Influence of dynamical screening of four-quarks interaction on the chiral phase diagram

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Investigation of QCD phase diagram in μ_B, T directions

- Requires non-preturbative treatment
- LQCD $\rightarrow$ first-principle calculations
- Effective models $\rightarrow$ QCD-like theories
  - Extension to large μ
  - Building intuitions $\rightarrow$ Complementary to more advanced methods

Other directions also possible, e.g. quark masses or magnetic field

This talk:

- Investigation of in-medium screening of four-quark interaction
- Chiral phase transition at finite μ [1] and B [2]

[1] MS, PM Lo, K. Redlich, C. Sasaki, 2309.03124
[2] PM Lo, MS, K. Redlich, C. Sasaki, Eur. Phys. J. A (2022) 58:172

Starting point → Chiral model inspired by Coulomb gauge QCD[3]

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m_0)\psi(x) + \int d^4y \rho^a(x) V^{ab}(x-y)\rho^b(y)$$

with

- $\rho^a(x) = \bar{\psi}(x)\gamma^0 T^a \psi(x) \rightarrow$ color quark current
- $V^{ab}(x-y) \rightarrow$ Interaction potential

This work → Contact interaction, gap equation:

$$M = m_0 + C_F V_0 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(1 - N_{th}(E,\mu) - \bar{N}_{th}(E,\mu)\right)$$

The same form as the NJL model if $C_F V_0 \rightarrow 4 N_c N_f (2 G_{NJL})$

- NJL → Scalar-scalar interaction

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + G_{NJL}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2\right]$$

- Current model → Vector-vector interaction
  - Systematic improvements possible → dressing by polarization

[3] See e.g. P. M. Lo, E. S. Swanson Phys. Rev. D **81** 034030 (2010)

Dressing by polarization, ring diagram approximation

$$\tilde{V}_0^{-1} = V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p}) \quad \Rightarrow \quad \tilde{V}_0 = \frac{1}{V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p})}$$

Static limit

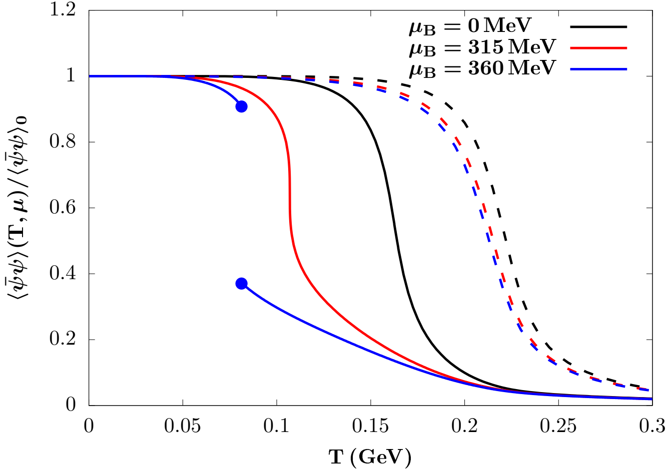
$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0)$$

Screening → Medium-dependent coupling

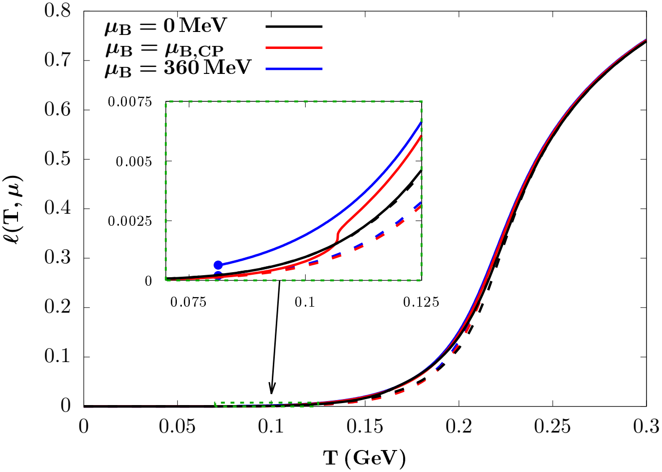
Contact interaction → No confinement

- Polyakov loop → Statistical confinement
- $\Pi_{00}(M) \to \Pi_{00}(M, \ell, \bar{\ell})$
- Regulates the screening strength

