The imprint of conservation laws on correlated particle production

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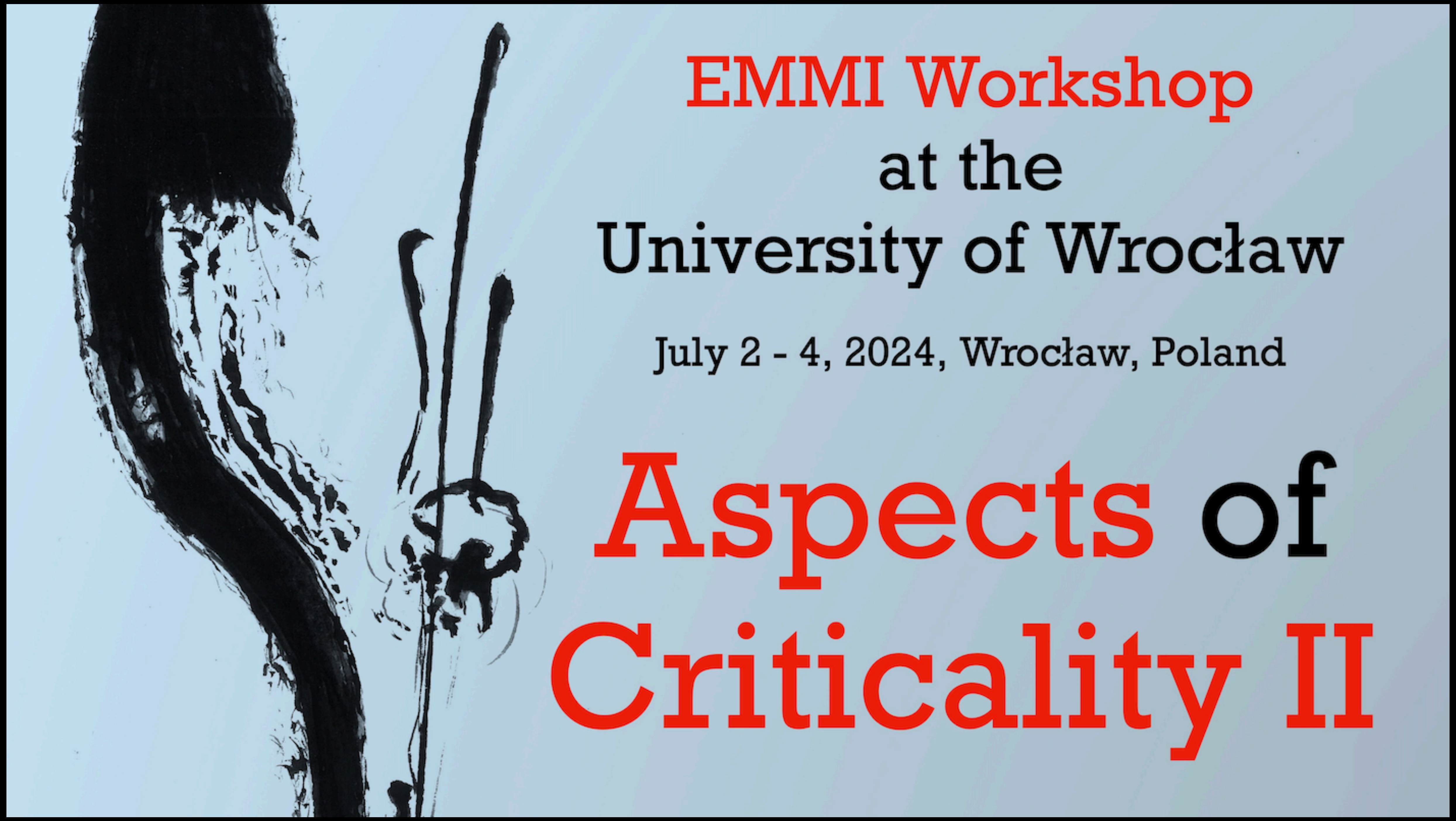
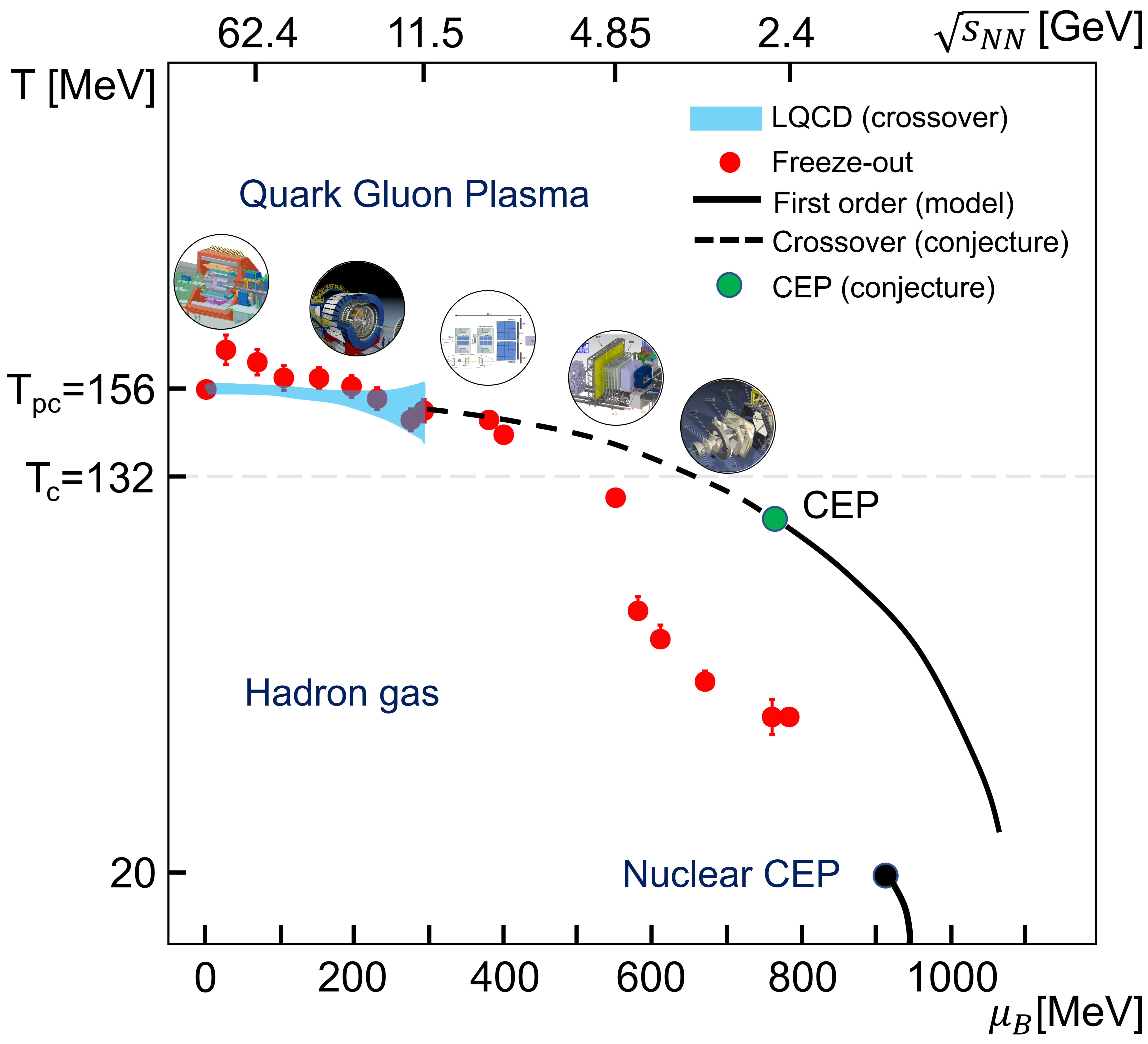
In collaboration with: P. Braun-Munzinger, B. Friman, K. Redlich, J. Stachel



Deciphering the phases with fluctuations/correlations



E-by-E fluctuations are predicted within Grand Canonical Ensemble

direct link to EoS

$$\frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3} = \frac{1}{VT^3}\frac{\partial^n lnZ(V,T,\mu_B)}{\partial\left(\mu_B/T\right)^n} \equiv \hat{\chi}_n^B$$

for a thermal system of fixed volume V and temperature T

κ_n - cumulants (measurable in experiment)

$\hat{\chi}_n^B$ - susceptibilities (e.g. from lQCD)

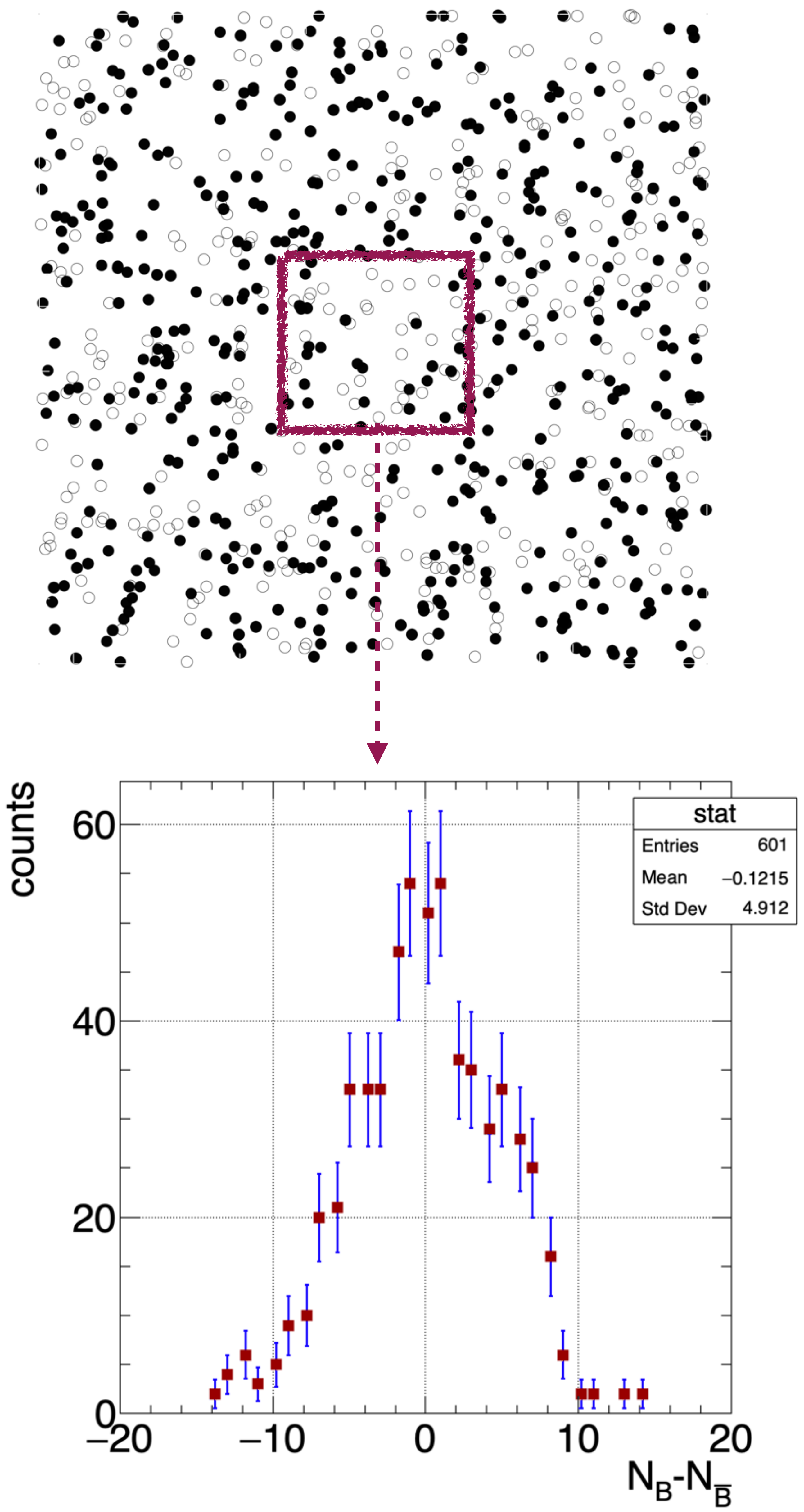
IMPORTANT

$T_{pc}^{LQCD} = 156.5 \pm 1.5$ MeV

$T_{FO}^{ALICE} = 156.5 \pm 1.5 \pm 3$ MeV (sys)

decoding the phase structure of matter with cumulants of multiplicity distributions

Measurements vs. theoretical calculations



- $\Delta N = N_B - N_{\bar{B}}$ occurs with probability $p(\Delta N)$ (measured)
- r^{th} order central moment: $\mu_r = \sum_{\Delta N} (\Delta N - \langle \Delta N \rangle)^r p(\Delta N)$
- first 4 cumulants: $\kappa_1 = \langle \Delta N \rangle$, $\kappa_2 = \mu_2$, $\kappa_3 = \mu_3$, $\kappa_4 = \mu_4 - 3\mu_2^2$
- **advantage:** sensitive to small (critical) signals
- **disadvantage:** sensitive to any non-critical contributions

in GCE

$$\frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3} = \frac{1}{VT^3} \frac{\partial^n lnZ(V,T,\mu_B)}{\partial \left(\mu_B/T\right)^n} \equiv \hat{\chi}_n^B$$

Minimal baseline: Ideal Gas EoS + GCE

particles (Poisson)

$$\frac{\kappa_m}{\kappa_n} = 1$$

net-particles (Skellam)

$$\frac{\kappa_{2m}}{\kappa_{2n}} = \frac{\langle N \rangle + \langle \bar{N} \rangle}{\langle N \rangle + \langle \bar{N} \rangle} = 1, \quad \frac{\kappa_{2m}}{\kappa_{2n+1}} = \frac{\langle N \rangle + \langle \bar{N} \rangle}{\langle N \rangle - \langle \bar{N} \rangle}$$

Outline

- Conservation laws within Canonical Ensemble
  - Global conservations
  - Local conservations (correlations)
- Do we understand the NEW (BESII) STAR data?
  - Chasing for proton clusters

Fluctuations in Canonical Ensemble

$$Z_B(V,T) = \sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B(2\, z\sqrt{\lambda_B \lambda_{\bar{B}}})$$

- B net-baryon number, conserved in each event
- I_B modified Bessel function of the first kind
- $z_B, z_{\bar{B}}$ single particle partition functions for baryons, anti baryons
- $\lambda_B, \lambda_{\bar{B}}$ auxiliary parameters for calculating cumulants of baryons, anti baryons

P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901

$$\frac{\kappa_2(B-\bar{B})}{\langle n_B + n_{\bar{B}}\rangle} = 1 - \boxed{\frac{\alpha_B\langle n_B\rangle + \alpha_{\bar{B}}\langle n_{\bar{B}}\rangle}{\langle n_B + n_{\bar{B}}\rangle} + \left(z^2 - \langle N_B\rangle\langle N_{\bar{B}}\rangle\right)\frac{(\alpha_B - \alpha_{\bar{B}})^2}{\langle n_B + n_{\bar{B}}\rangle}}$$

canonical suppression

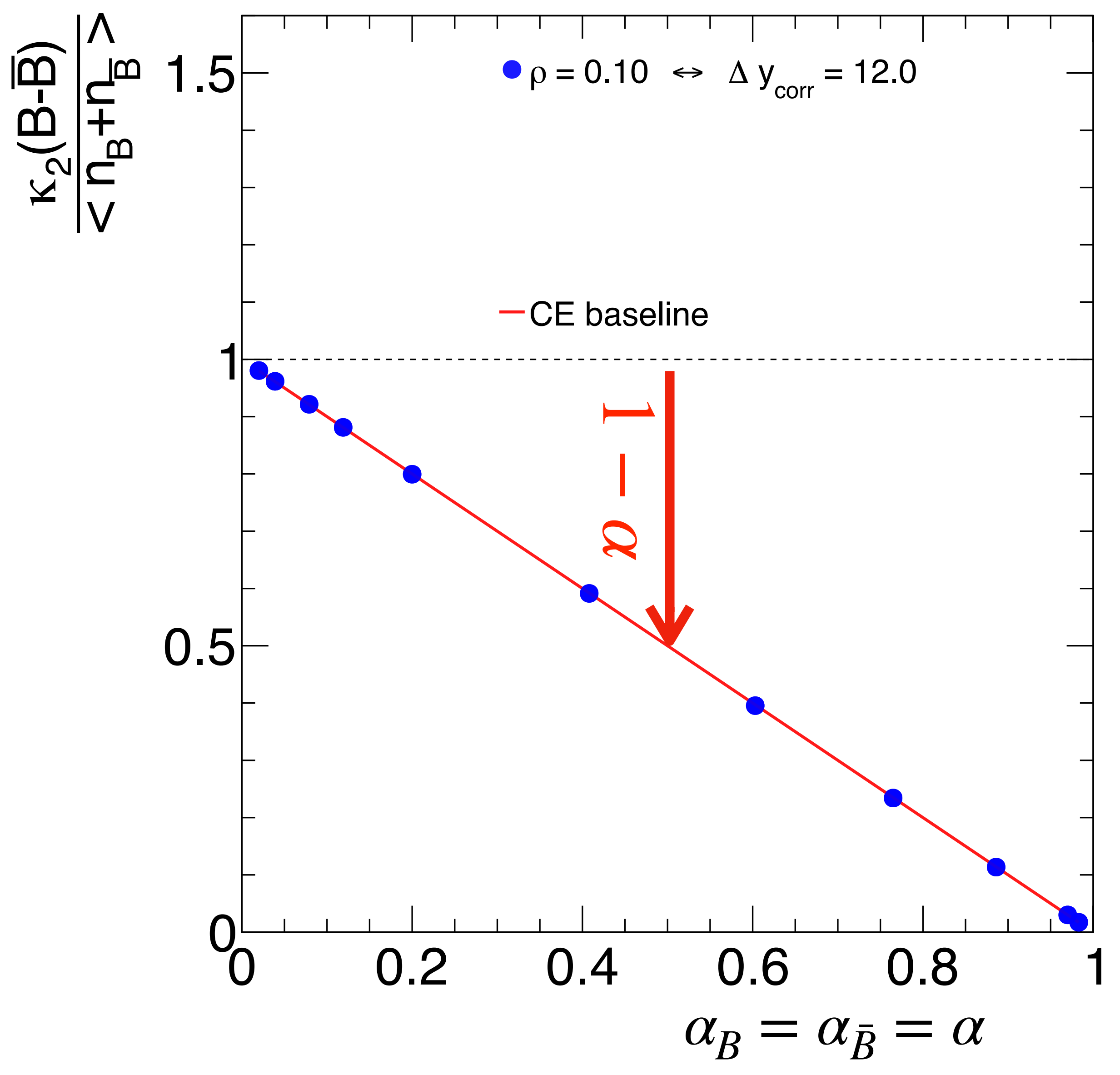
$\langle N_B\rangle, \langle N_{\bar{B}}\rangle$ - in 4π

$\langle n_B\rangle, \langle n_{\bar{B}}\rangle$ - inside acceptance

$\alpha_B = \langle n_B\rangle/\langle N_B\rangle$ - acceptance for B

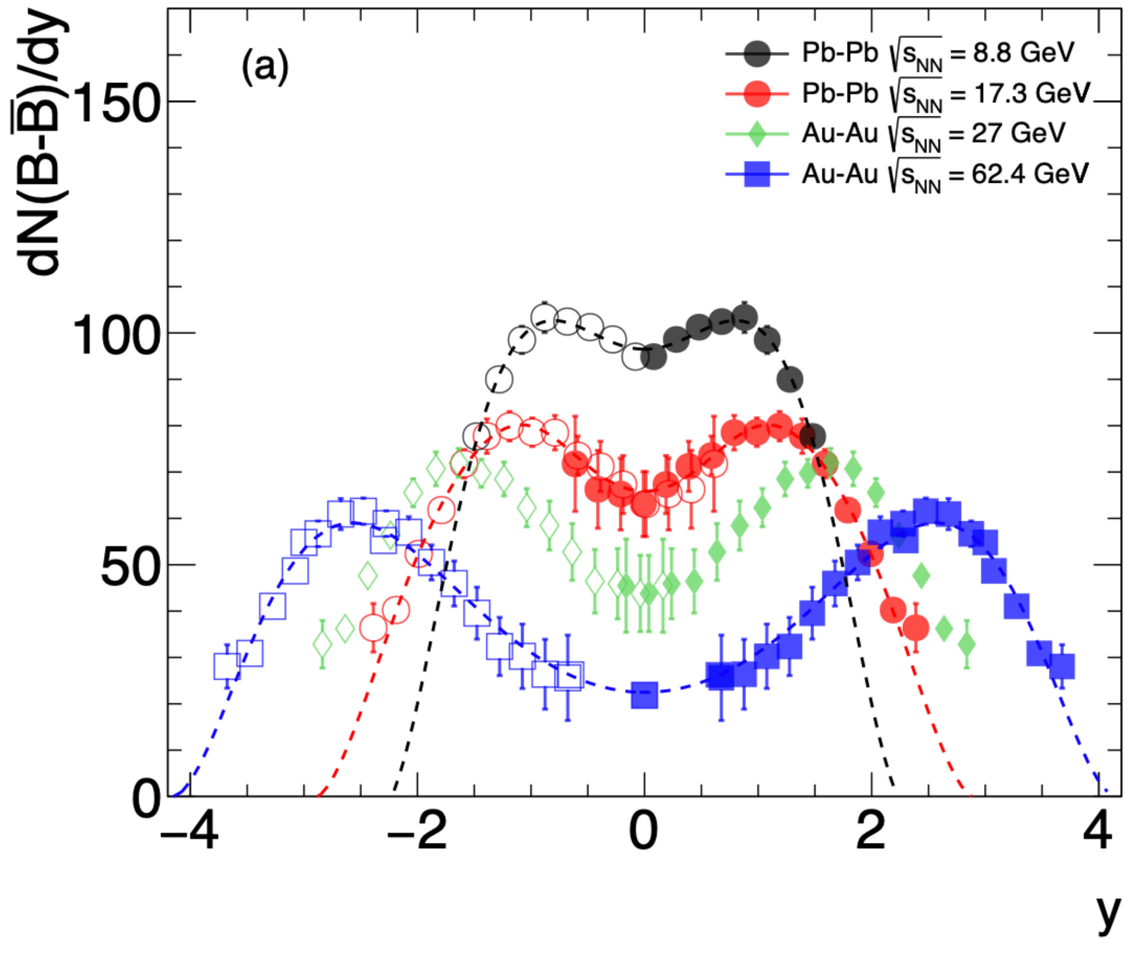
$\alpha_{\bar{B}} = \langle n_{\bar{B}}\rangle/\langle N_{\bar{B}}\rangle$ - acceptance for $\bar{B}$

