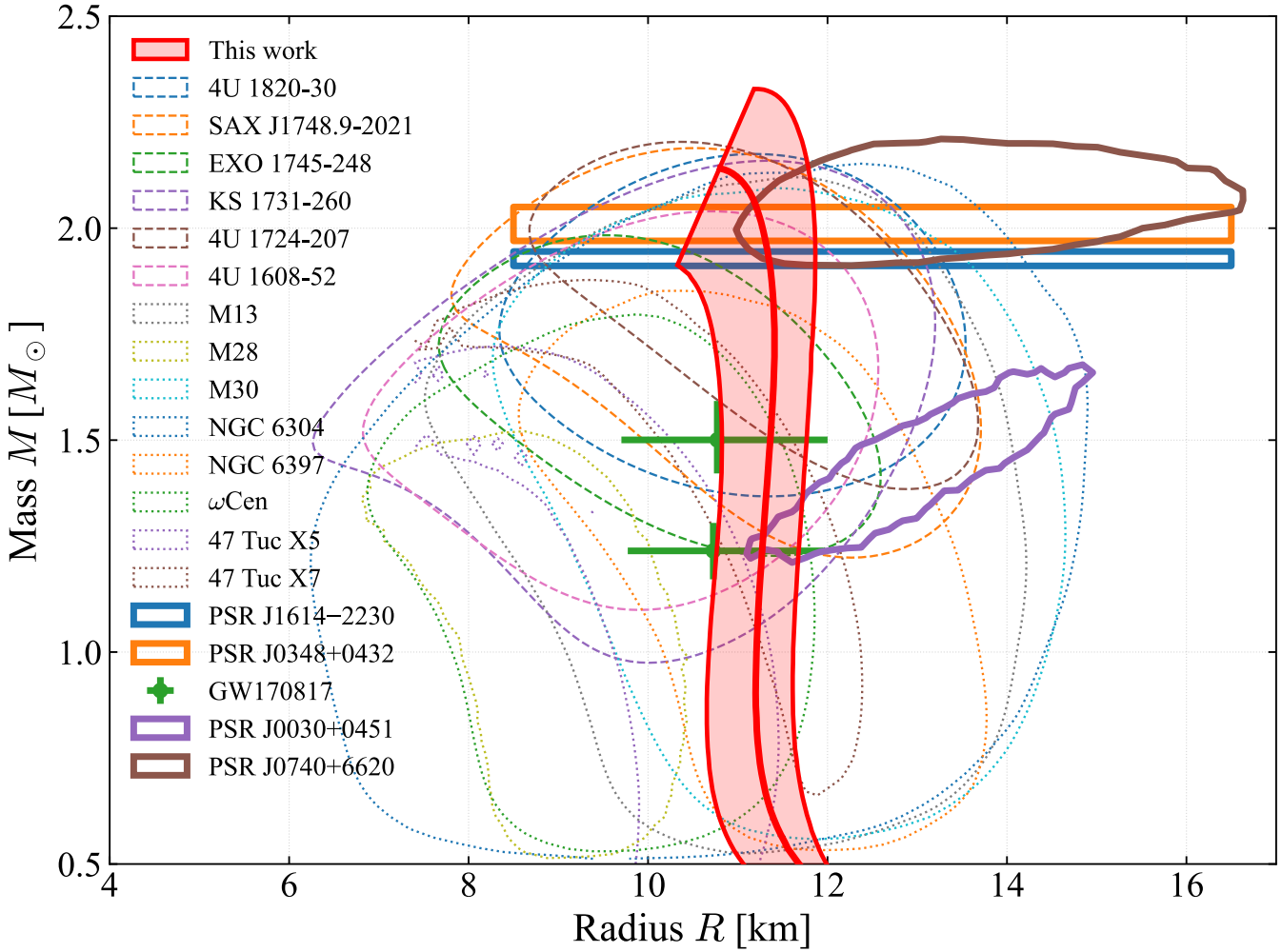

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