Vacuum term → Proper-time regularization

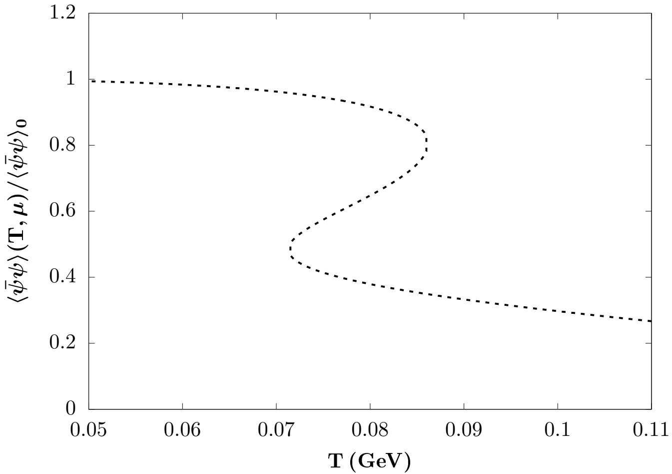


Solid lines: screening; dashed lines: no screening



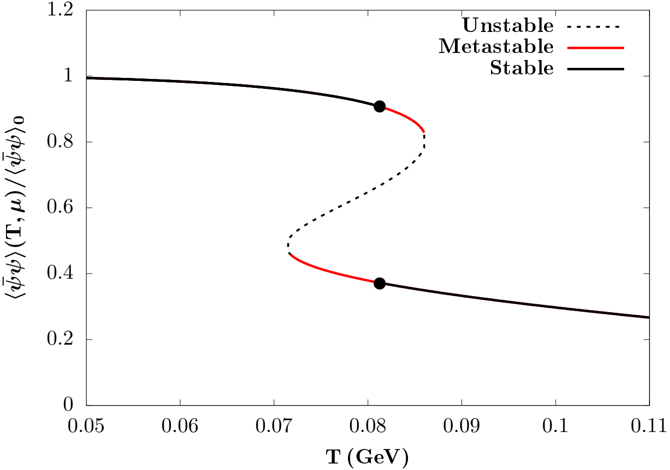
Polyakov loop weakly modified by screening

- No backreaction of ring diagram on Polyakov loop gap equations



Gap equations $\rightarrow$ multiple solutions

- Mean-field $\rightarrow$ Stable solution from global minimum of the potential
- Current model $\rightarrow$ DSE, start from gap equations



Gap equations → multiple solutions

- Mean-field → Stable solution from global minimum of the potential
- Current model → DSE, start from gap equations

Curent model

- DSE → gap equations

$$f_1(\vec{\phi}) = 0, ..., f_n(\vec{\phi}) = 0$$

$\vec{\phi} = (M, \ell, \bar{\ell})$

- No potetnial to look for a minimum
- Strategy → construct potential from gap eqauations

Goal → find a "potential" such that

$$\frac{\partial U(\phi_1, \phi_2, ...\phi_n)}{\partial \phi_i} = f_i(\phi_1, \phi_2, ...\phi_n)$$

Possible if

$$\frac{\partial f_i(\phi_1, \phi_2, ...\phi_n)}{\partial \phi_k} = \frac{f_k(\phi_1, \phi_2, ...\phi_n)}{\partial \phi_i}$$

Current model → NOT satisfied

- No backreactoin of ring on Polyakov loop sector

Current model $\rightarrow$ No potential exist

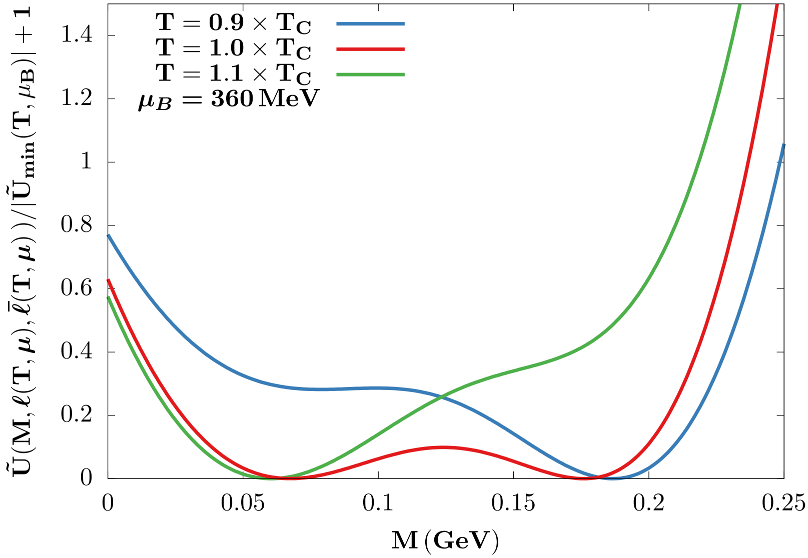
- Approximation needed
- Focus on chiral sector, Polyakov loop as a background