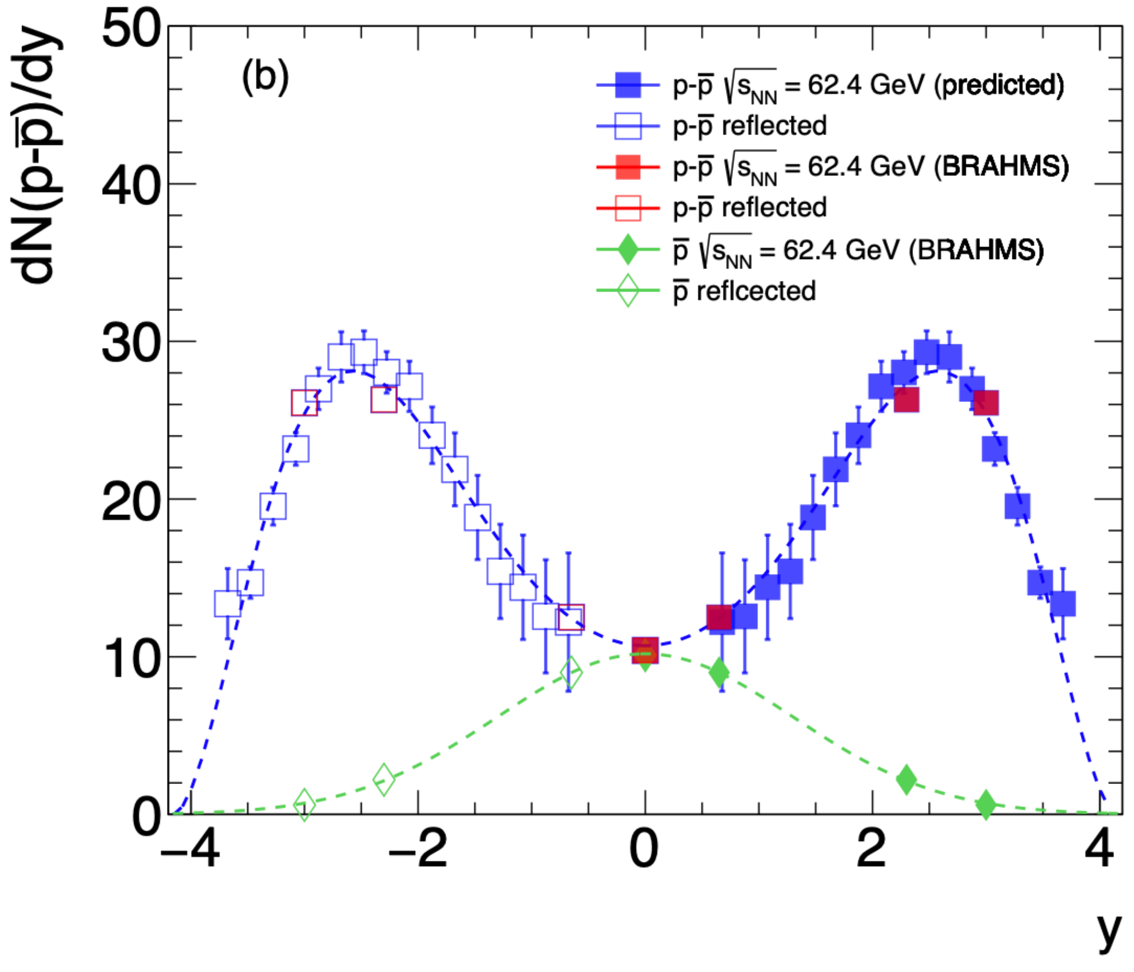
z - single baryon partition function



Fixing input parameters

$$Z_B(V,T) = \sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B(2z\sqrt{\lambda_B \lambda_{\bar{B}}})$$

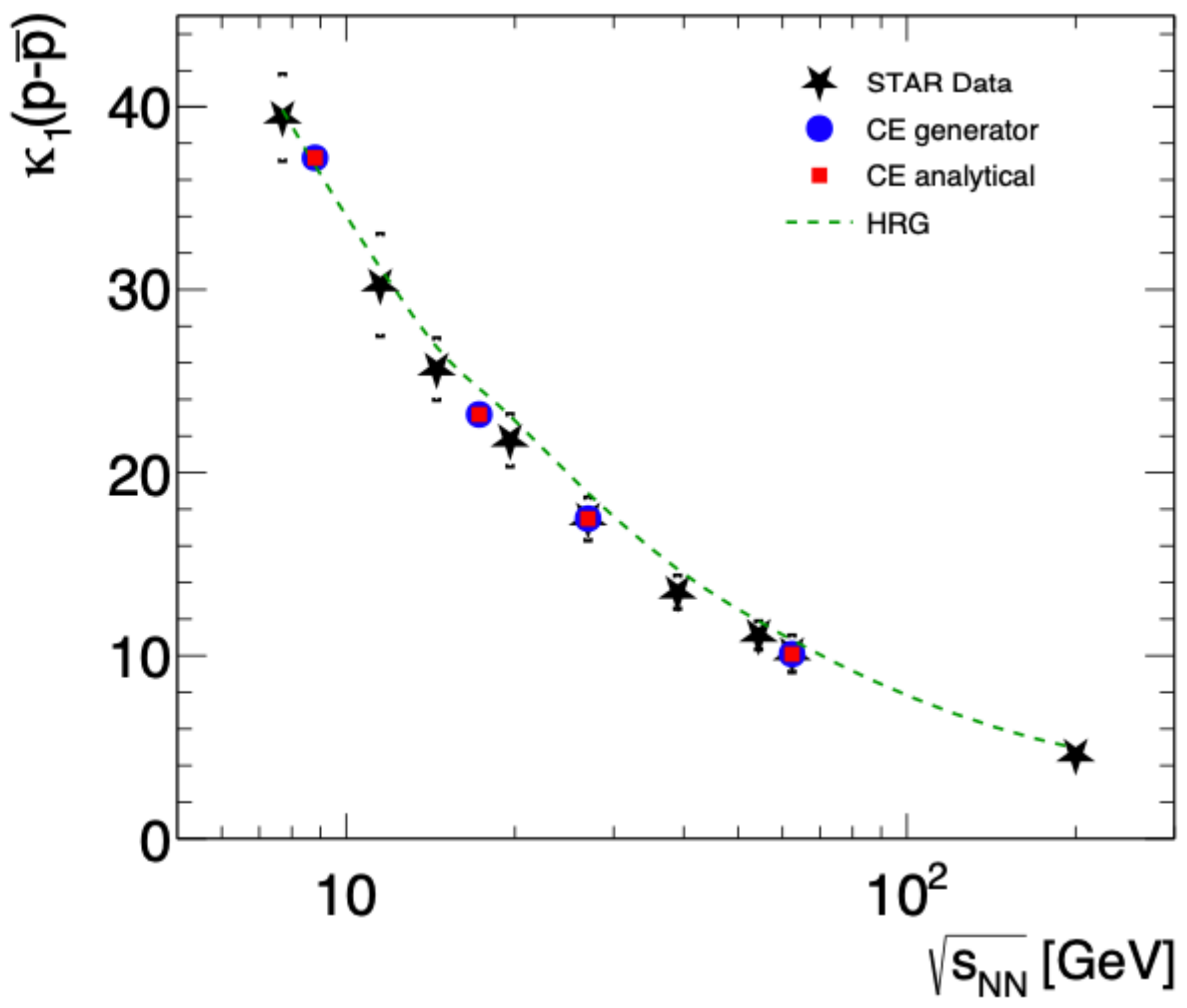




fixing B

- **baryon rapidity distributions**
- **measured mean multiplicities**

fixing $\alpha_{B,\bar{B}}$



fixing z

$z = \sqrt{z_B z_{\bar{B}}}$ **is calculated by solving**

$$\langle N_B \rangle = \lambda_B \frac{\partial \ln Z_B}{\partial \lambda_B}\bigg|_{\lambda_B,\,\lambda_{\bar{B}}\,=\,1} = z\frac{I_{B-1}(2z)}{I_B(2z)}$$

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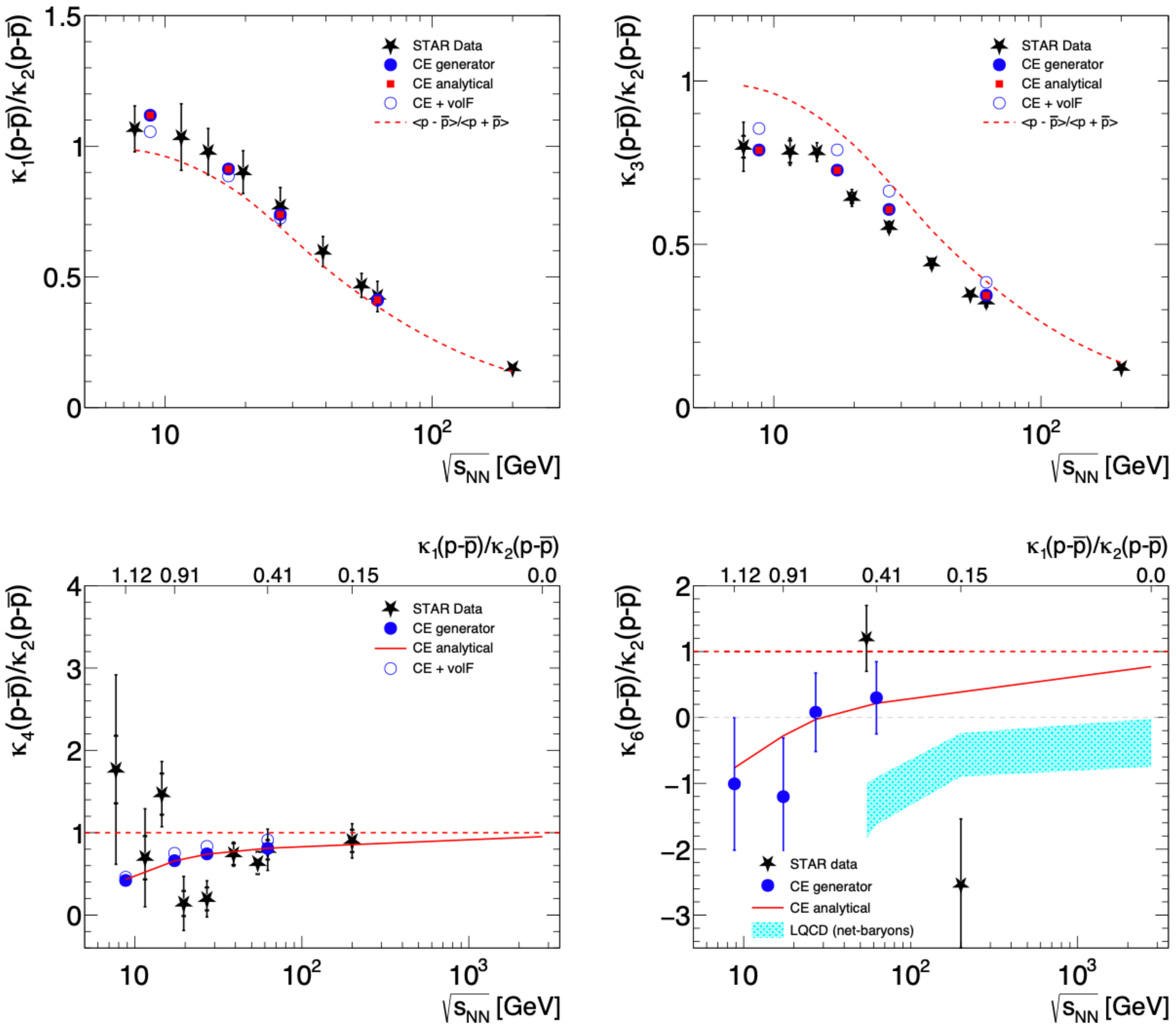
Comparison to OLD (BESI) STAR data



- **remarkable agreement between calculations and the STAR data is obvious**

artefact of a fixed acceptance in rapidity

for higher energies the ratios approach the HRG baseline

- **significant reduction of κ_6/κ_2 going from positive values at LHC to negative values at lower energies**

- **LQCD results for κ_6/κ_2 are negative for all energies (<u>for net-baryons</u>)**

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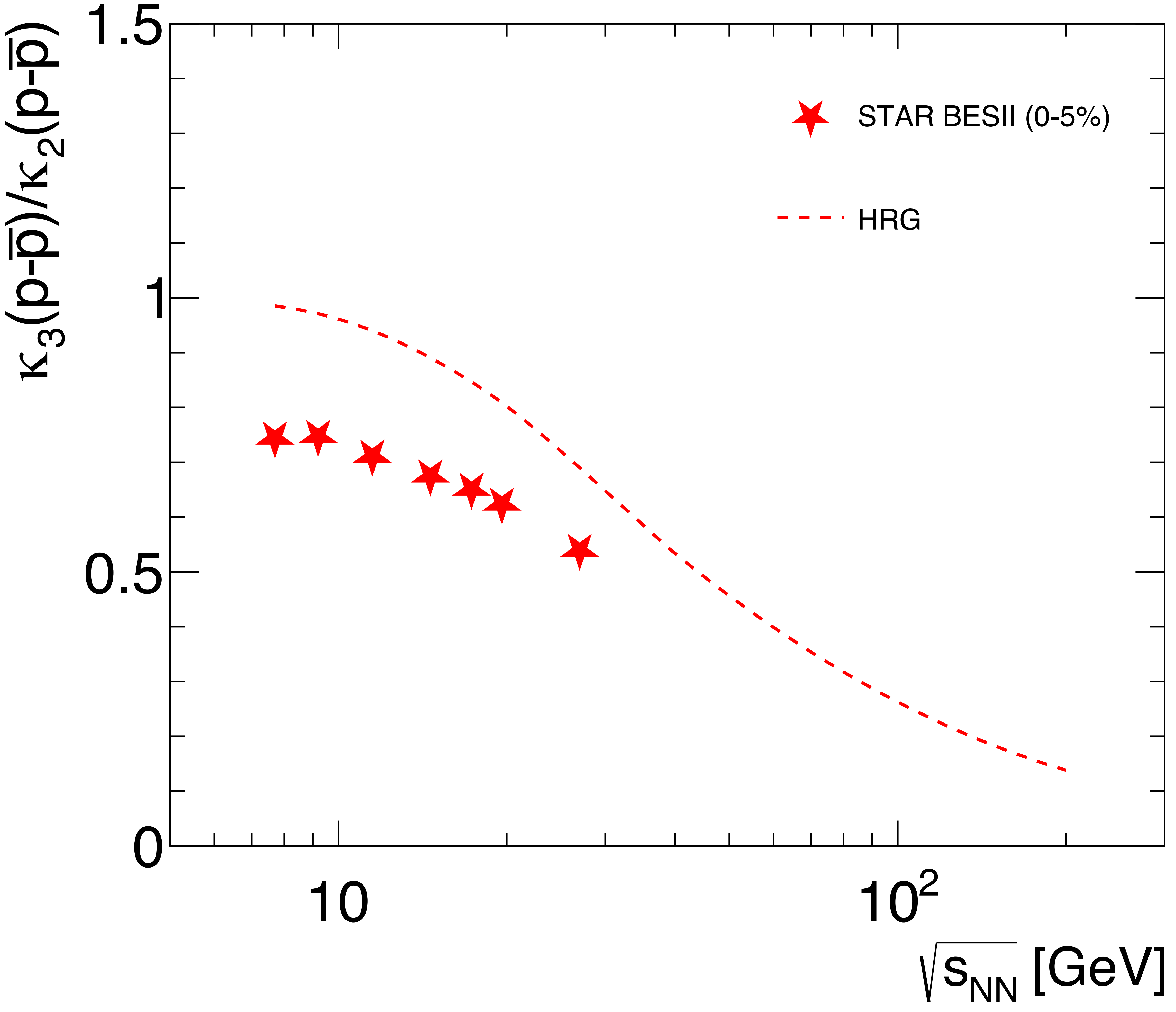
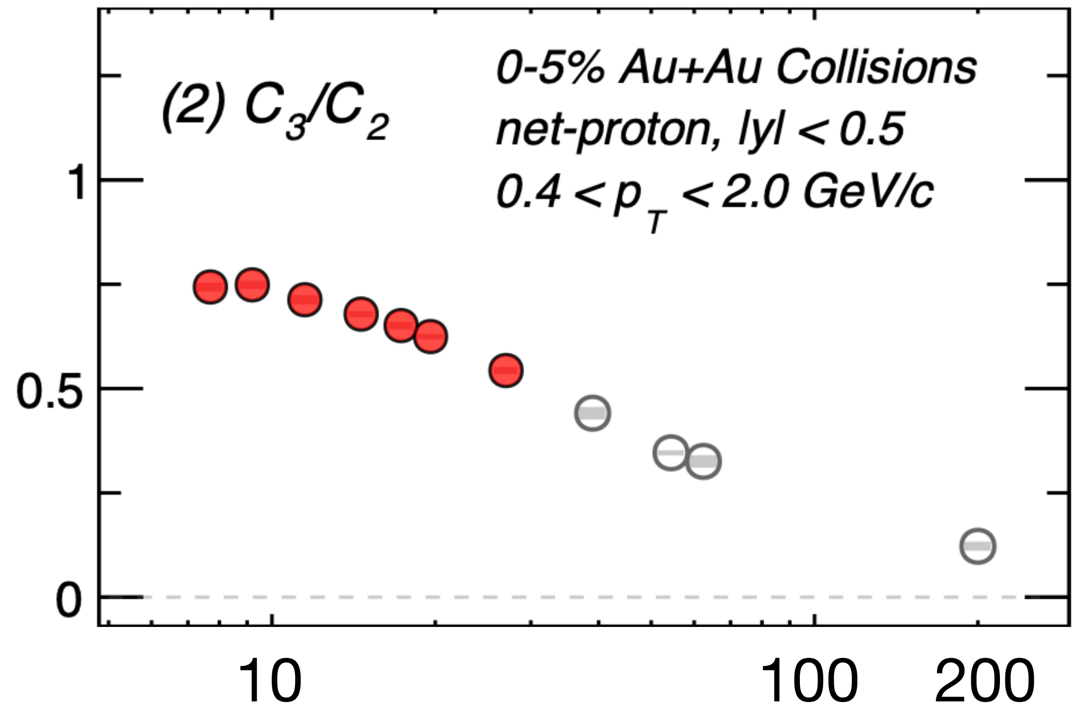
NEW (BESII) STAR DATA

κ_3/κ_2 of net-protons



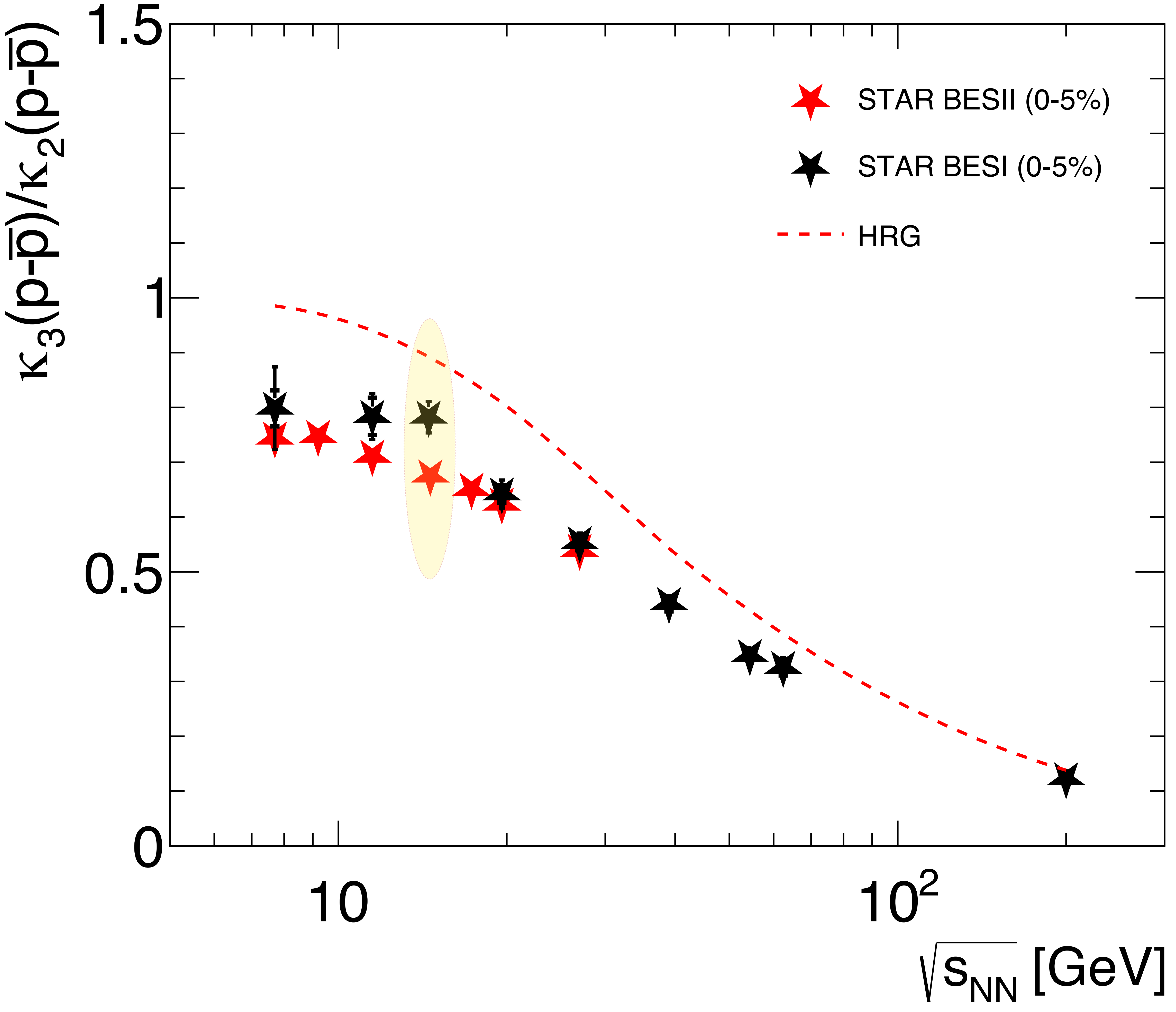
NEW STAR data points are digitised from the pdf plot!