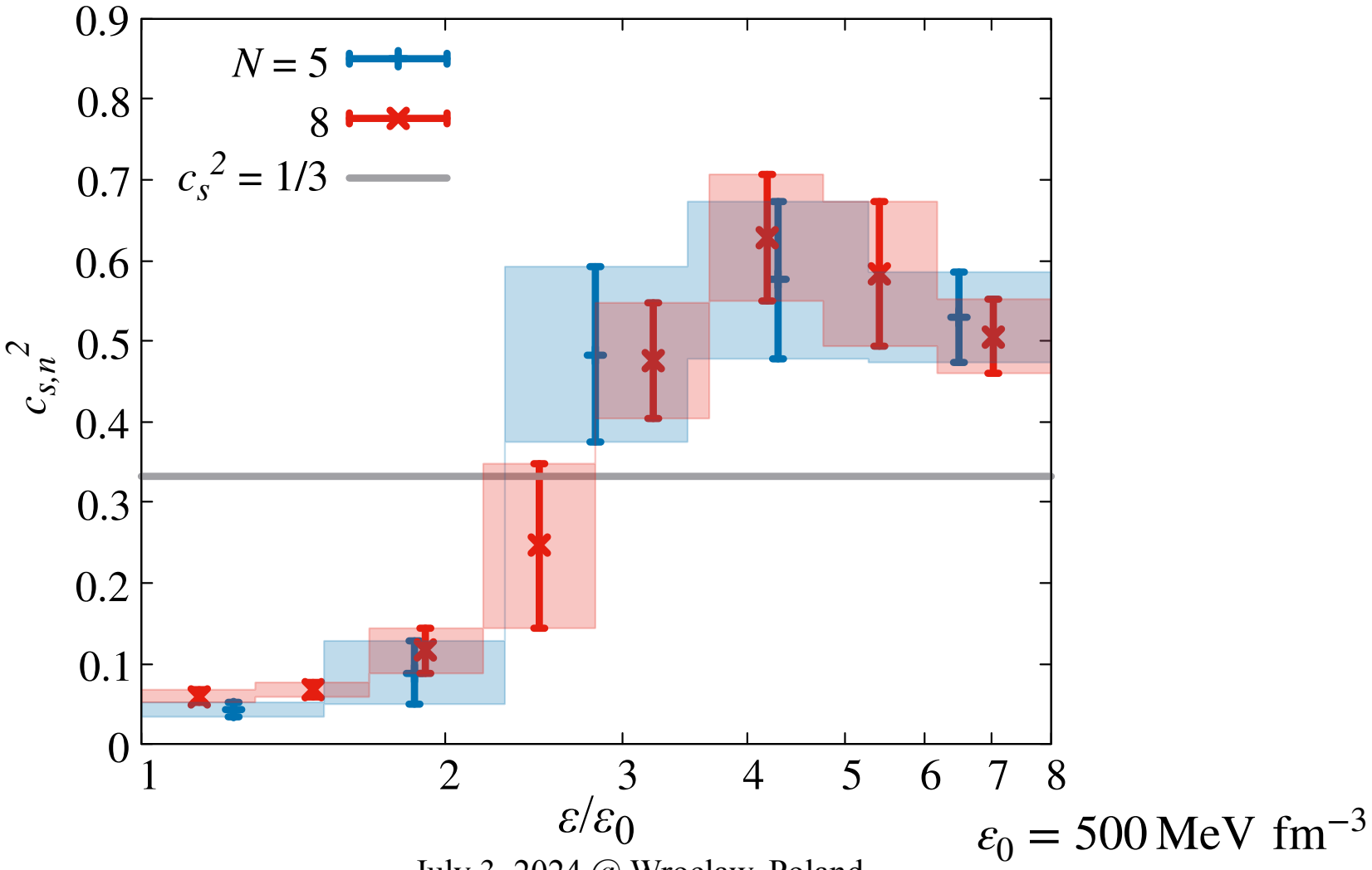
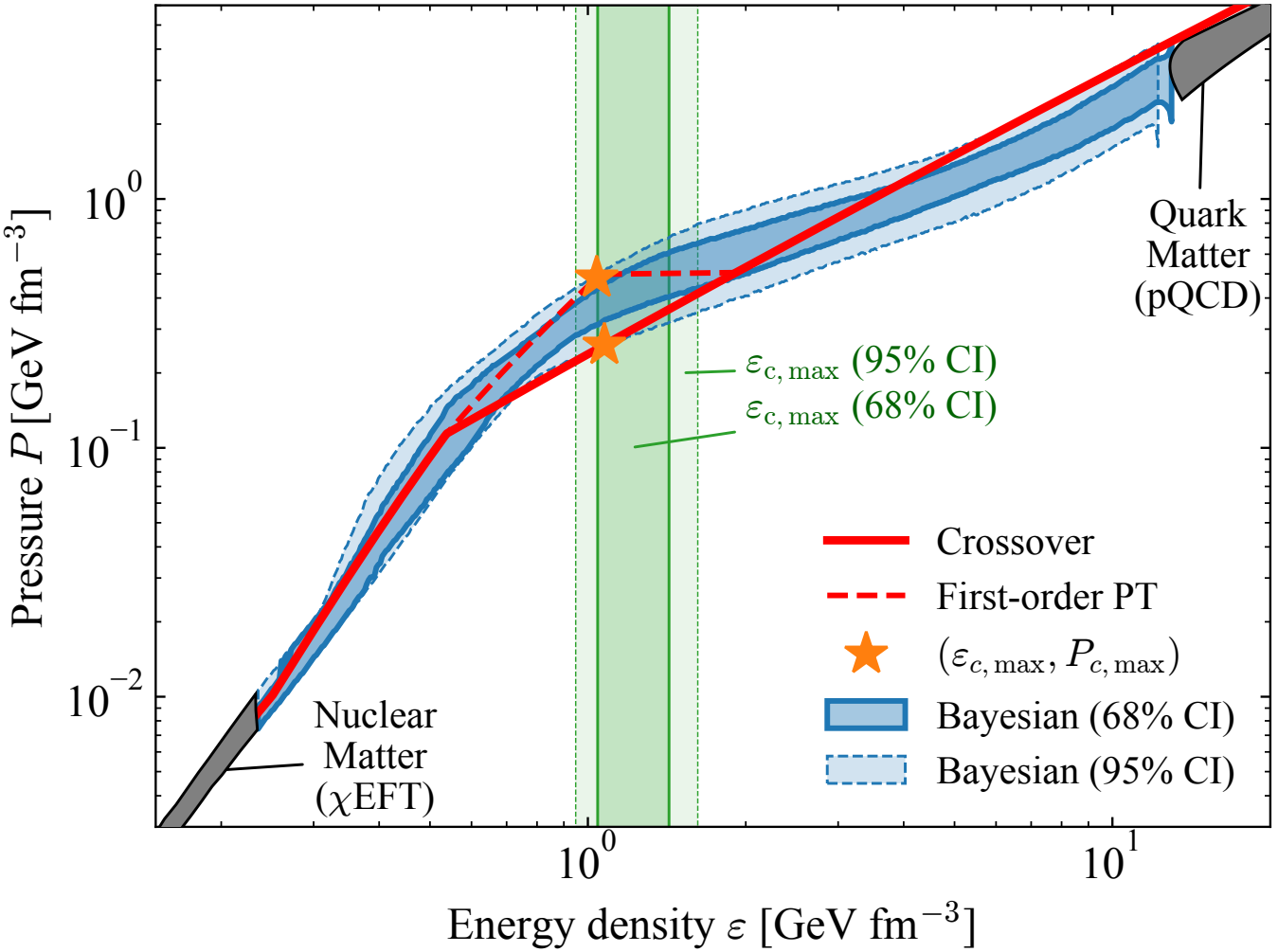