Approximate potential:

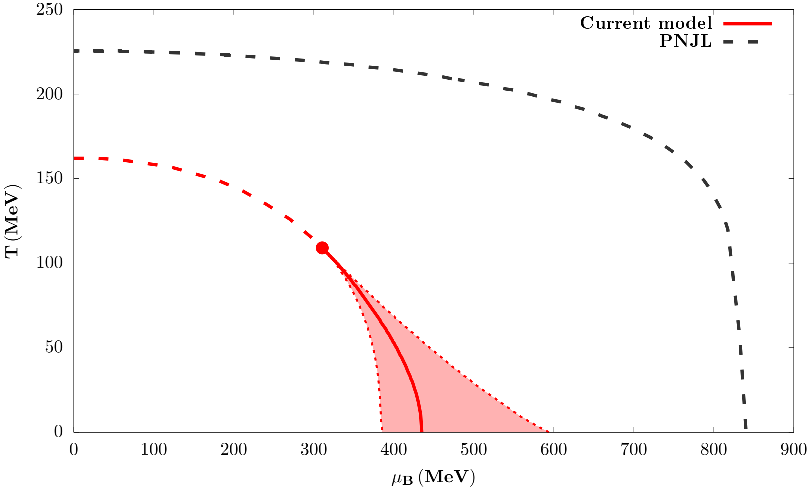
$$\begin{aligned}\tilde{U}(M,\ell(T,\mu),\bar{\ell}(T,\mu)) &= U_0(m_0,\ell(T,\mu),\bar{\ell}(T,\mu)) \\ &+ 4N_cN_f\int_{m_0}^{M} F(M',\ell(T,\mu),\bar{\ell}(T,\mu))dM',\end{aligned}$$

with F from gap equation

$$F(M,\ell,\bar{\ell}) = \frac{M-m_0}{C_F\tilde{V}(M,\ell,\bar{\ell})} - M\left(I_{vac}(M) - I_{thermal}(M,\ell,\bar{\ell})\right)$$



$\mathbf{T = 0.9 \times T_C}$
$\mathbf{T = 1.0 \times T_C}$
$\mathbf{T = 1.1 \times T_C}$
$\boldsymbol{\mu_B = 360\,\mathrm{MeV}}$
$\tilde{\mathbf{U}}(\mathbf{M}, \boldsymbol{\ell}(\mathbf{T}, \boldsymbol{\mu}), \bar{\boldsymbol{\ell}}(\mathbf{T}, \boldsymbol{\mu}))/|\tilde{\mathbf{U}}_{\mathbf{min}}(\mathbf{T}, \boldsymbol{\mu}_{\mathbf{B}})| + \mathbf{1}$
$\mathbf{M\,(GeV)}$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.05
0.1
0.15
0.2
0.25



Proper-time regularization

Screening: crossover → CEP → 1st order phase transition

PNJL: crossover

Contact interaction → Regularization scheme dependence (both models!)

Why study QCD in strong magnetic field?

May be important for phenomenology:

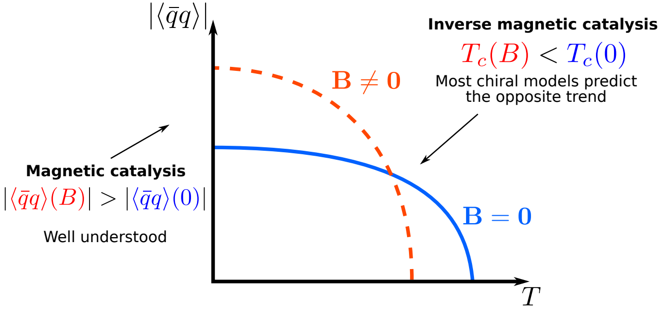
- Non-central heavy-ion collisions (eB up to $15m_\pi^2$ [1])
- Magnetars

Additional parameter to study QCD under extreme conditions

- Can be probed directly in LQCD simulations
- Possibility to test effective models