A. Pandav, CPOD 2024

Notation: $C_i \rightarrow \kappa_i$

NEW vs. OLD STAR DATA, κ_3/κ_2



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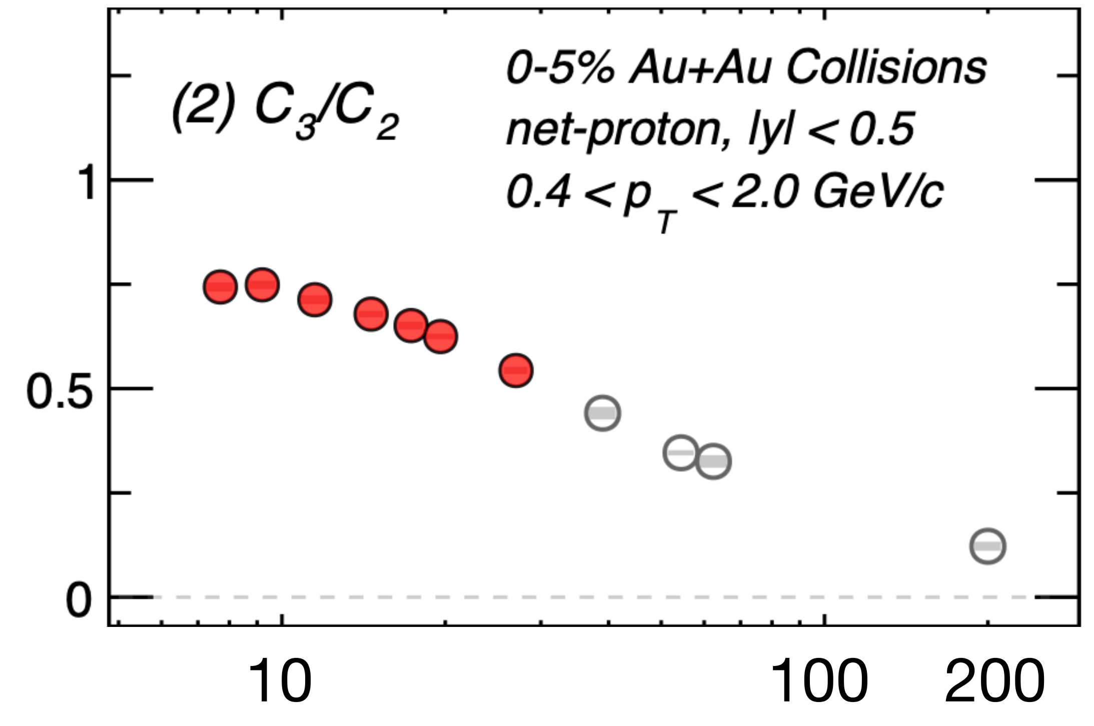
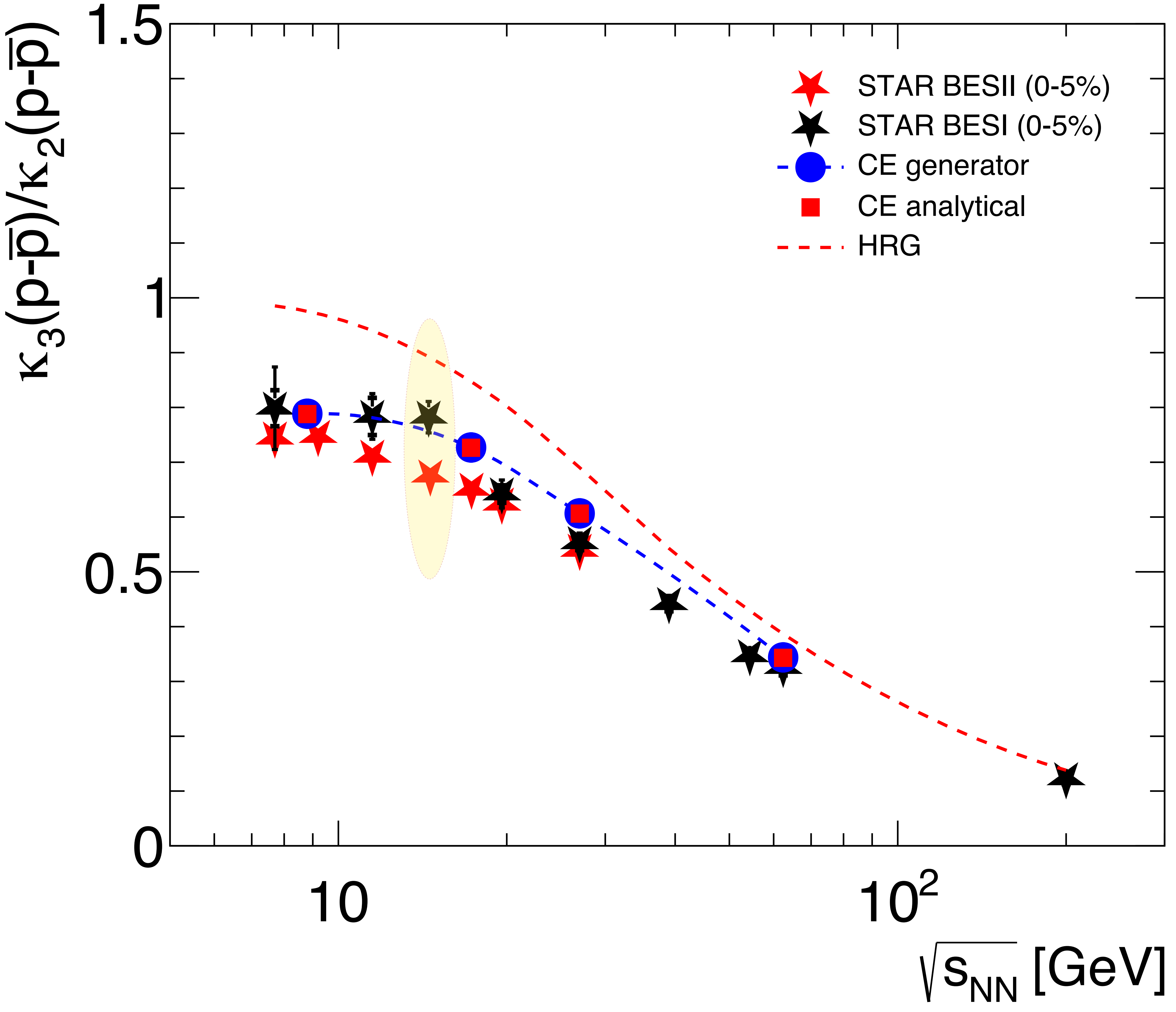


A. Pandav, CPOD 2024

Notation: $C_i \rightarrow \kappa_i$

- **The NEW data are systematically below the OLD ones**
  - **difference at 14.5 GeV is significant!**

OLD vs. NEW STAR DATA, κ_3/κ_2 (Comparison to CE baseline)



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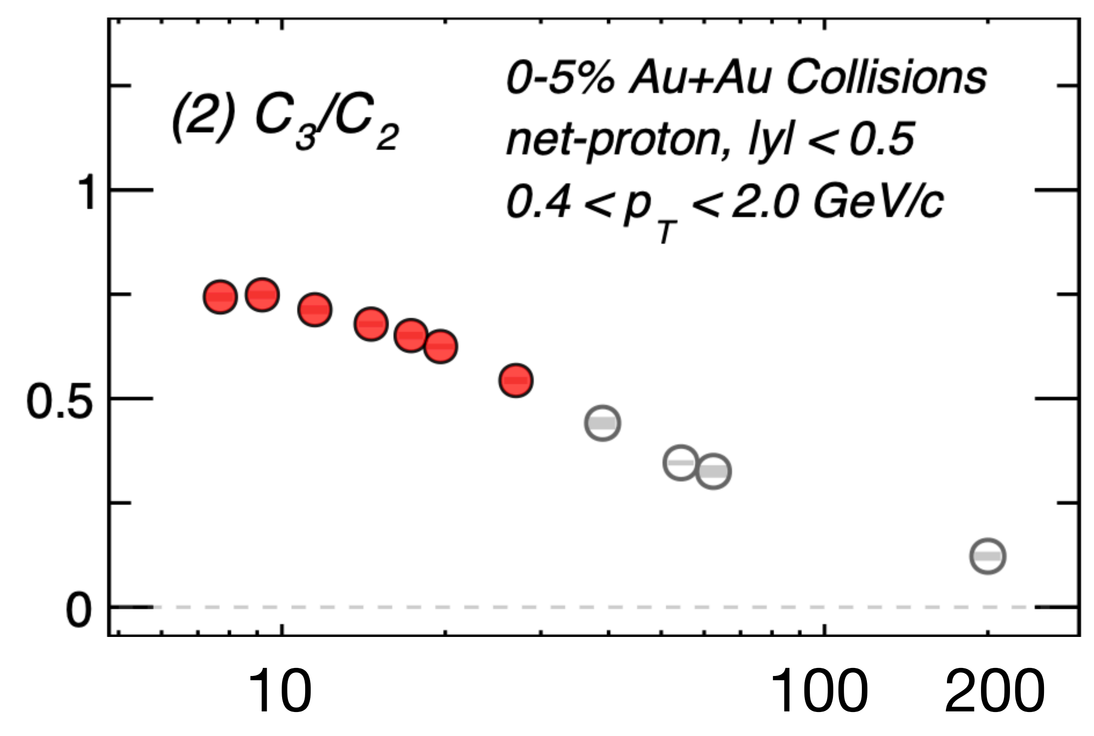


A. Pandav, CPOD 2024

Notation: $C_i \rightarrow \kappa_i$

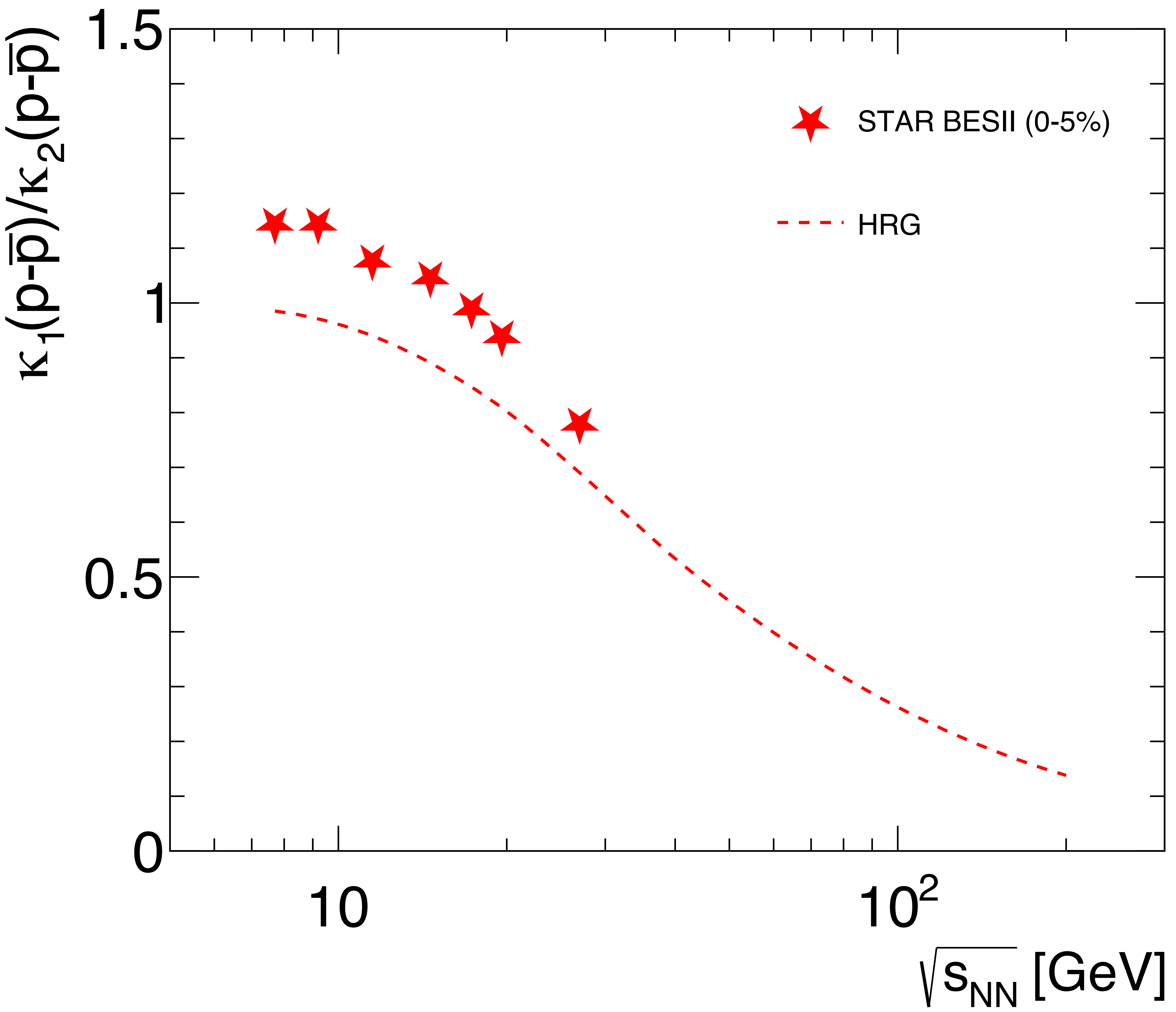
- **The NEW data are systematically below the OLD ones**
  - **difference at 14.5 GeV is significant!**
  - **systematically below the CE baseline**

See also: V. Vovchenko, V. Koch, Ch. Shen PRC 105 (2022), 1, 014904

NOTE: The baseline is calculated based on OLD (BESI) STAR multiplicities!

NEW (BESII) STAR DATA

κ_1/κ_2 of net-protons



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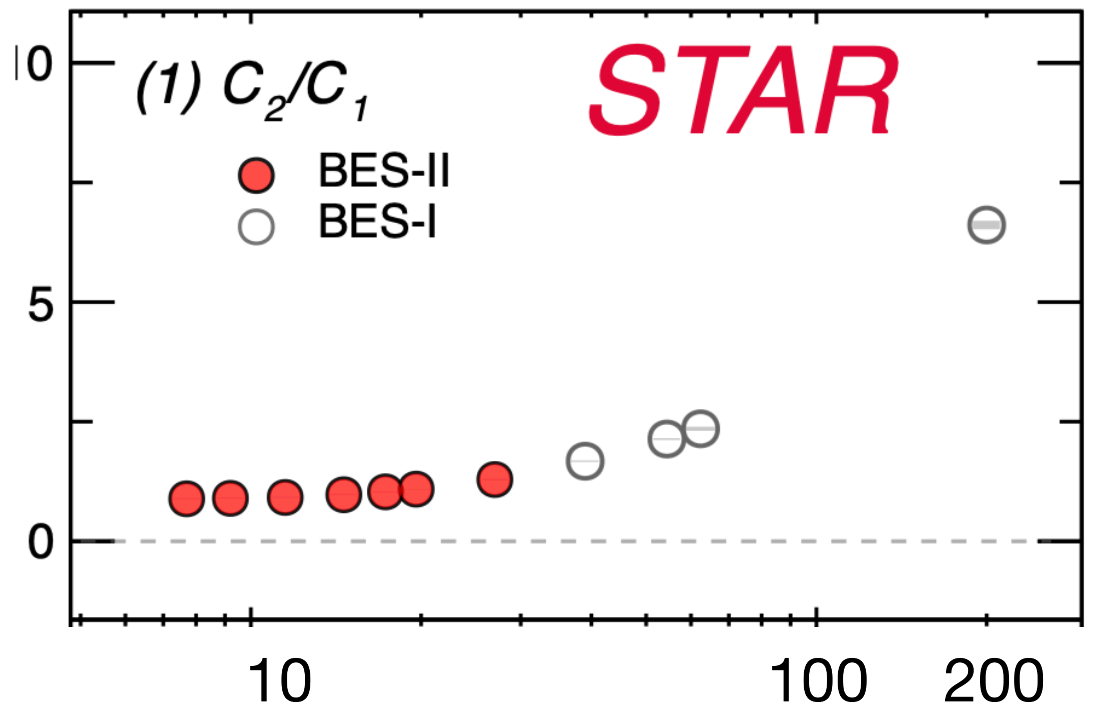
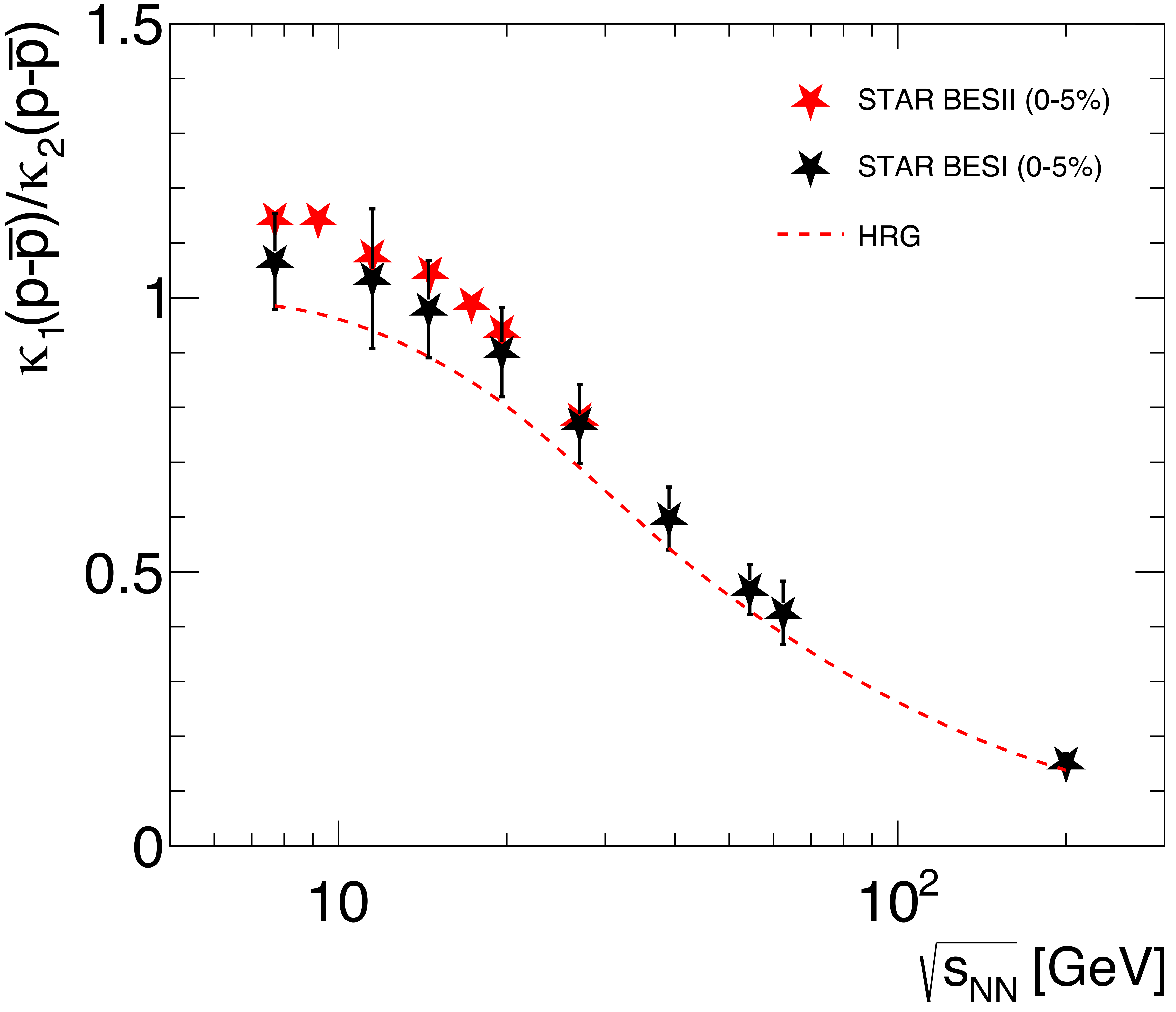