Crossover vs. 1st-order PT

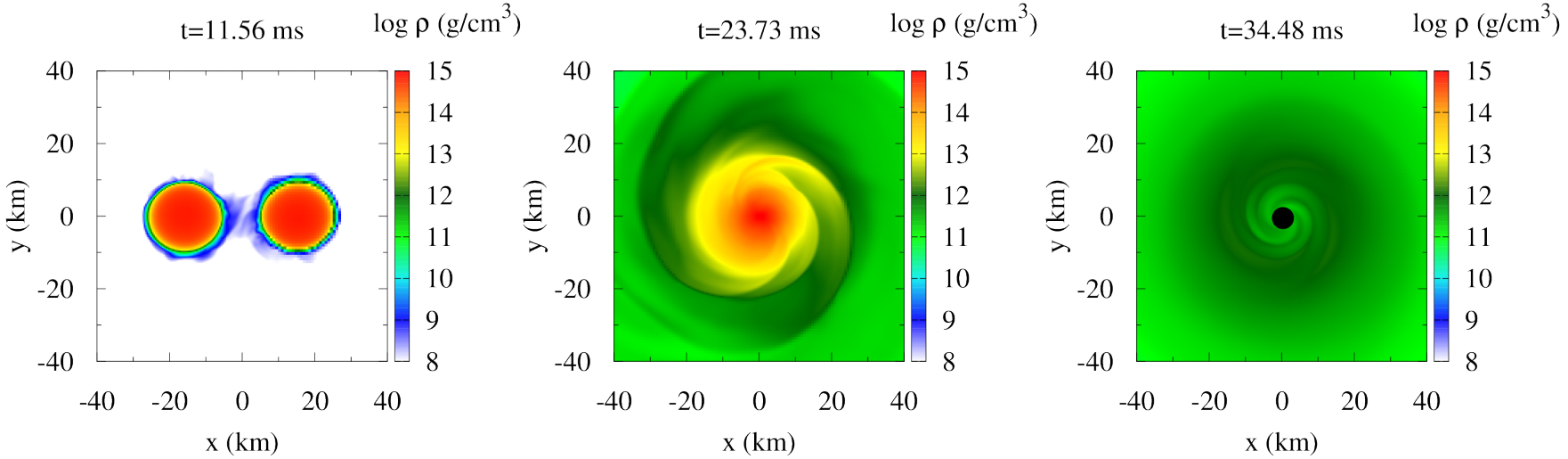
Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)