A. Pandav, CPOD 2024

Note: We prefer to plot C_1/C_2

Notation: $C_i \rightarrow \kappa_i$

OLD vs. NEW STAR DATA, κ_1/κ_2



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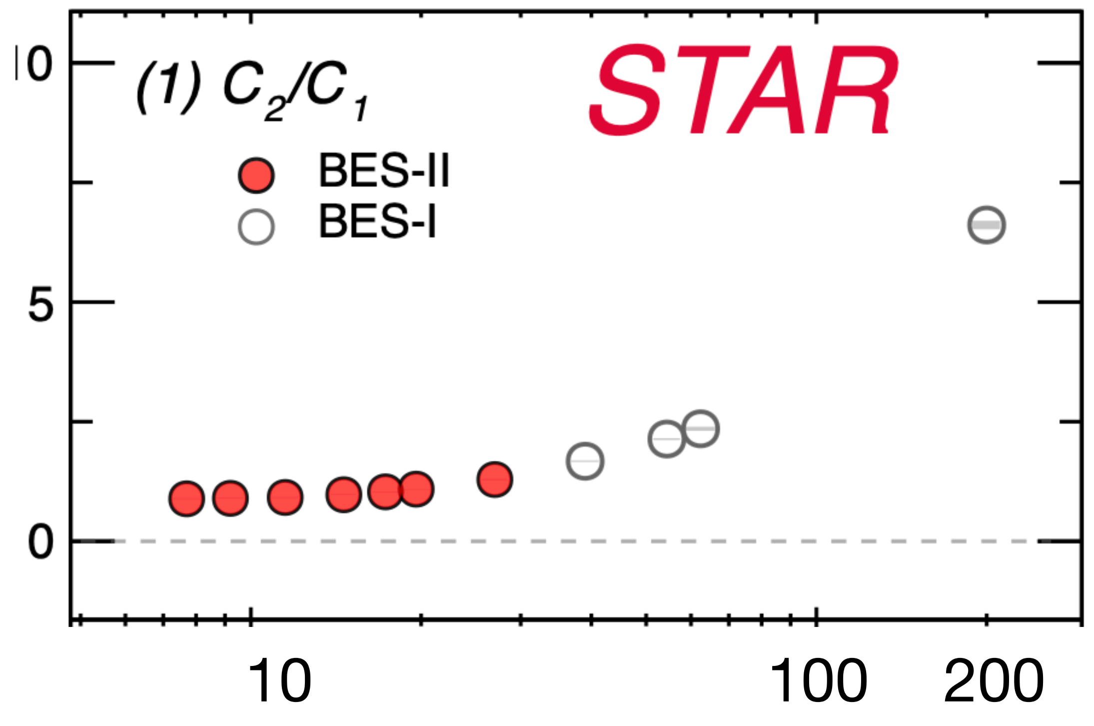


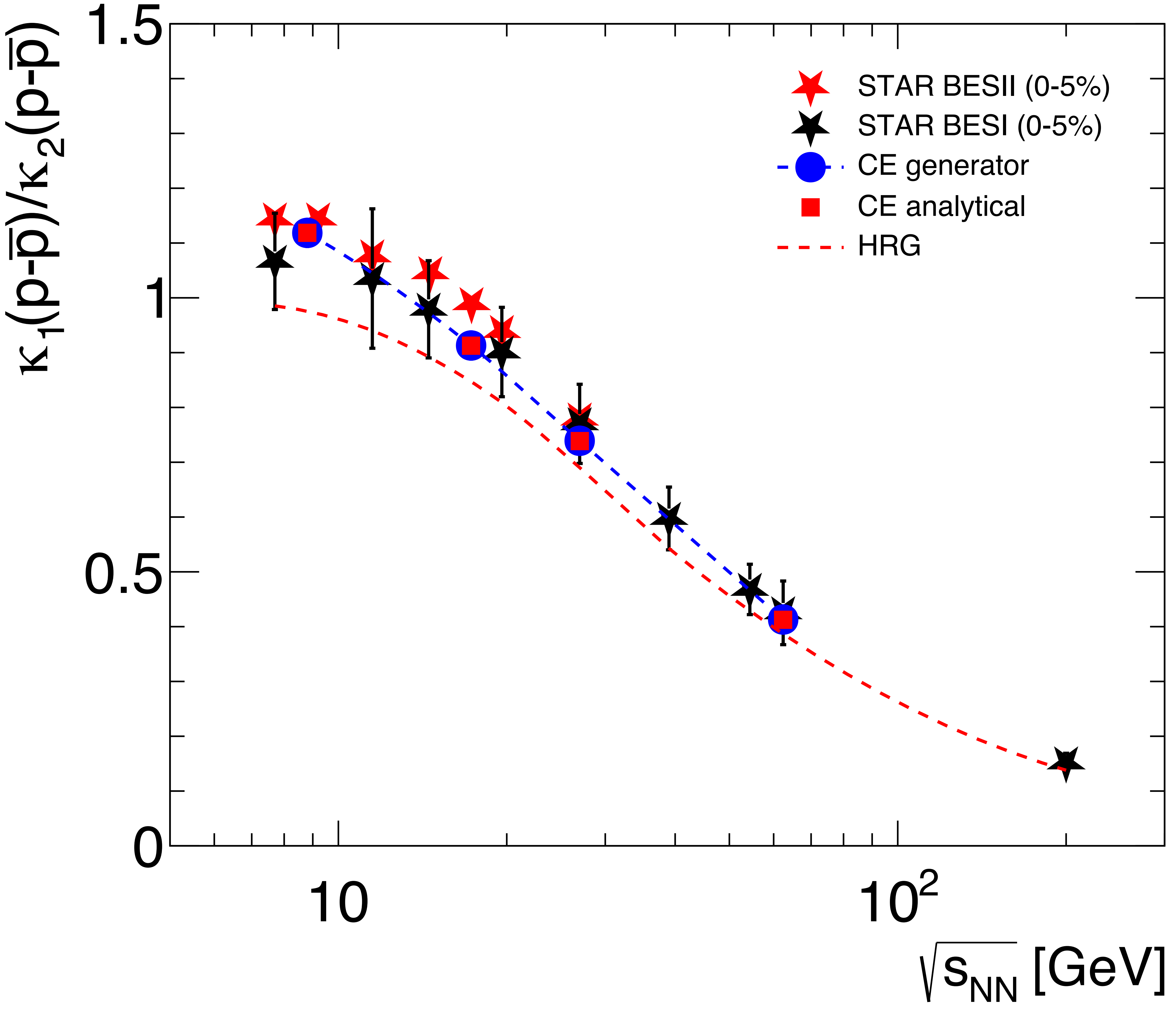
A. Pandav, CPOD 2024

Note: We prefer to plot C_1/C_2

Notation: $C_i \rightarrow \kappa_i$

- **The NEW data are systematically <u>above</u> the OLD ones**

OLD vs. NEW STAR DATA, κ_1/κ_2 (Comparison to CE baseline)



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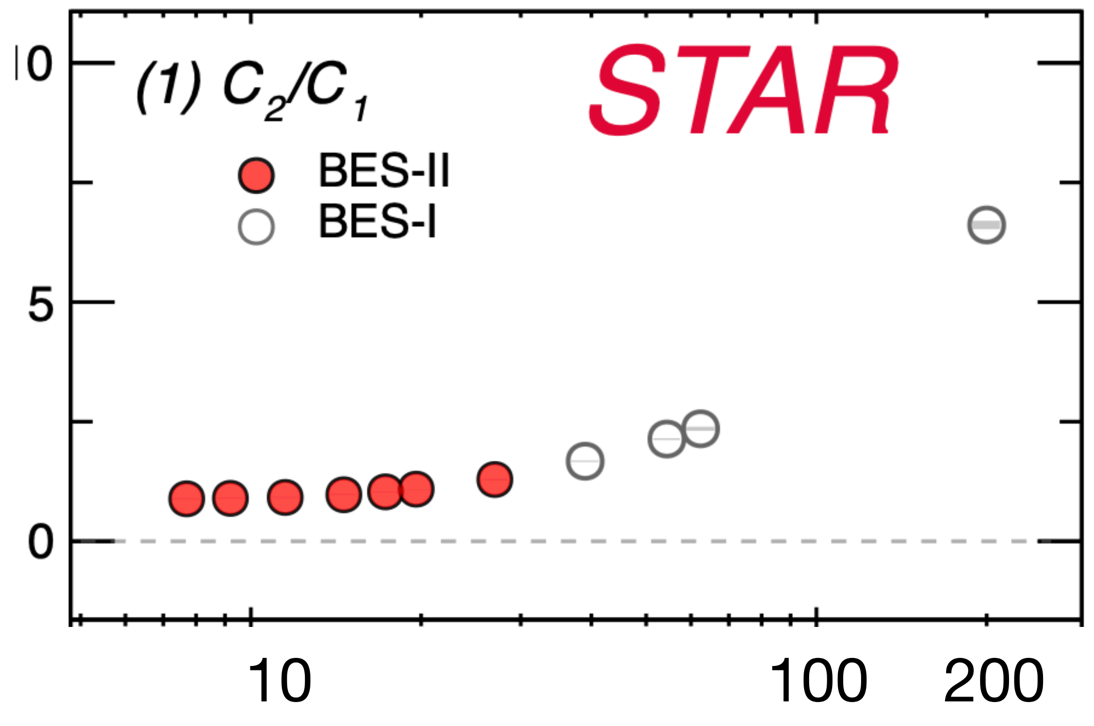


A. Pandav, CPOD 2024

Note: We prefer to plot C_1/C_2

Notation: $C_i \rightarrow \kappa_i$

- **The NEW data are systematically <u>above</u> the OLD ones**
  - **systematically <u>above</u> the CE baseline**

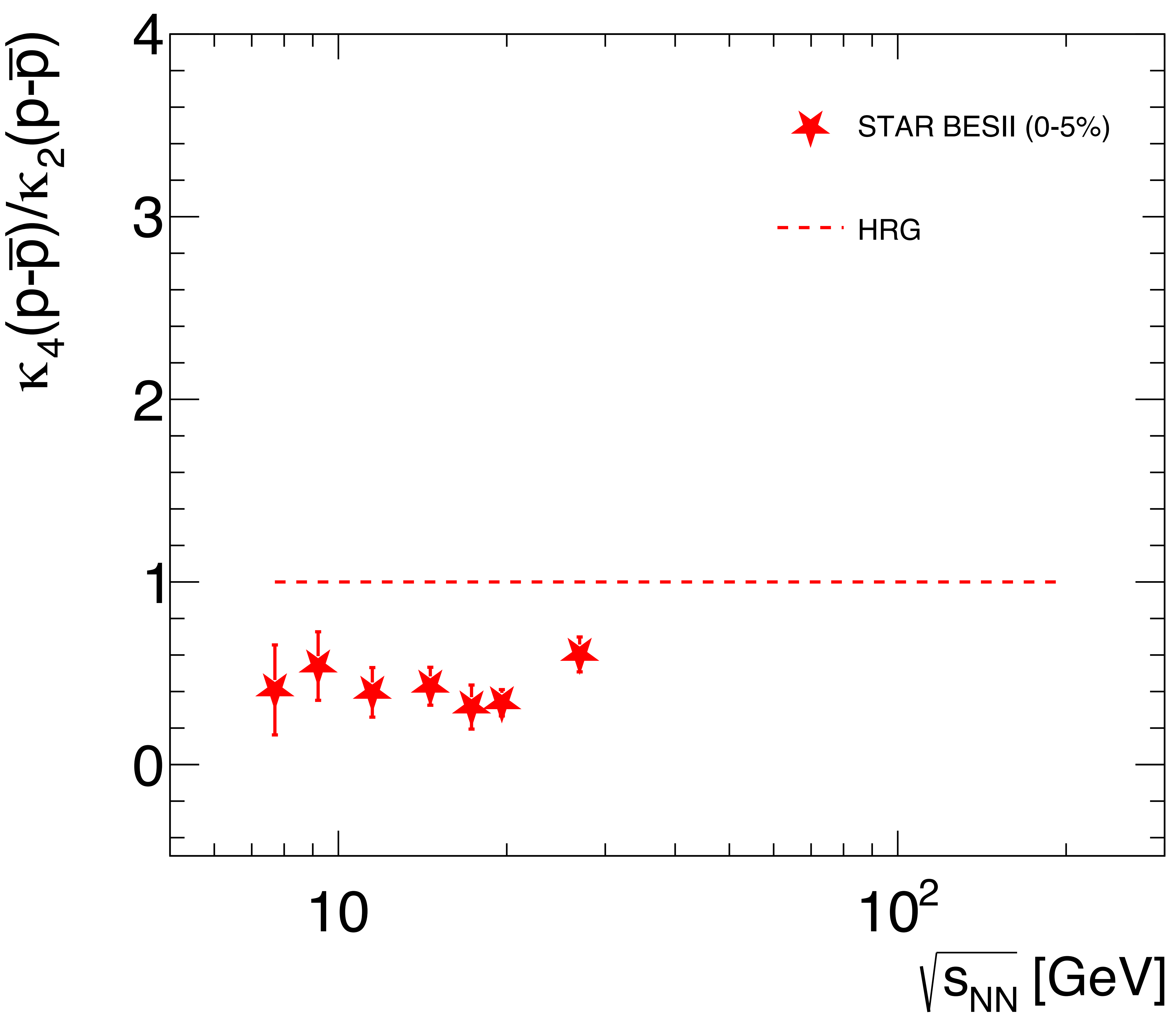
See also: V. Vovchenko, V. Koch, Ch. Shen PRC 105 (2022), 1, 014904

NOTE: The baseline is calculated based on OLD (BESI) STAR multiplicities!

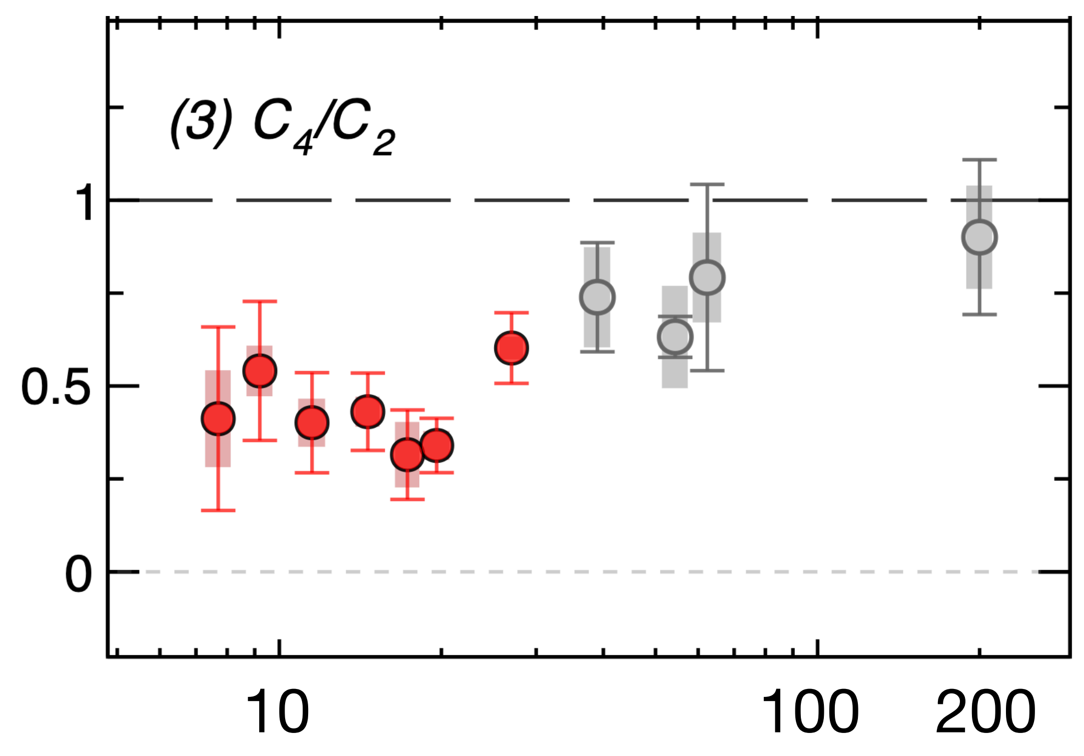
NEW (BESII) STAR DATA

κ_4/κ_2 of net-protons

NEW STAR DATA, κ_4/κ_2



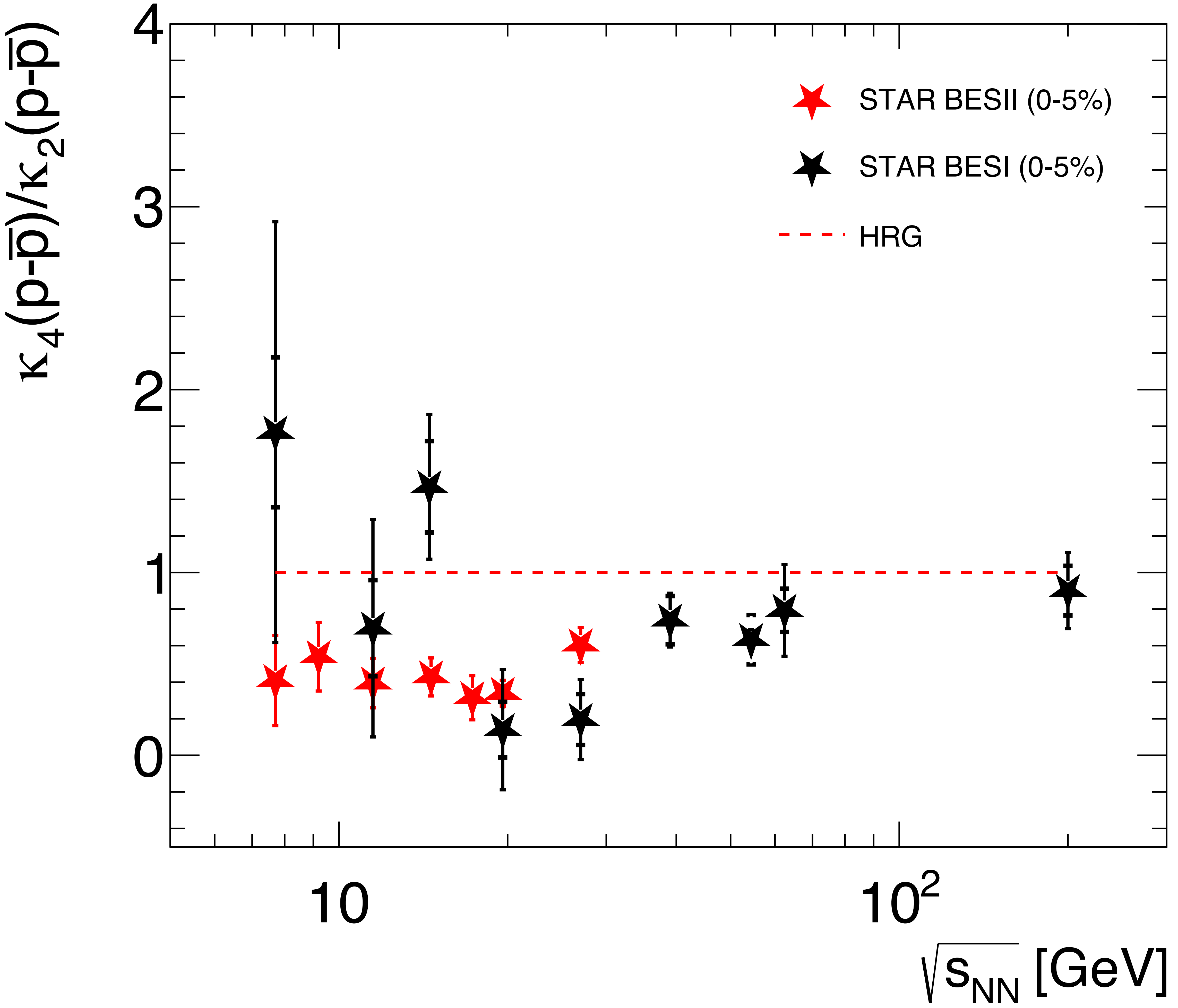
NEW STAR data points are digitised from the pdf plot!



A. Pandav, CPOD 2024

Notation: $C_i \rightarrow \kappa_i$

OLD vs. NEW STAR DATA, κ_4/κ_2



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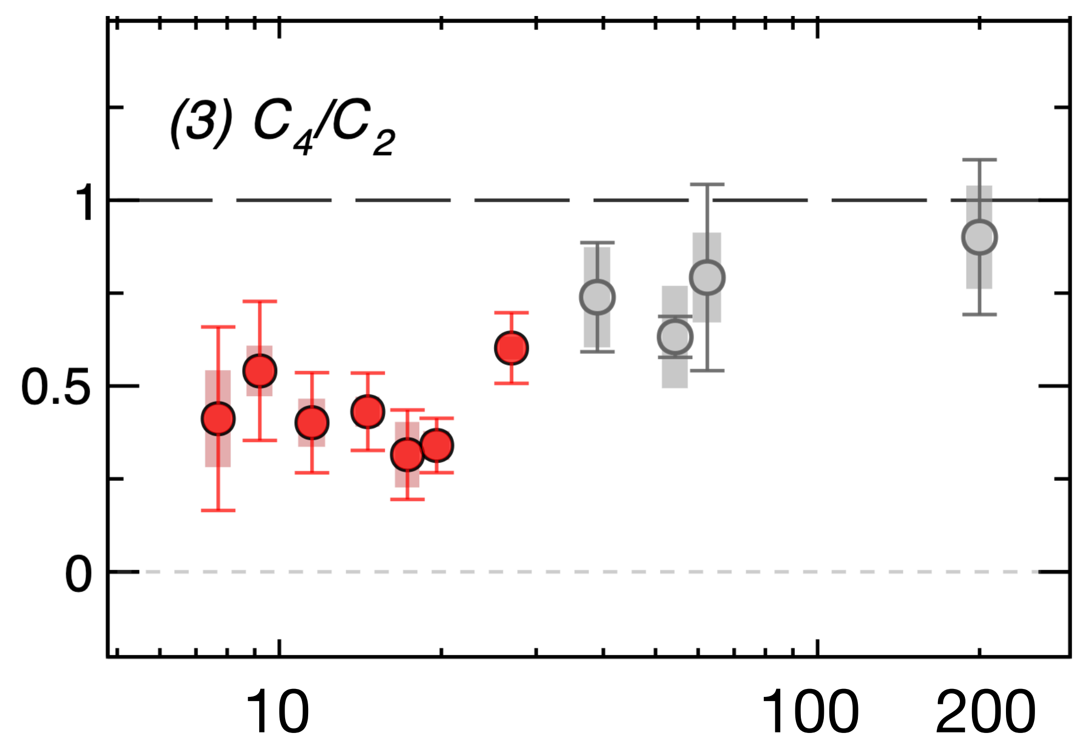


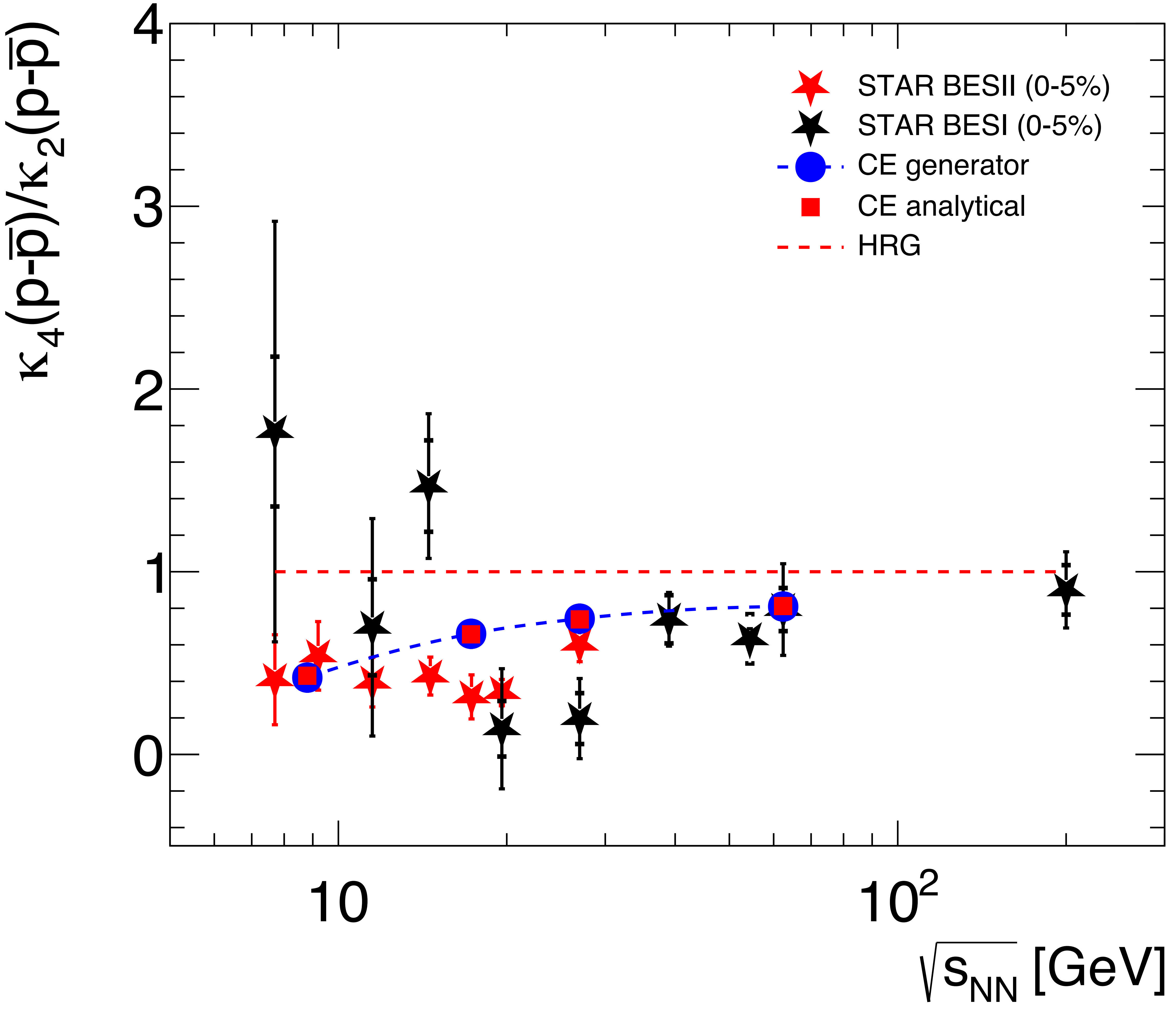
A. Pandav, CPOD 2024

Note: We prefer to plot C_1/C_2

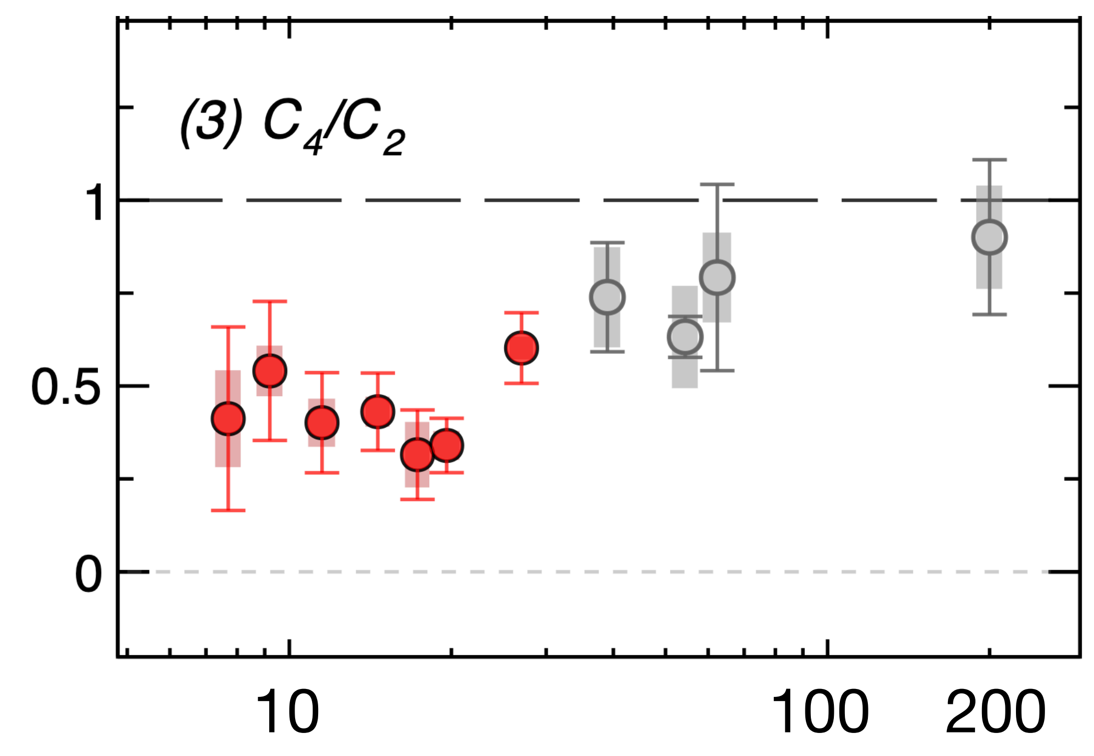
Notation: $C_i \rightarrow \kappa_i$

- **The NEW data with significantly reduced uncertainties**

OLD vs. NEW STAR DATA, κ_4/κ_2 (Comparison to CE baseline)



NEW STAR data points are digitised from the pdf plot!



A. Pandav, CPOD 2024

Note: We prefer to plot C_1/C_2

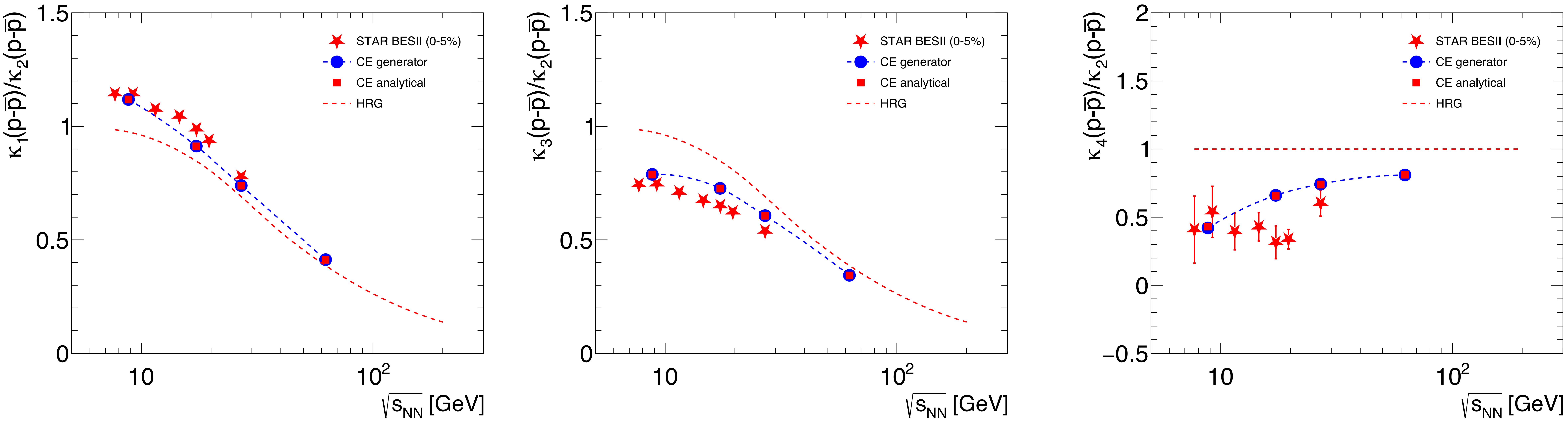
Notation: $C_i \rightarrow \kappa_i$

- **The NEW data with significantly reduced uncertainties**
  - **deviation from the CE baseline is more significant**

See also: V. Vovchenko, V. Koch, Ch. Shen PRC 105 (2022), 1, 014904

NOTE: The baseline is calculated based on OLD (BESI) STAR multiplicities!

NEW STAR data, cumulants of net-protons (summary)

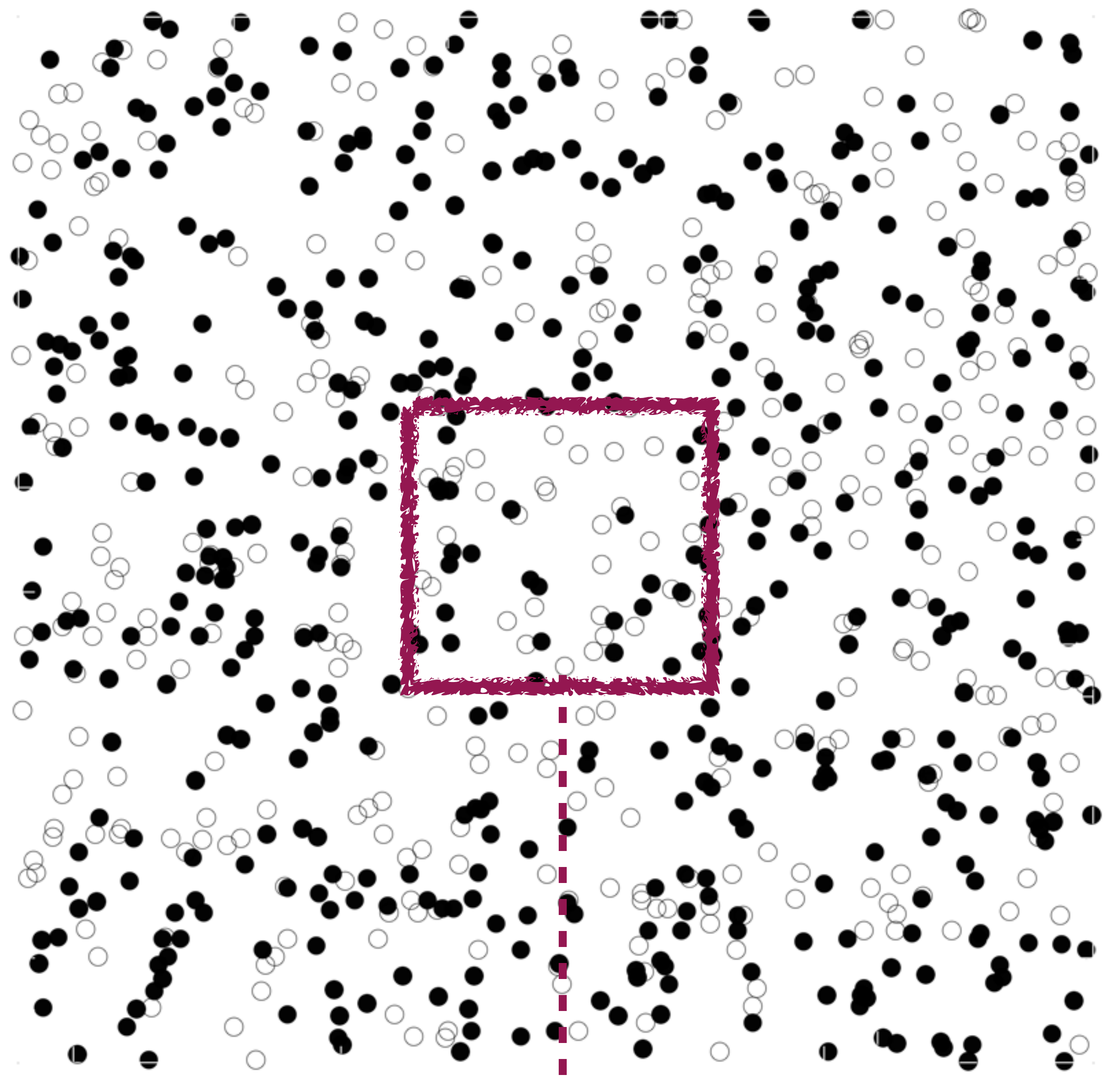


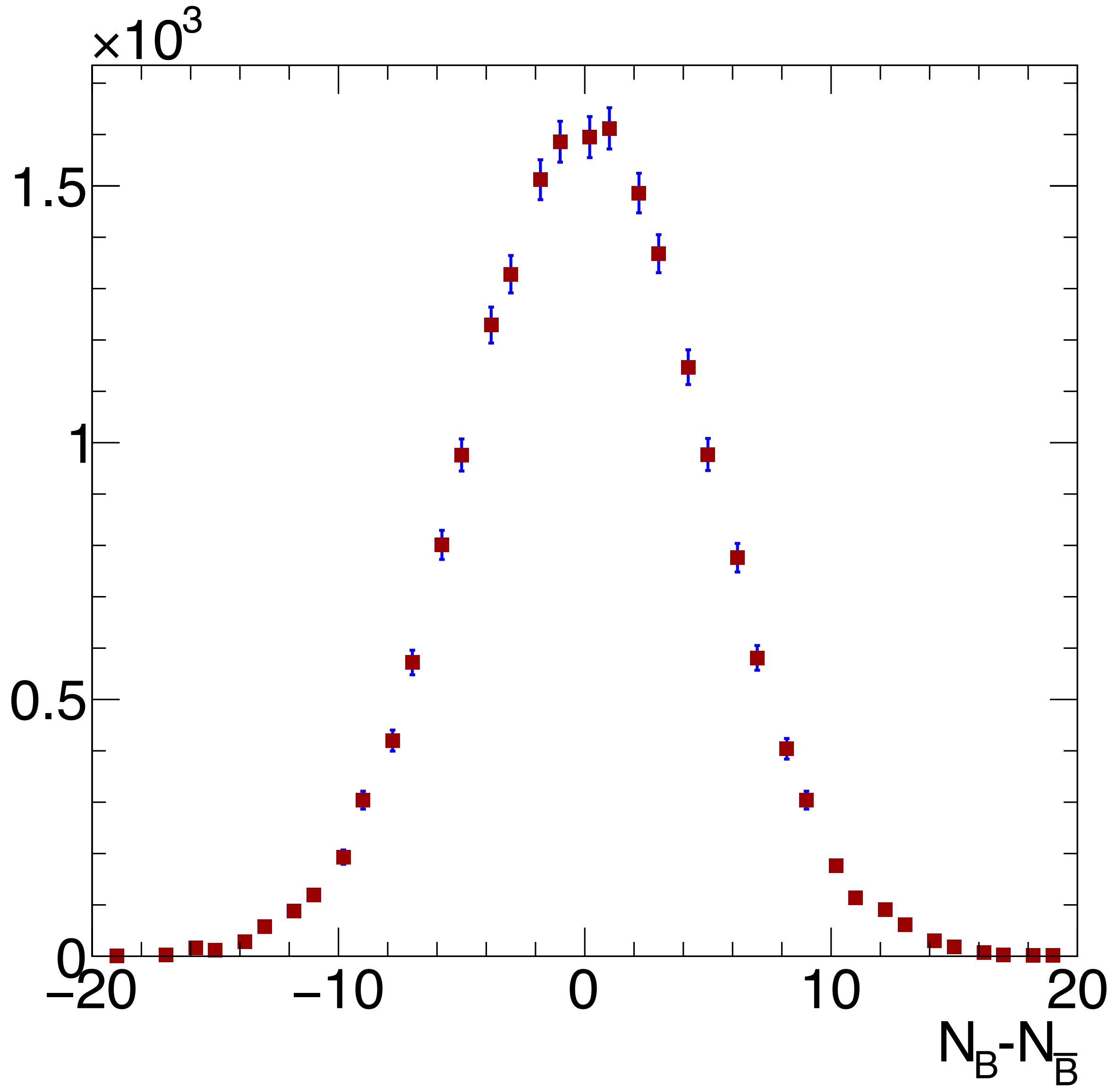
Canonical baselines show systematic deviations from NEW (BESII) STAR data

NOTE: The baseline is calculated based on OLD (BESI) STAR multiplicities!