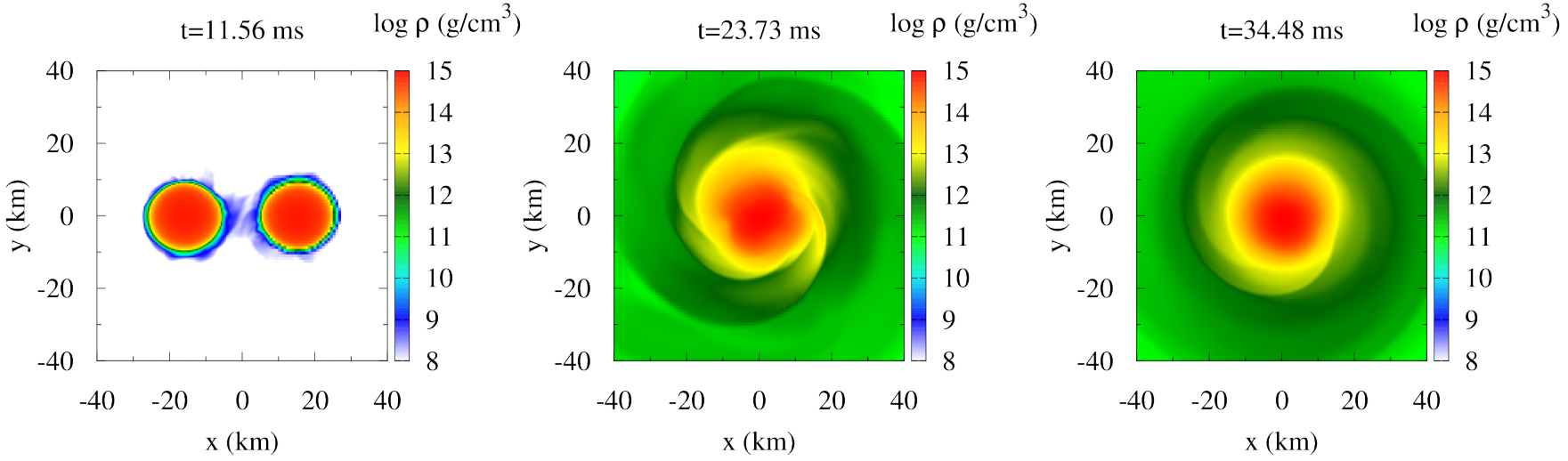
Crossover vs. 1st-order PT

Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)

CO



PT



Crossover vs. 1st-order PT

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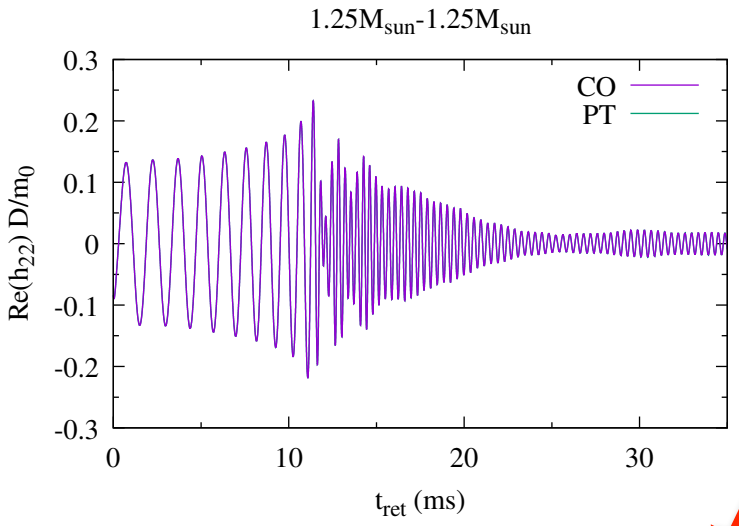
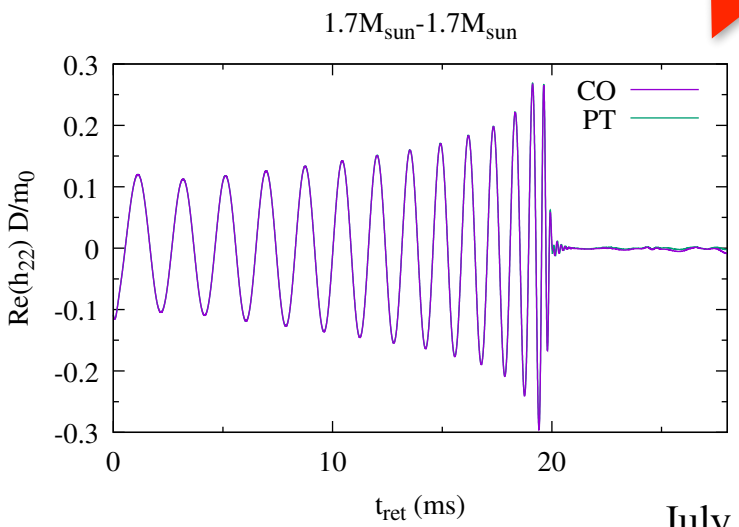
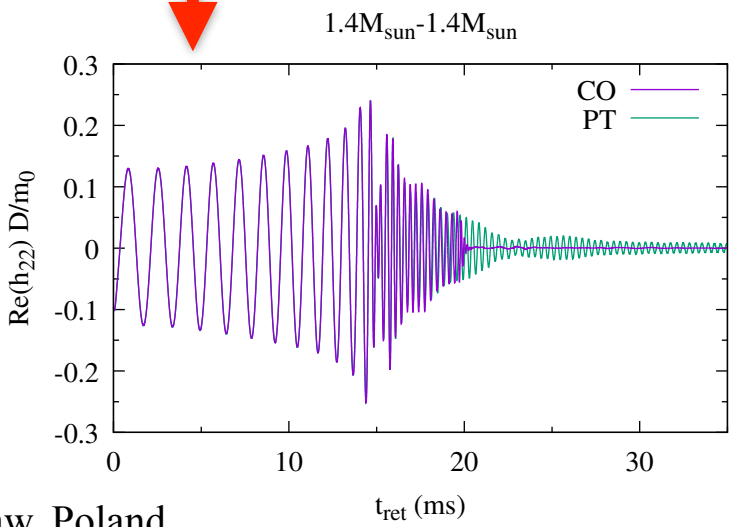


Light System (No BH)

Heavy System (Both BH)

GW170817 (Discriminable!)