Introducing local correlations

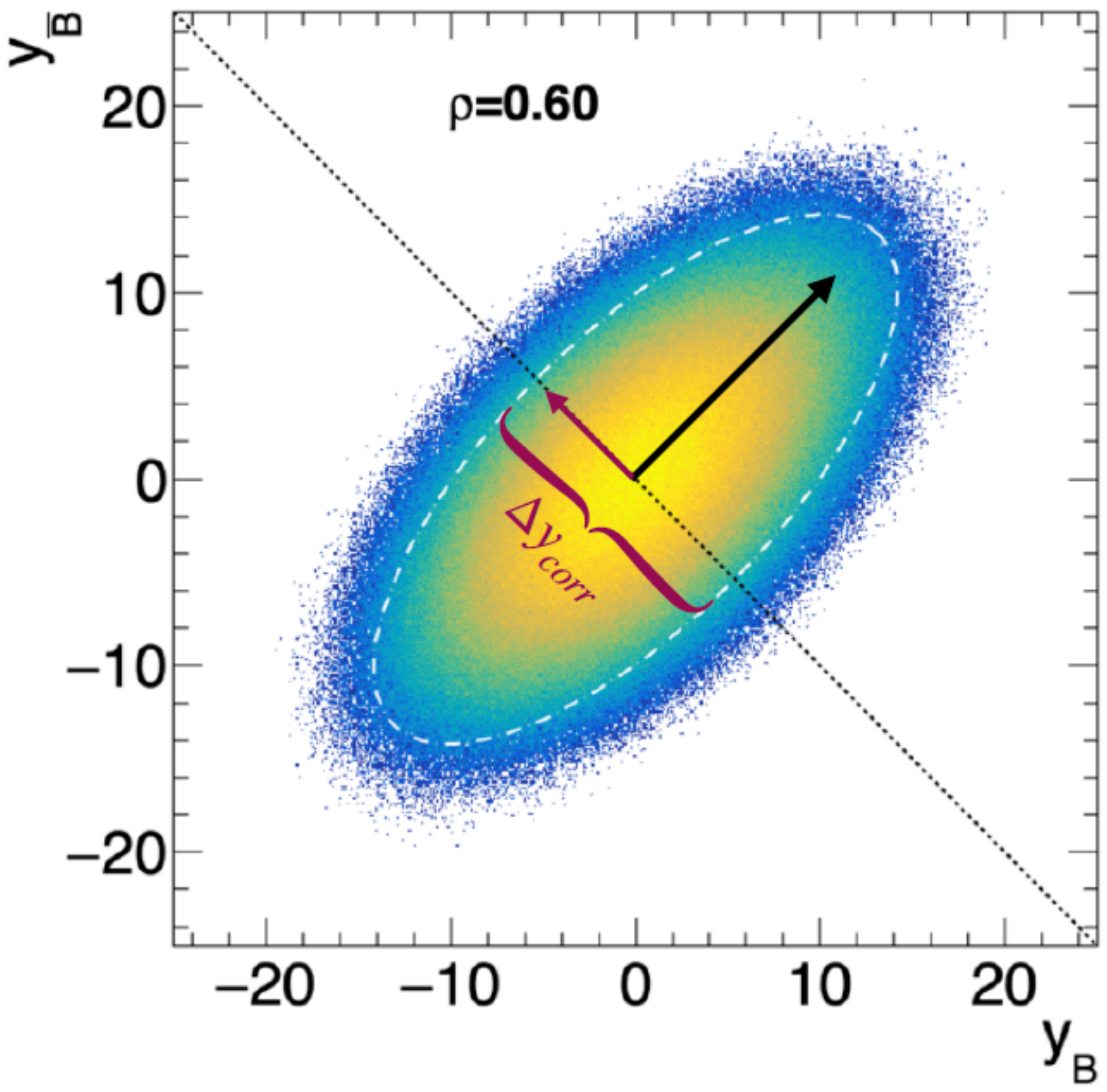
Implementation of local correlations





- exploiting **C**anonical **E**nsemble in the full phase space
  - no fluctuations in 4π (like in experiments)

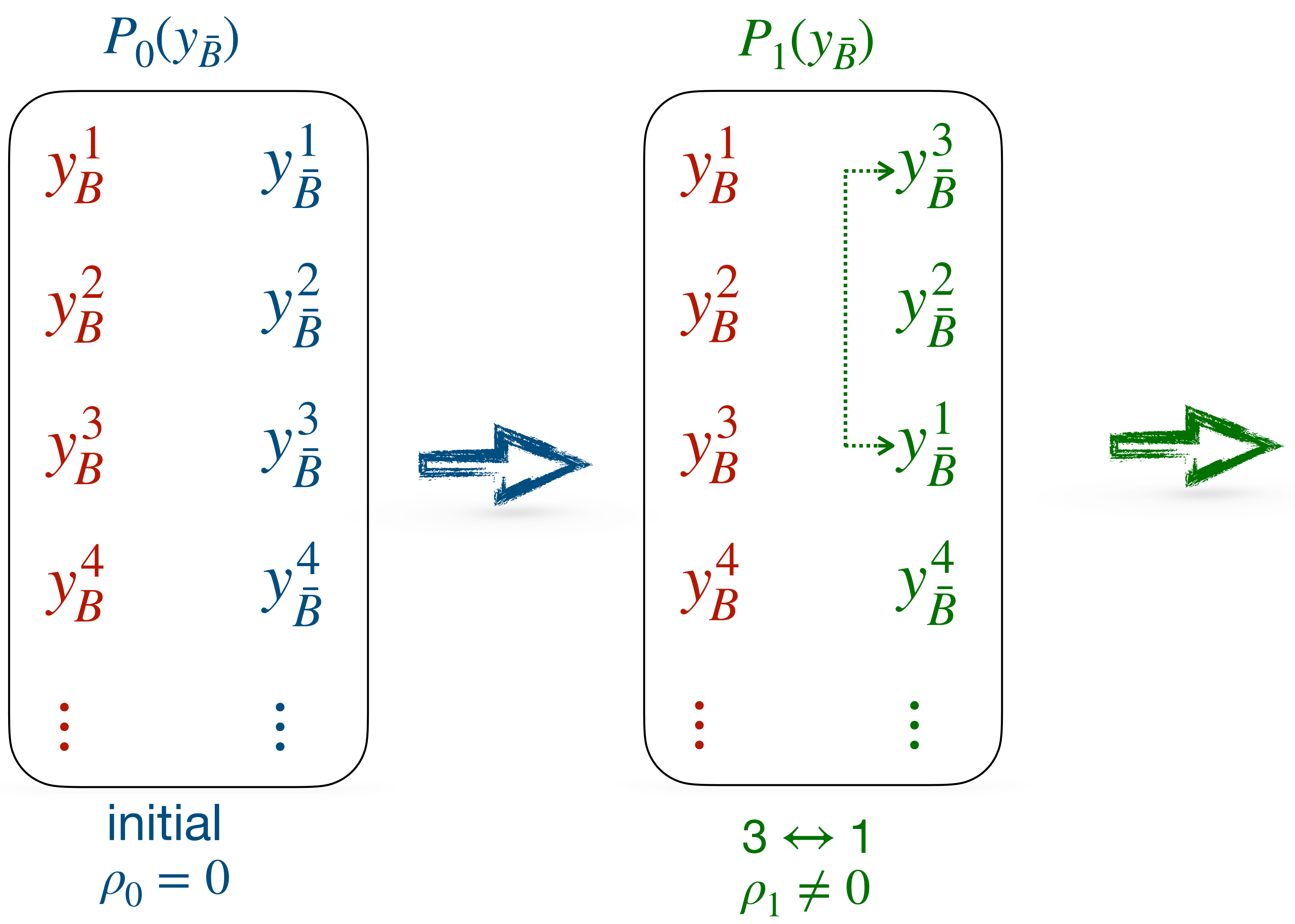
Local conservations: correlations in rapidity space



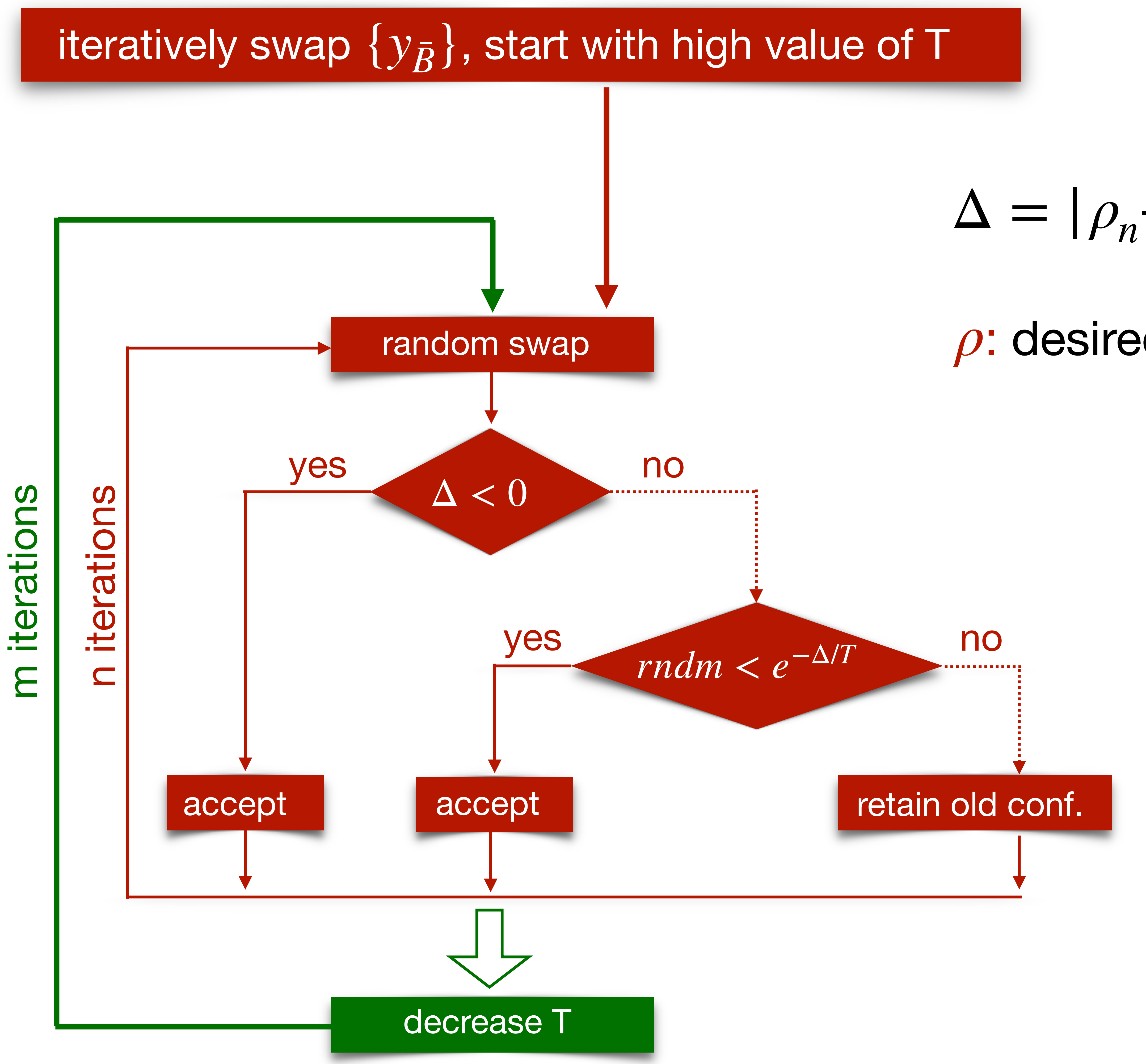
P. Braun-Munzinger, K. Redlich, A.R., J. Stachel, e-Print: 2312.15534 [nucl-th]

Correlations and the Metropolis algorithm

start with uncorrelated $\{y_B\}$, $\{y_{\bar{B}}\}$



$$\rho_n = \frac{cov[y_B, P_n(y_{\bar{B}})]}{\sigma_{y_B}\sigma_{y_{\bar{B}}}}$$



stop if accepted rate is too small (< 1%)

$$\Delta = |\rho_n - \rho| - |\rho_{n-1} - \rho|$$

ρ: desired corr. coefficient

P. Braun-Munzinger, K. Redlich, A.R., J. Stachel, e-Print: 2312.15534 [nucl-th]

Canonical Ensemble + correlations

$$Z_B(V,T) = \sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B(2z\sqrt{\lambda_B \lambda_{\bar{B}}})$$

B net baryon number, conserved in each event

I_B modified Bessel function of the first kind

$z_B, z_{\bar{B}}$ single particle partition functions for baryons, anti baryons

$\lambda_B, \lambda_{\bar{B}}$ auxiliary parameters for calculating cumulants of baryons, anti baryons

baryon number conservation (CE partition function) + **correlations in y space** (Metropolis algorithm) →

$N_B, N_{\bar{B}}, y_B, y_{\bar{B}}$ event 1

$N_B, N_{\bar{B}}, y_B, y_{\bar{B}}$ event 2

… …

$N_B, N_{\bar{B}}, y_B, y_{\bar{B}}$ event n

Input from experiments

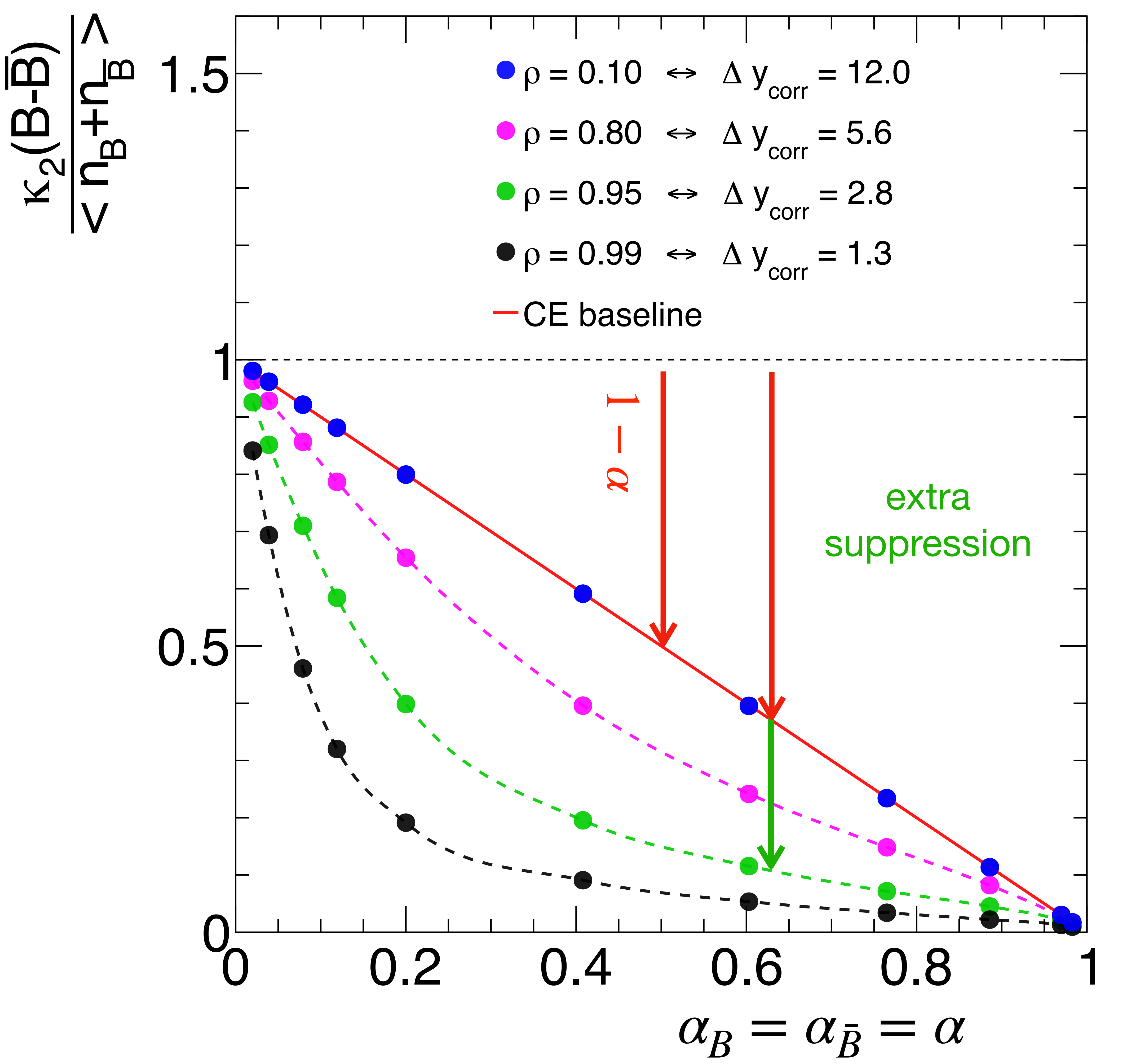
- baryon rapidity distributions
- measured (canonical) $\langle N_B \rangle$, $\langle N_{\bar{B}} \rangle$

$z = \sqrt{z_B z_{\bar{B}}}$ **is calculated by solving**

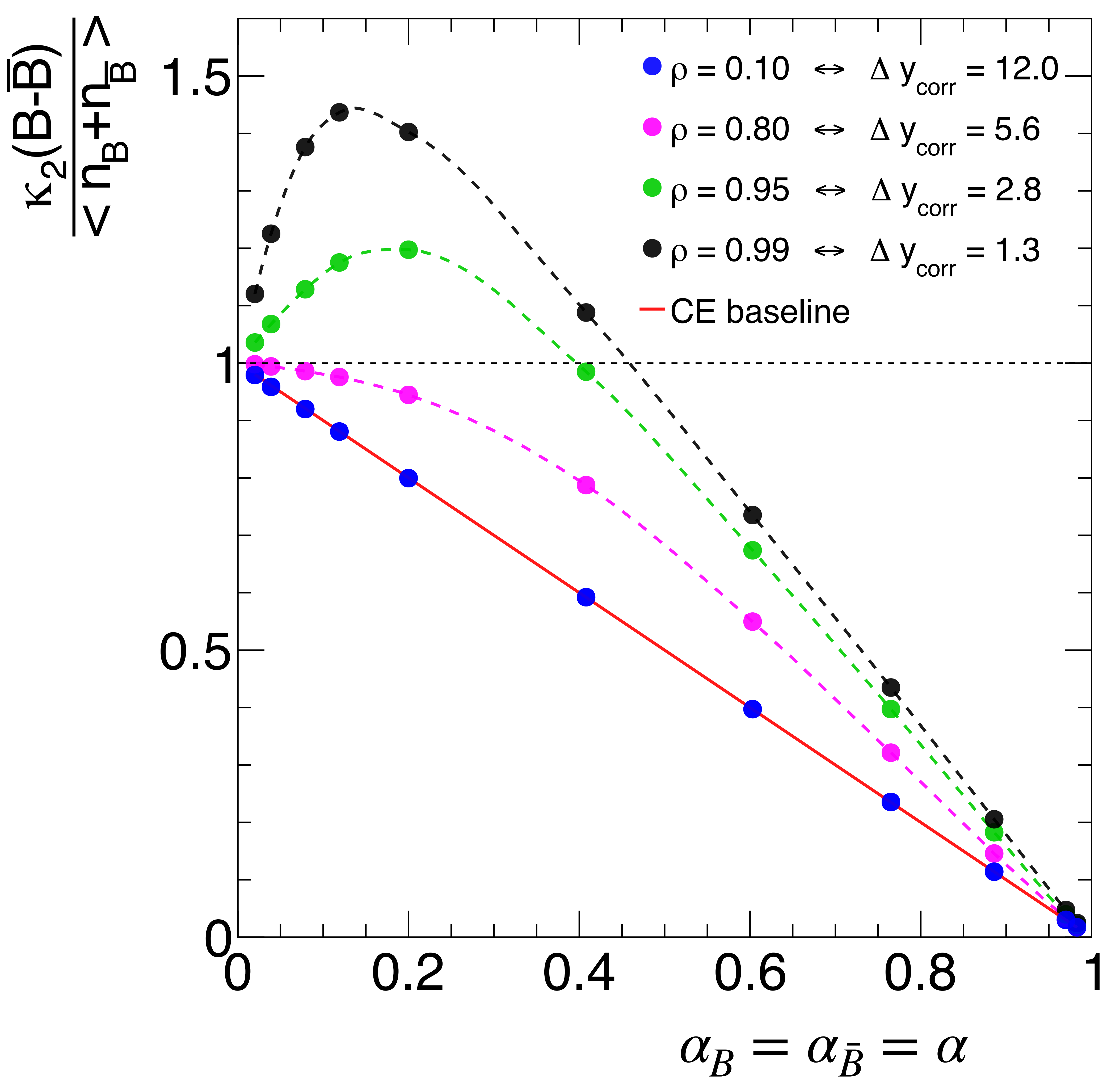
$$\langle N_B \rangle = \lambda_B \frac{\partial \ln Z_B}{\partial \lambda_B}\bigg|_{\lambda_B, \lambda_{\bar{B}} = 1} = z\frac{I_{B-1}(2z)}{I_B(2z)}$$

Results on $B-\bar{B}$, $B-B$ and $\bar{B}-\bar{B}$ correlations, ALICE energy

Correlations between $B-\bar{B}$



Correlations between $B-B$ and $\bar{B}-\bar{B}$



P. Braun-Munzinger, K. Redlich, A.R., J. Stachel, e-Print: 2312.15534 [nucl-th]

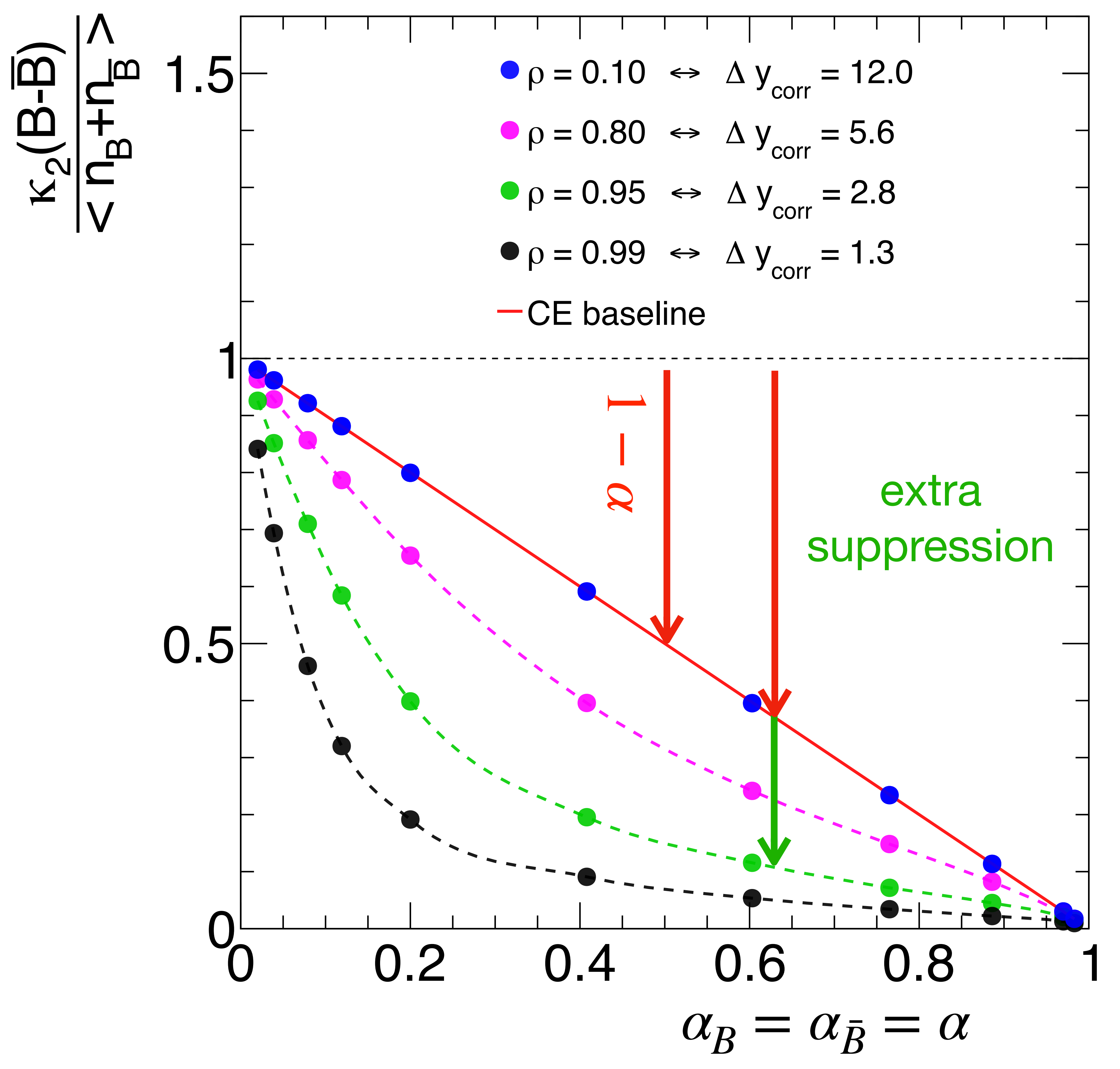
correlations between like-sign particles leads to cluster formation. Fluctuations increase.