Crossover vs. 1st-order PT

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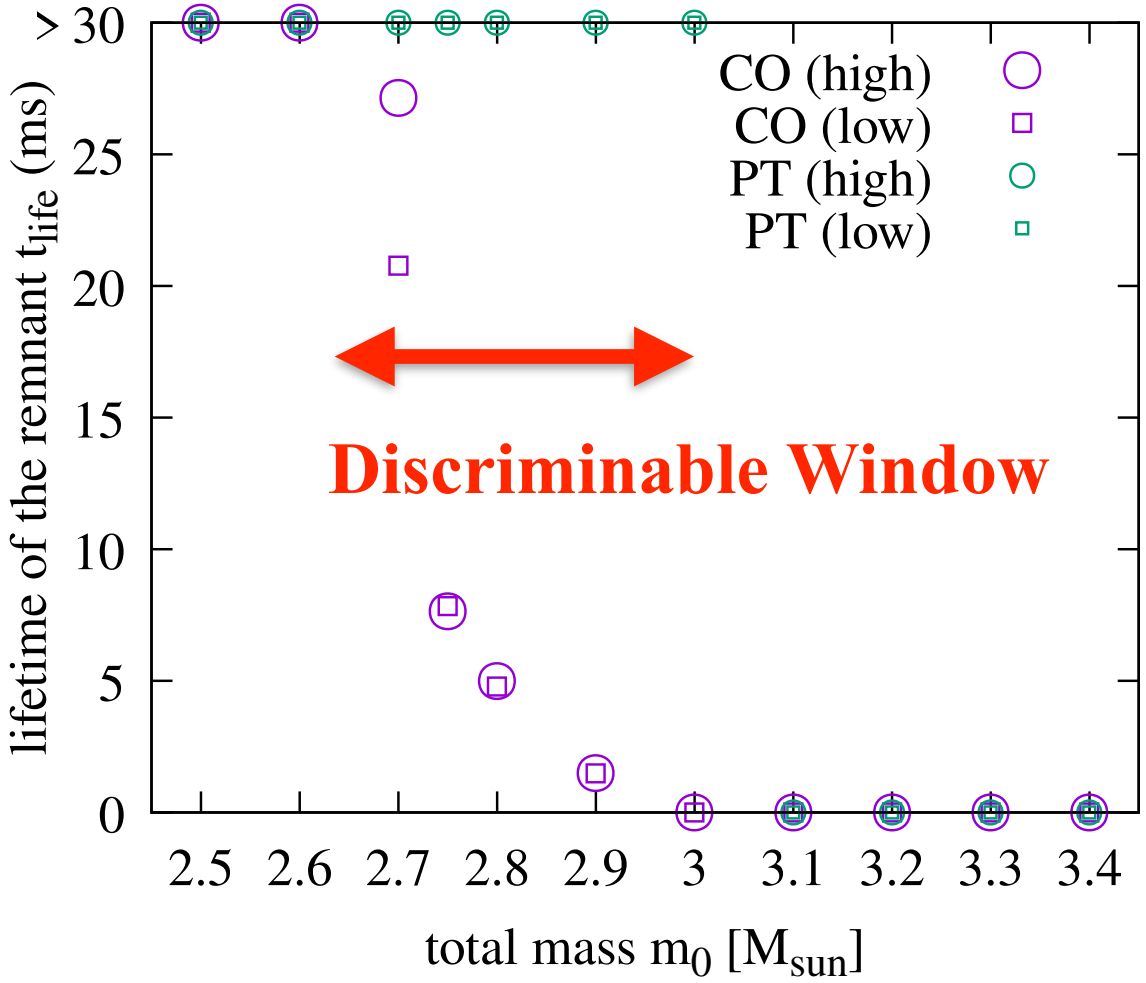


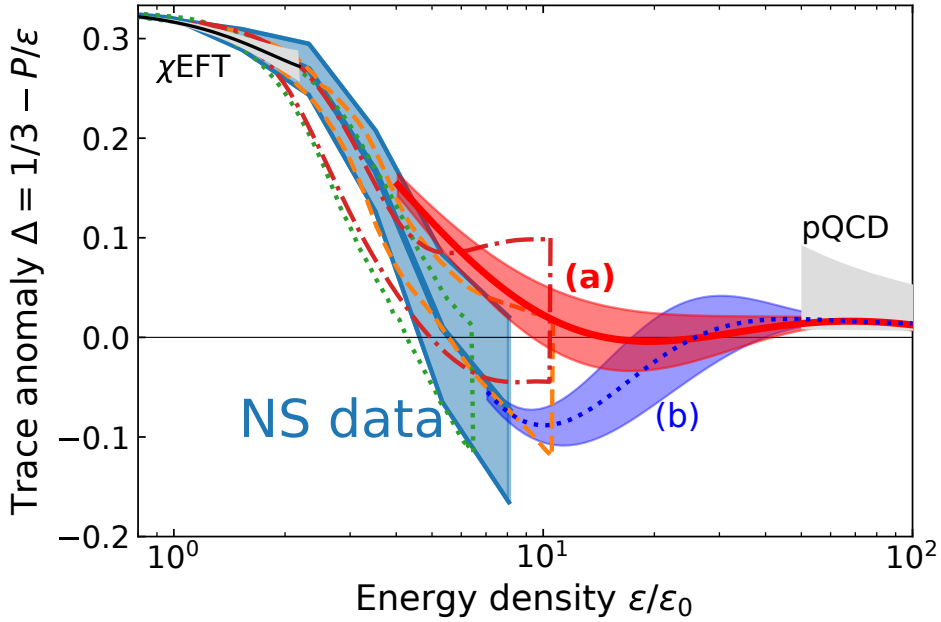
Keep in mind

Gravitational wave simulation has uncertainty other than EoS

- **resolution dependence (mesh size ~ 100-300m),**
  mass ratio *q* of the binary system,
  thermal effect (index), etc.

Conformality

Fujimoto-Fukushima-Praszalowicz-McLerran (2022)



$$\Delta \propto \varepsilon - 3p$$

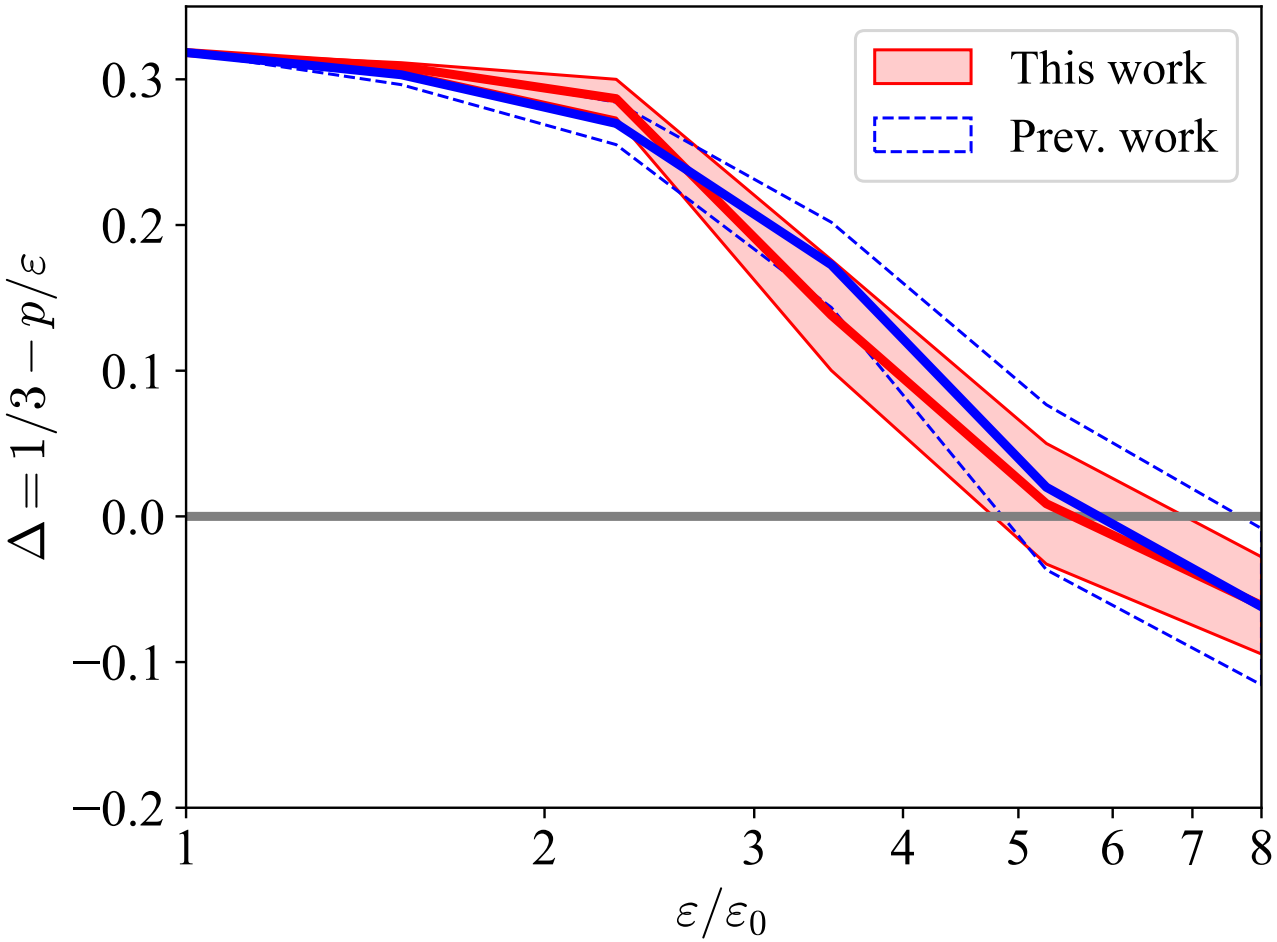
$$\propto \frac{d}{d\mu}\left(\frac{p}{\mu^4}\right)$$

Thermodynamic d.o.f.