Canonical baselines vs. ALICE data

P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

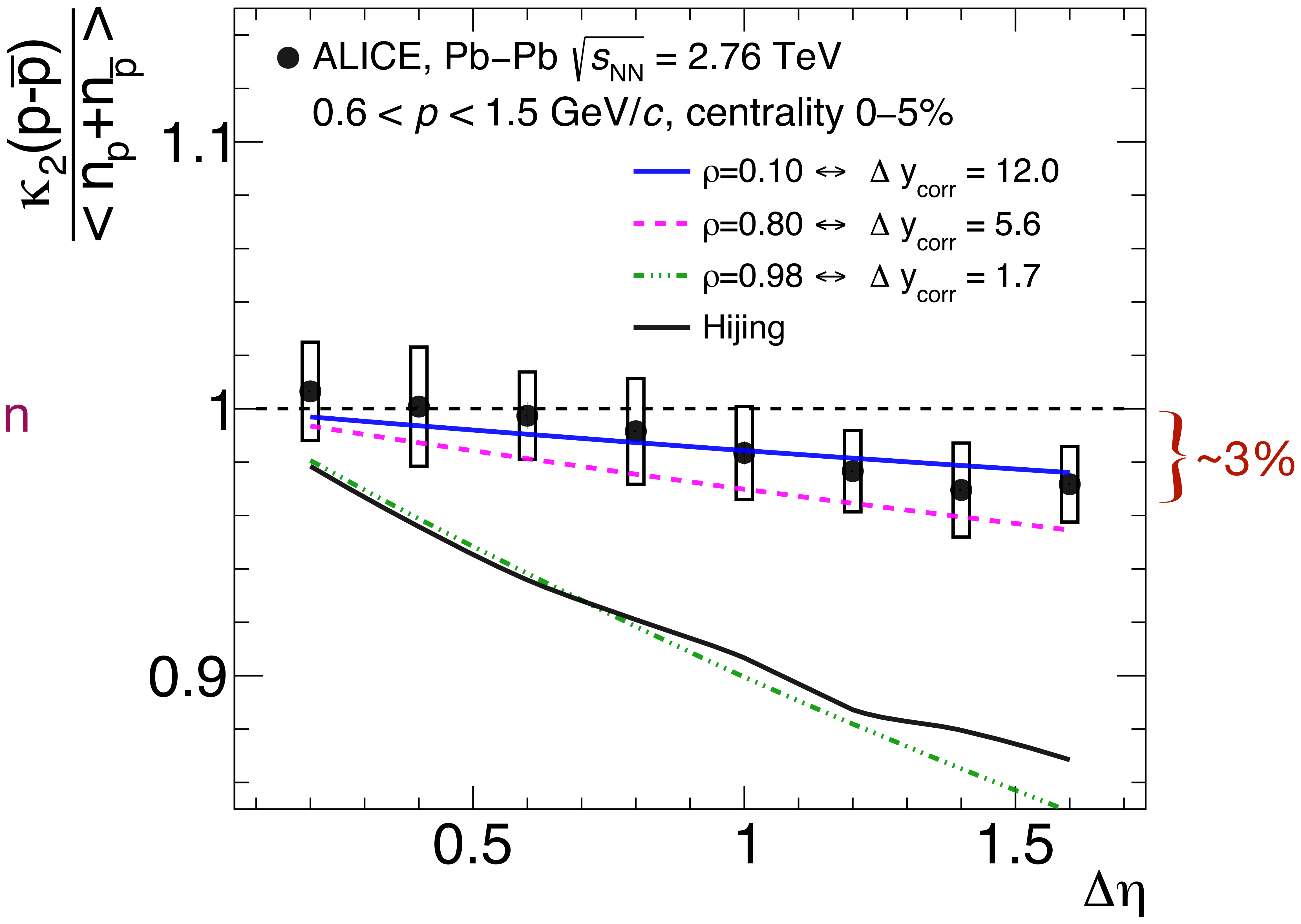
P. Braun-Munzinger, K. Redlich, A.R., J. Stachel, e-Print: 2312.15534 [nucl-th]

A.R., NPA 967 (2017) 453-456 **ALICE:** Phys. Lett. B 807 (2020) 135564; Phys. Lett. B (2022) 137545



A.R., P. Braun-Munzinger, J. Stachel, QM 2022

essential to constrain baryon production mechanism



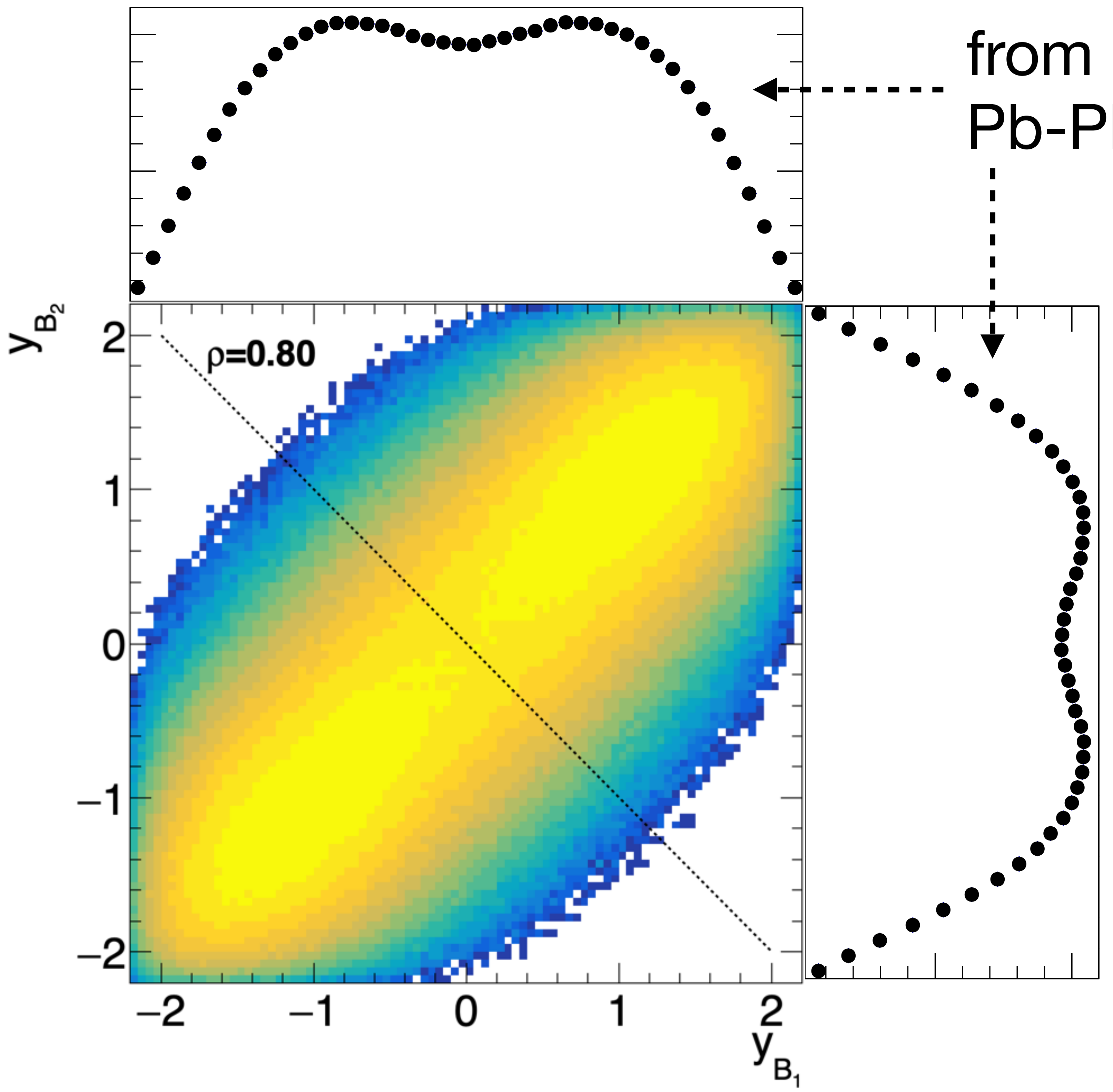
- Alice data: best description with $\rho = 0.1$ ($\Delta y_{corr} = 12$) ↔ **Long range correlations**
- Calls into question baryon production mechanism in Hjing (Lund String Fragmentation)
  - Hijing results suggest $\rho = 0.98$ ($\Delta y_{corr} = 1.7$) ↔ **Strong local correlations**

Chasing for proton clusters

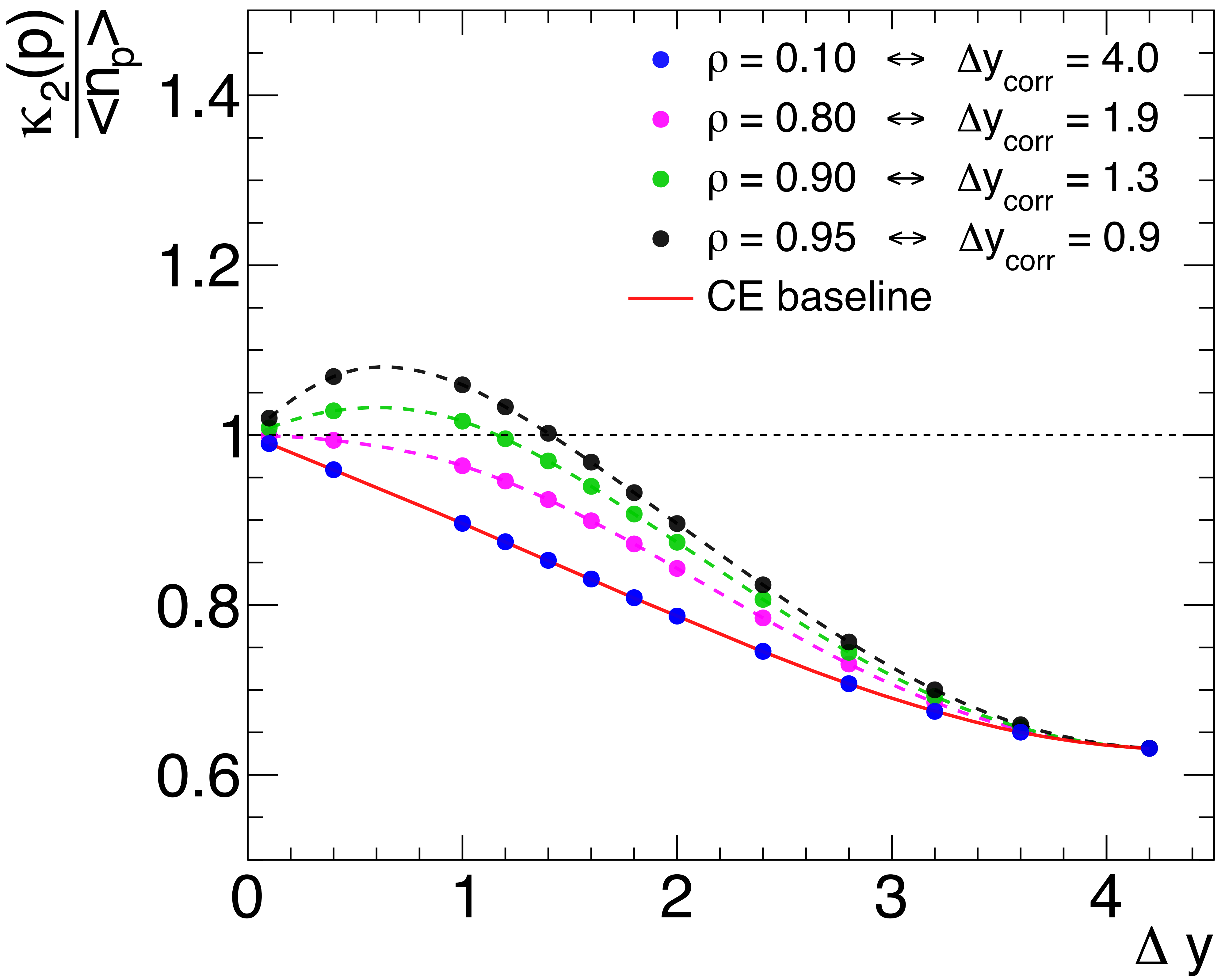
Chasing for proton clusters

proton clusters and cumulants A. Bzdak, V. Koch, V. Skokov, Eur.Phys.J.C 77 (2017) 5, 288

correlations between baryons



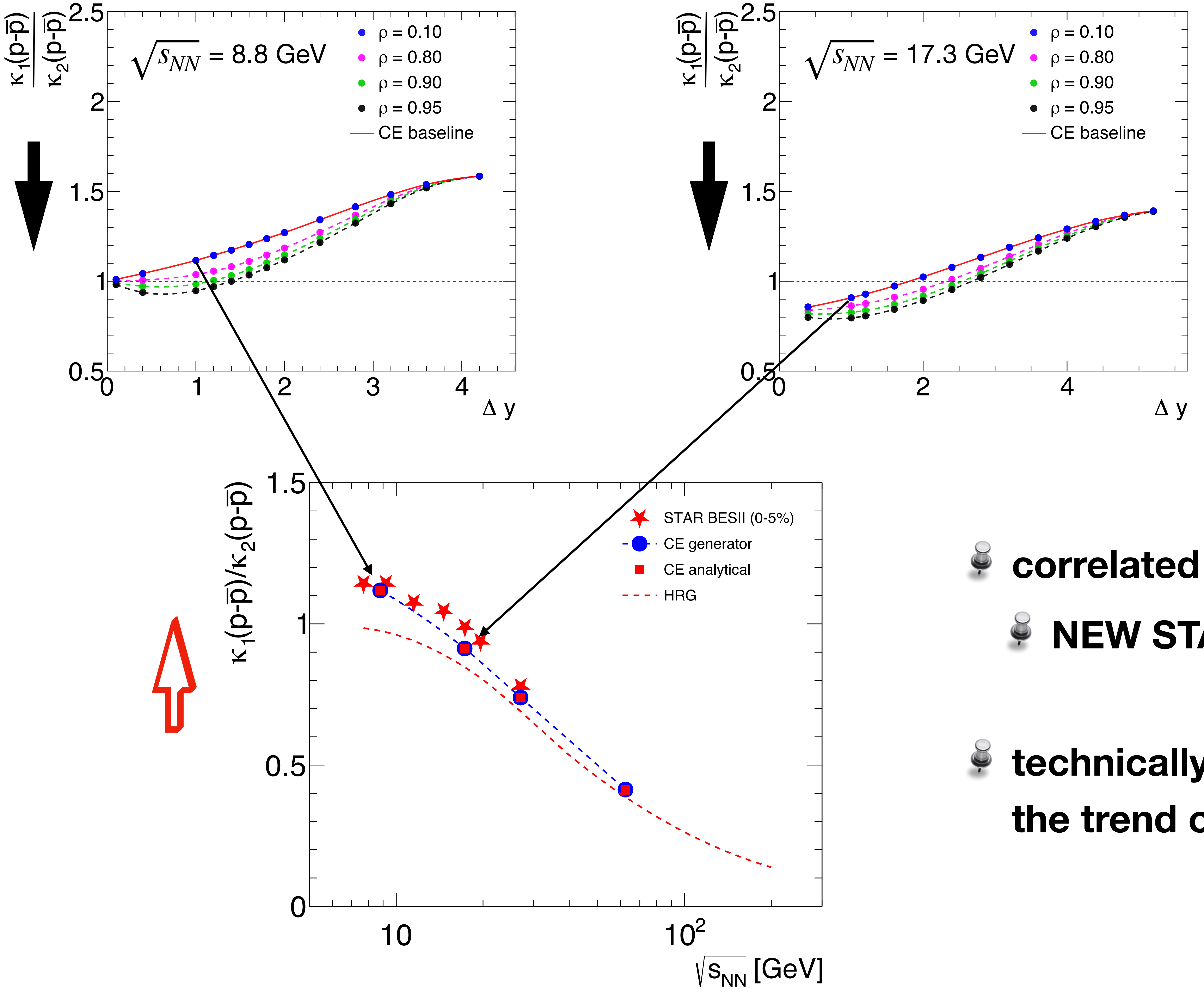
predictions for $\kappa_2(p)/\langle n_p \rangle$



CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

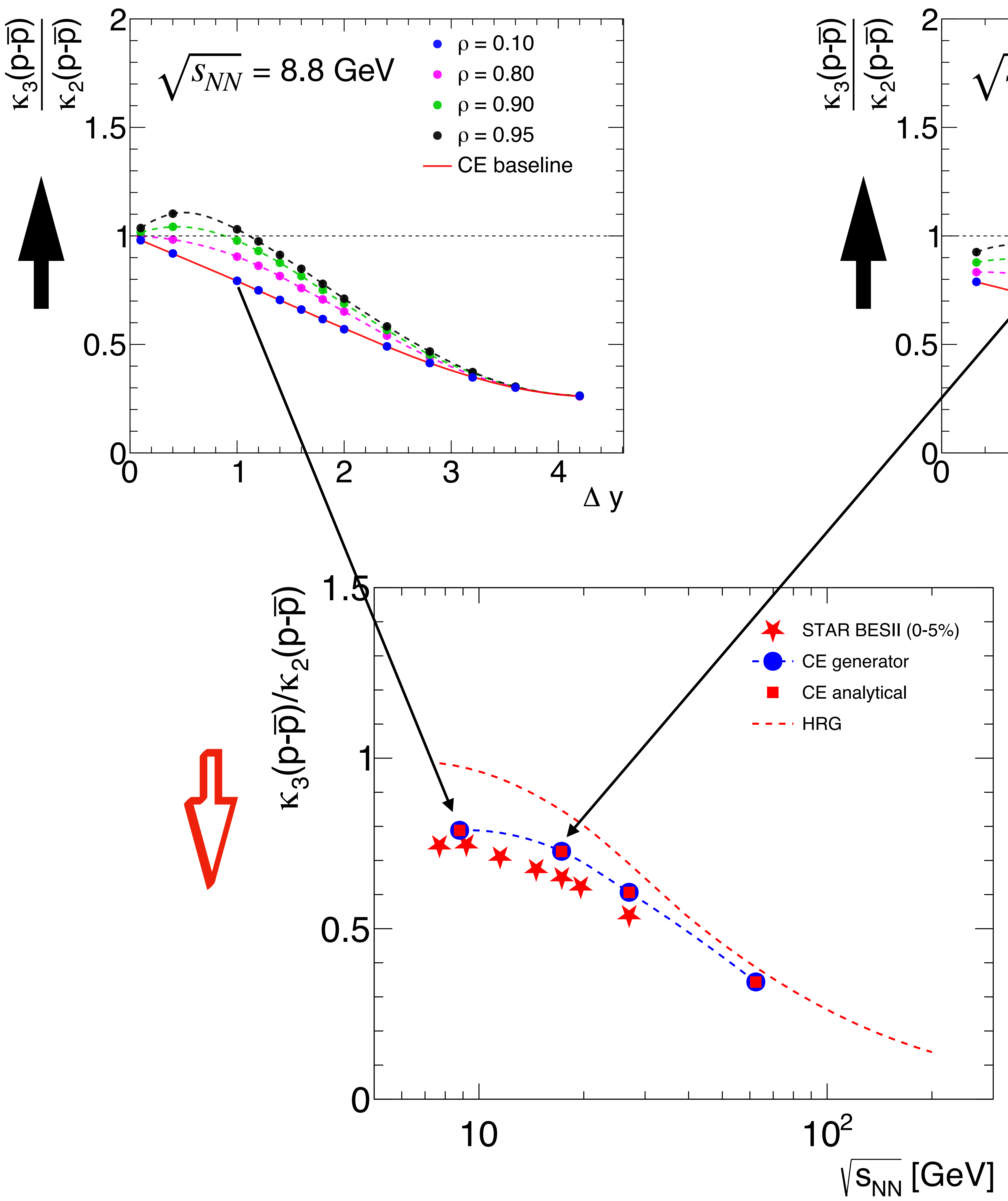
- for large values of ρ and small values of Δy it is more probable to treat protons **in pairs**
- this process increases the finally measured proton number fluctuations