Negative trace anomaly implies *decreasing* d.o.f. at higher density?

Conformality

Fujimoto-Fukushima-Kamata-Murase (2018-2024)

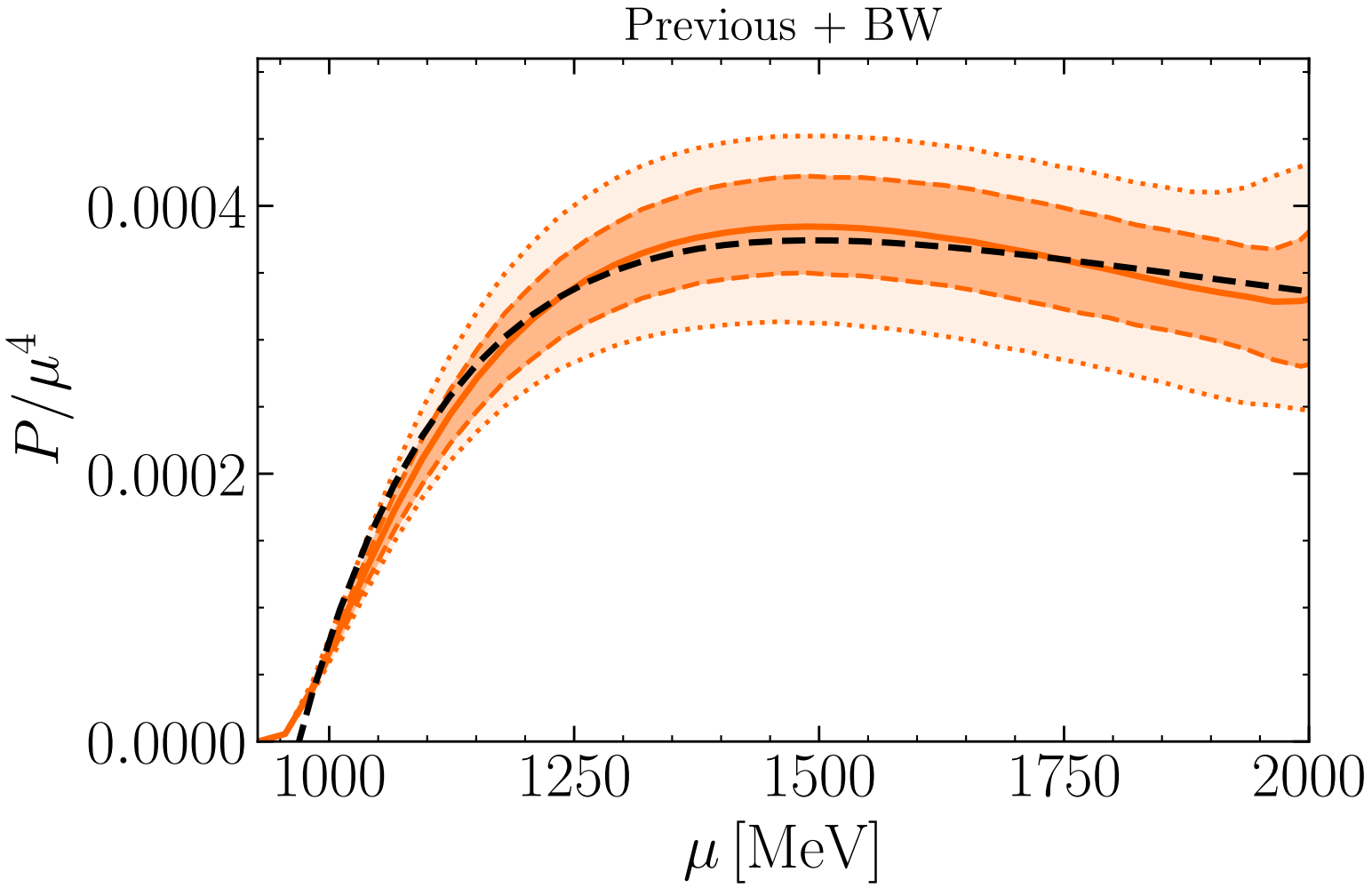


New data slightly pushes down.

Seems to go negative?