Cumulants vs. Proton clustering, κ_1/κ_2



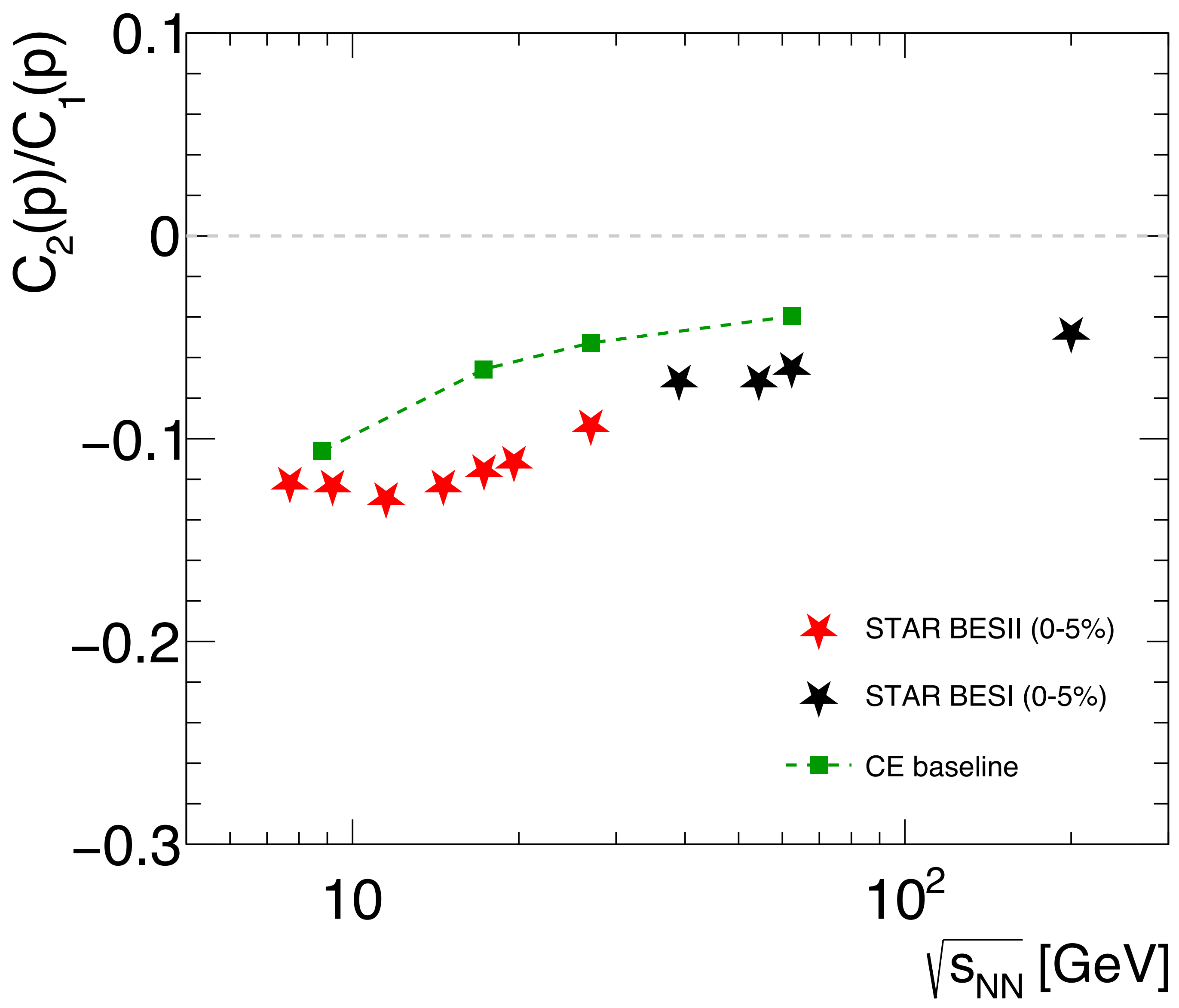
- correlated proton production suppresses the baseline
  - NEW STAR data shows opposite behaviour
- technically anti-correlations (repulsion) could catch the trend of the data (in progress)

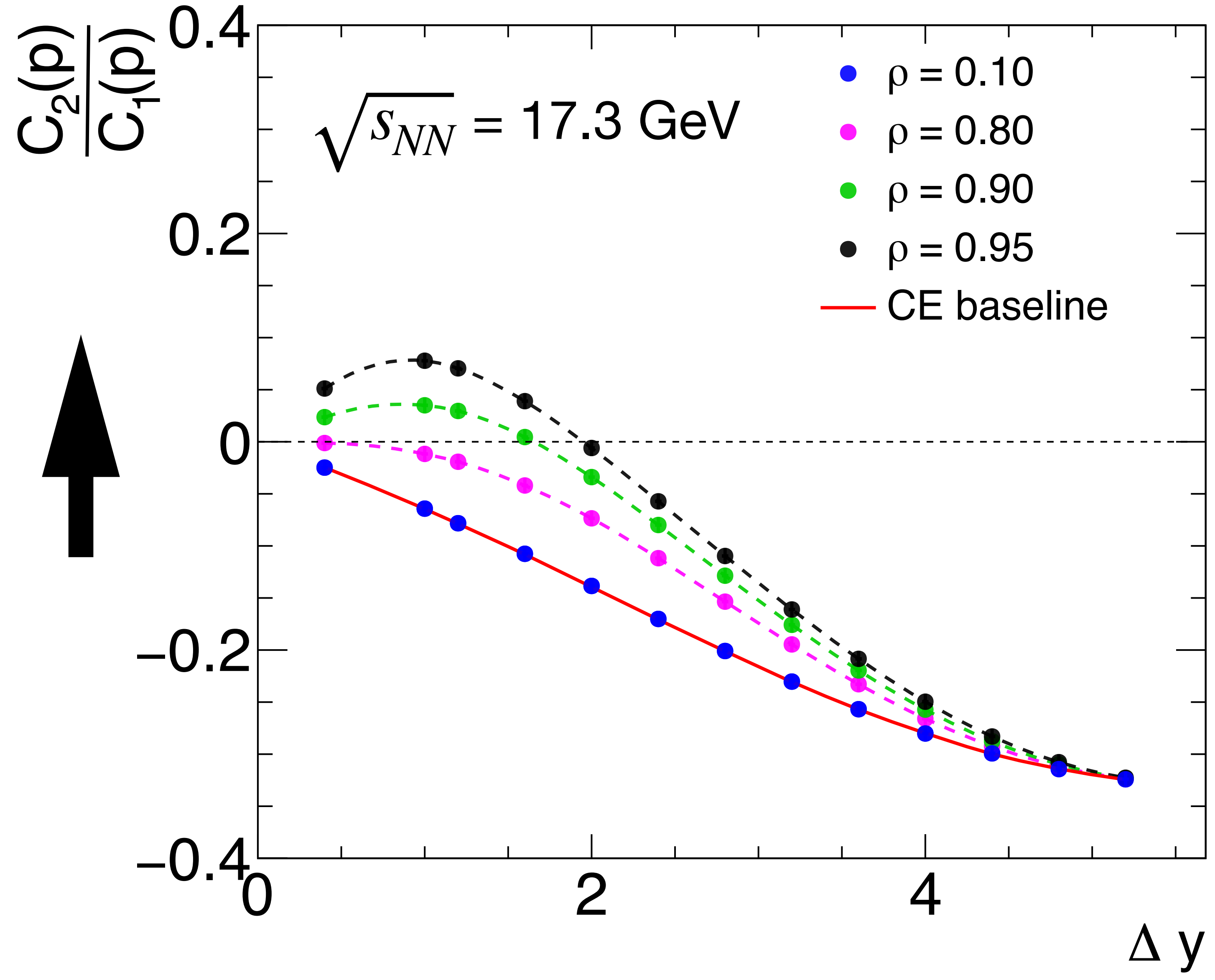
Cumulants vs. Proton clustering, κ_3/κ_2



- **correlated proton production increases the baseline**
  - **NEW STAR data shows opposite behaviour**
- **technically anti-correlations (repulsion) could catch the trend of the data (in progress)**

Factorial cumulant vs. Proton clustering





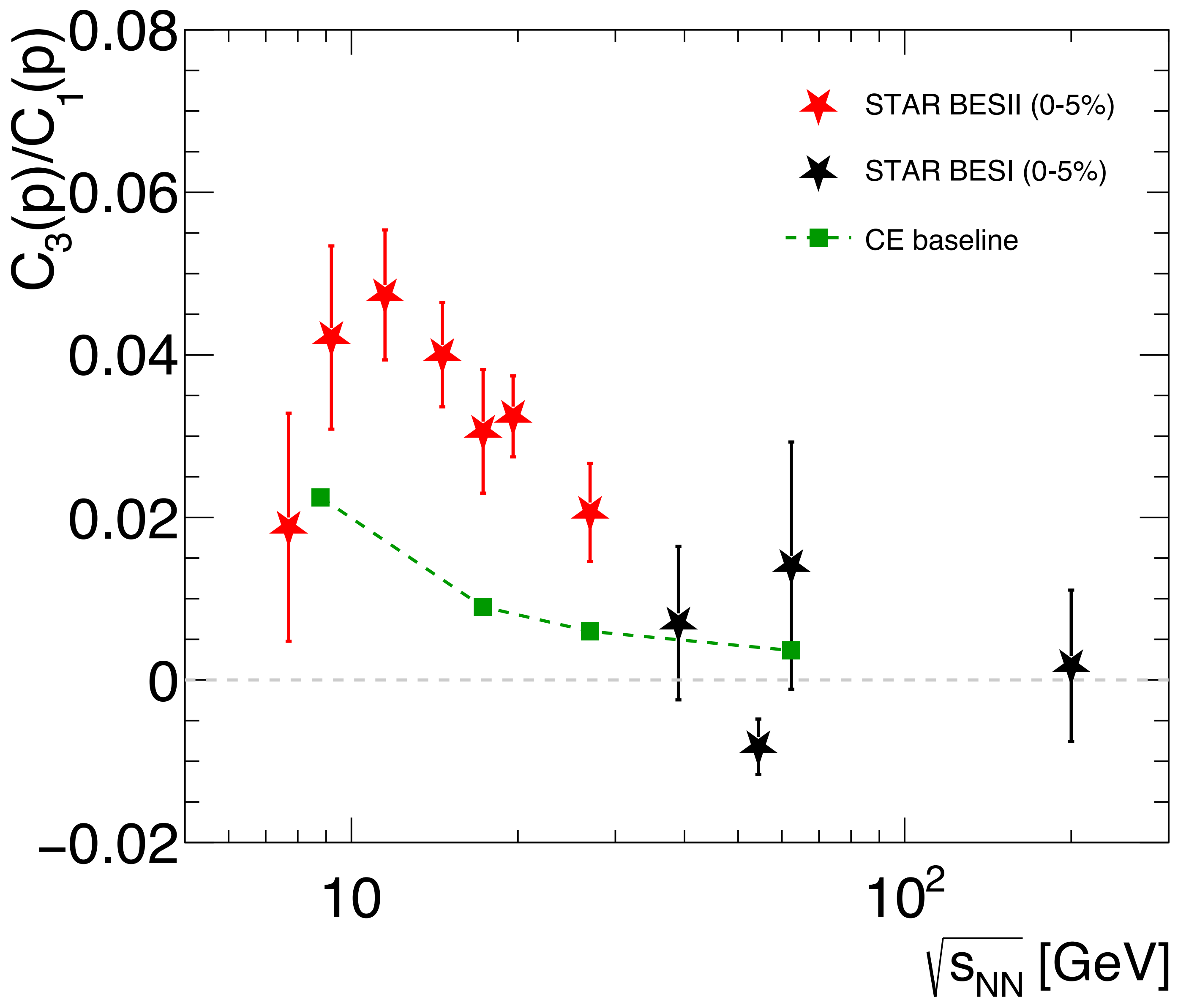
$$C_1(p) = \kappa_1(p) = \langle n_p \rangle$$

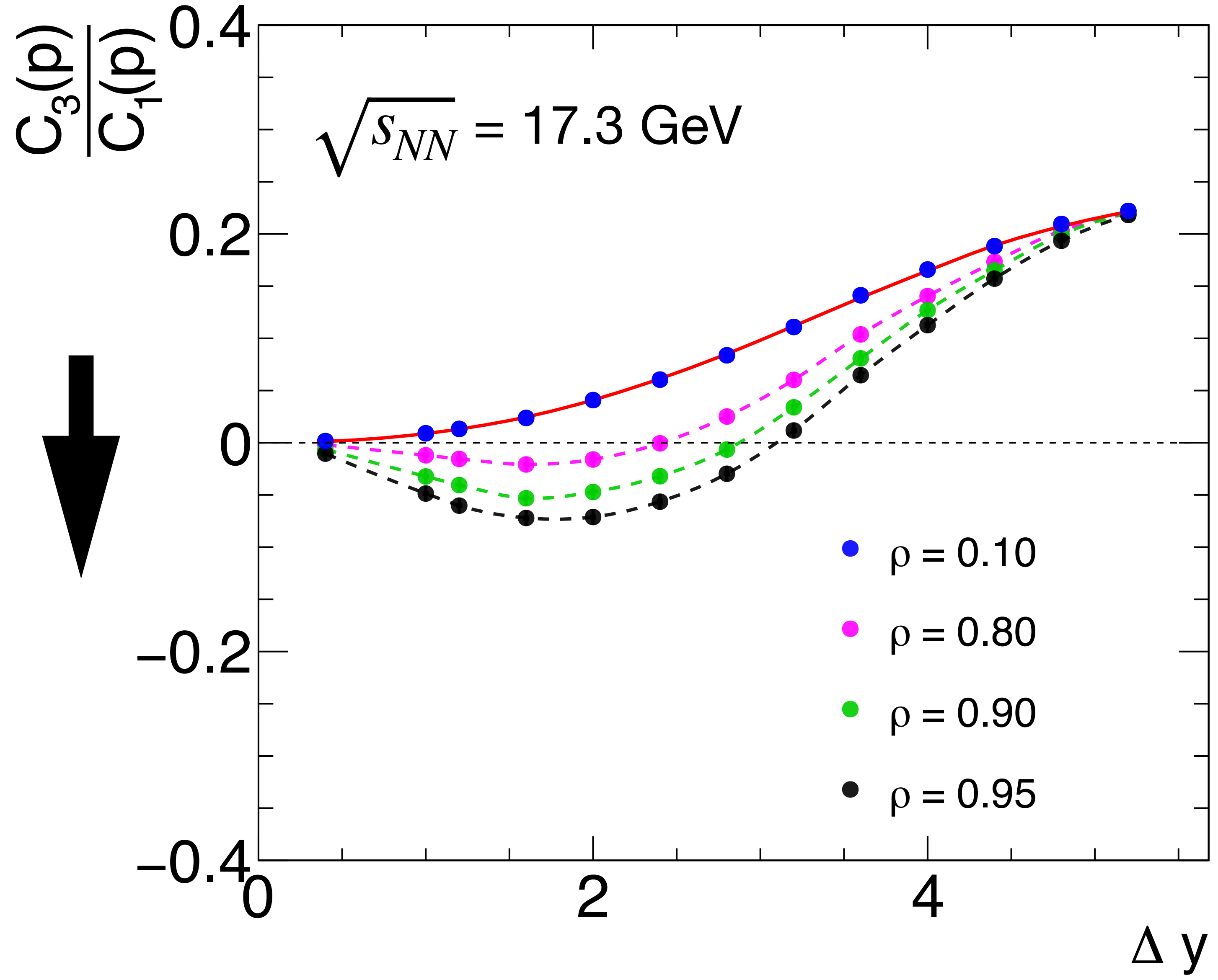
$$C_2(p) = -\kappa_1(p) + \kappa_2(p)$$

$$C_3(p) = 2\kappa_1(p) - 3\kappa_2(p) + \kappa_3(p)$$

$$C_4(p) = -6\kappa_1(p) + 11\kappa_2(p) - 6\kappa_3(p) + \kappa_4(p)$$

R. Holzmann, V. Koch, A. R., J. Stroth, 2403.03598 (2024)





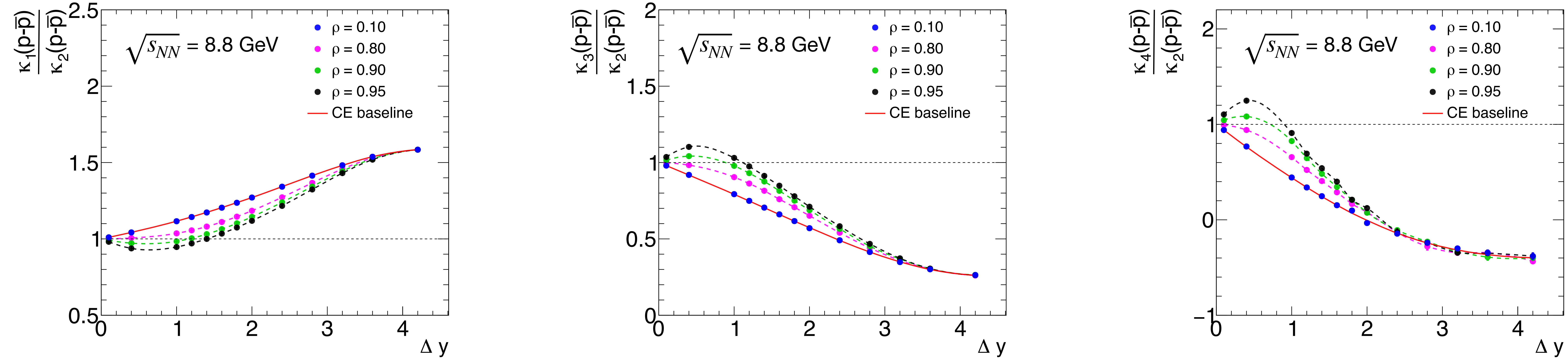
correlated proton production moves the baseline away from the STAR data

technically anti-correlations could solve the problem (in progress)

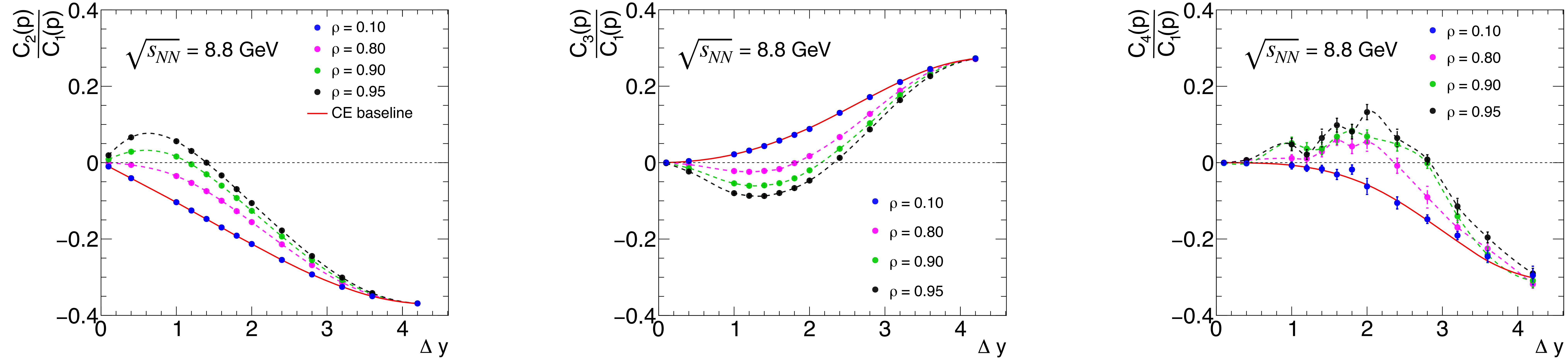
Predictions

Suggestion: Differential study of fluctuations, e.g., as a function of rapidity, pt and for each collision energy

cumulants



factorial cumulants



Summary

Fluctuations of conserved charges from event-to-event are fundamental/direct tools to study phase transitions

- ☑ **However, a number of non-critical contributions need to be accounted for**
  - ☑ **Modifications caused by conservation laws**
  - ☑ **Contributions from experimentally unavoidable participant/volume fluctuations**
    - ☑ **Not covered in this talk (for recent review see: R. Holzmann, V. Koch, A. R., J. Stroth, 2403.03598 (2024), NPA in print)**
- ☑ **Using Canonical Ensemble, accurate baselines are derived for cumulants of any order**
  - ☑ **Alice data prefers long range correlations, which is at odds with the Lund String Model**
  - ☑ **The NEW (BESII) STAR data show deviations from canonical baselines**
    - ☑ **The hypothesis of proton clustering moves baselines away from the STAR data**
    - ☑ **<u>The NEW STAR data suggest that repulsion (anti-correlation) dominates over attraction</u>**
      - ☑ **<u>The implementation is in progress</u>**

Differential measurements, including correlations, as a function of rapidity, pt, energy, etc., are needed


THANK YOU
FOR YOUR
ATTENTION