Conformality

Brandes-Fukushima-Iida-Yu (2024)



Conformality

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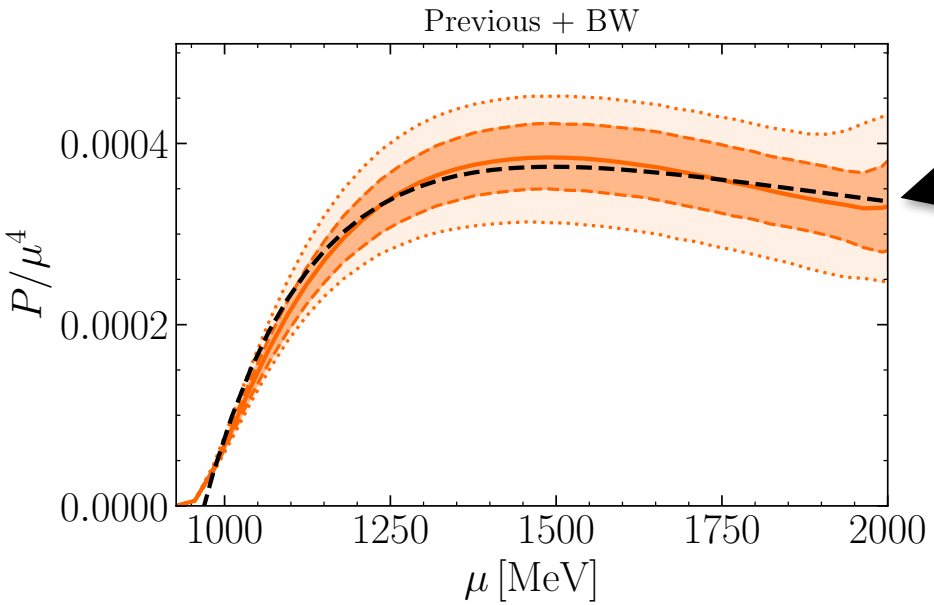


Fitting Formula

$$\frac{P}{\mu^4} = \frac{a}{\mu^4} + \frac{b}{\mu^2} + c$$

$a = -127.37\,\mathrm{MeV}\ \mathrm{fm}^{-3}$

$b = 876.27\,\mathrm{MeV}^2$

$c = 1.7816 \times 10^{-2}$

pQCD cannot generate…?

Condensation

Brandes-Fukushima-Iida-Yu (2024)

Suppose the vacuum has a bosonic condensate:

$$V(|\phi|^2) = m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4 + \frac{\eta}{6}|\phi|^6$$

$$-\mu^2|\phi|^2 \quad \text{from the kinetic term}$$

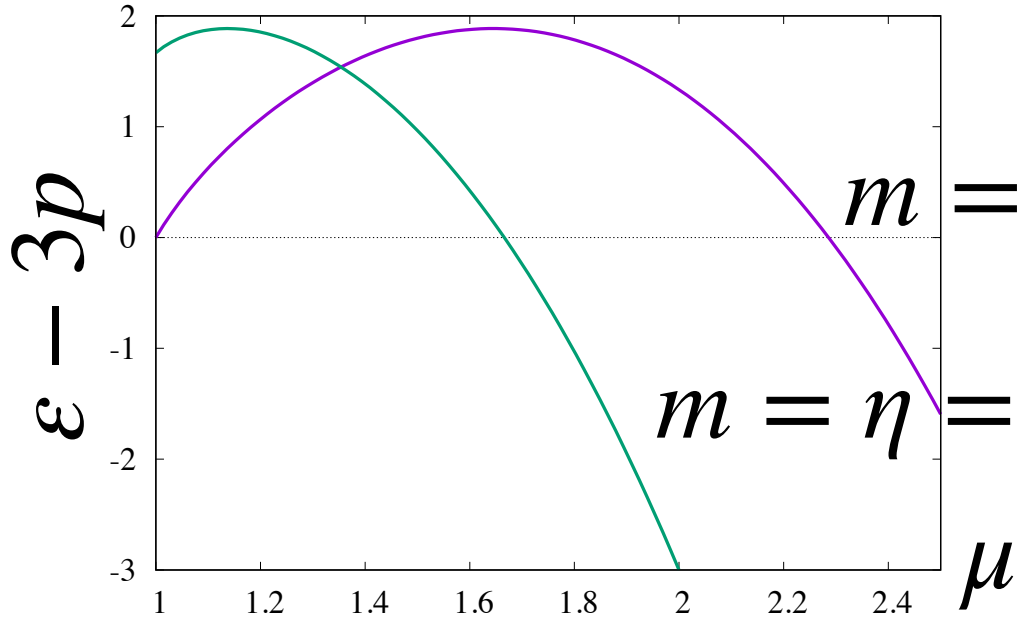
$$|\phi_c|^2 = \frac{1}{2\eta}\left[\sqrt{\lambda^2 + 8\eta(\mu^2 - m^2)} - \lambda\right]$$

Condensation

Brandes-Fukushima-Iida-Yu (2024)

Suppose the vacuum has a bosonic condensate:

$$\varepsilon - 3p = \frac{2}{3}(-\mu^2 + 4m^2)|\phi_c|^2 + \frac{\lambda}{3}|\phi_c|^4$$



$m = \lambda = \eta = 1$

$m = \eta = 1 \quad \lambda = -1$

In progress…

Summary

- **GW simulation can distinguish crossover and no-crossover (1st-order PT) EoSs.**
- **Total mass should be within a window.**
- **Trace anomaly goes negative… interpretation?**
- **Possibility to extract "condensate" from the trace anomaly from the observational data. (pQCD+CSC cannot give negative trace anomaly unless the CSC gap is unphysically large…?)**