[1] *V. Skokov et al., Int.J.Mod.Phys.A24:5925-5932,2009*

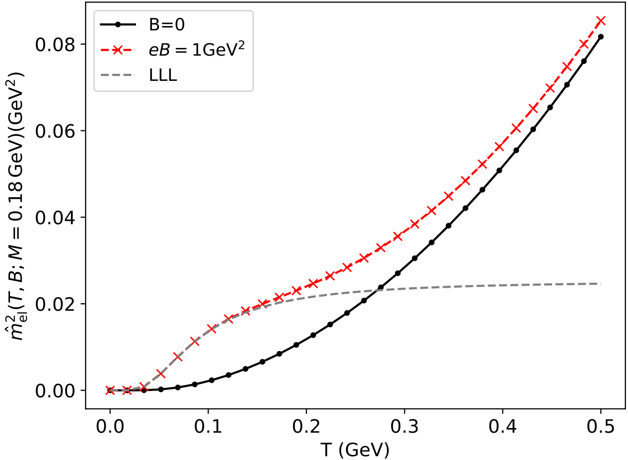
Schematic behavior of the quark condensate from first-principle numerical simulations



Opposite $T_C(B)$ for models and LQCD $\rightarrow$ Possible missing interactions

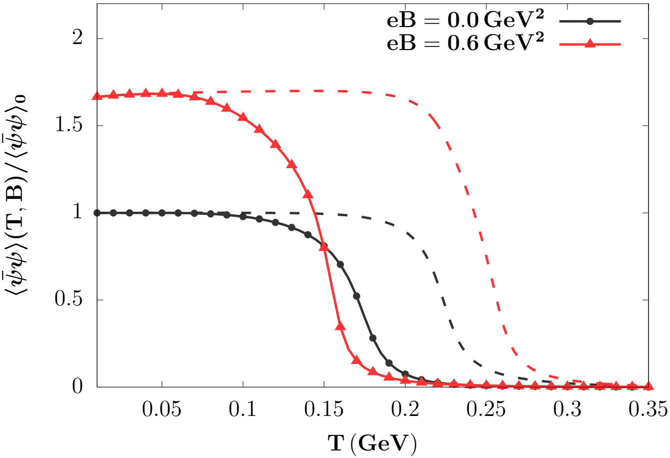
Screening?[4]

[4] 2107.05521, 2109.04439

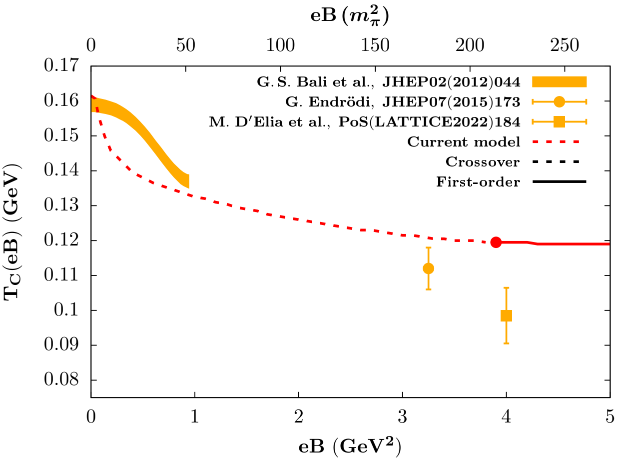


- LQCD $\rightarrow$ First-order transition for $4\,\mathrm{GeV}^2 < eB^{LQCD}_{crit.} < 9\,\mathrm{GeV}^2$ [5]
- Current model $\rightarrow$ $eB_{crit.} \approx 0.5\,\mathrm{GeV}^2 \ll eB^{LQCD}_{crit.}$
- Approximation: $\Pi(M, \ell, \bar{\ell}) \rightarrow \xi \times \Pi(\bar{M} \approx 0.130\,\mathrm{MeV}, \ell, \bar{\ell})$

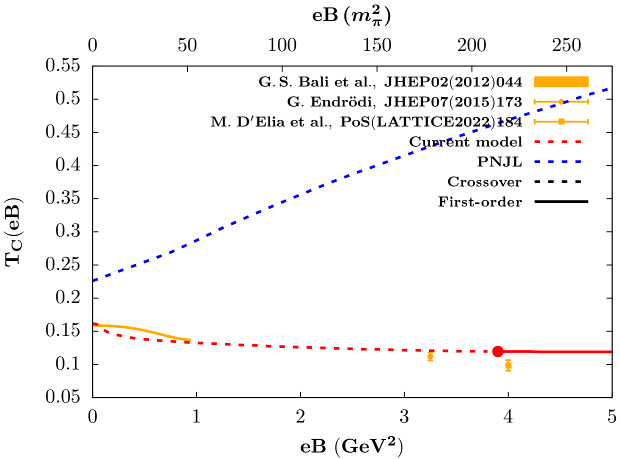
[5] M. D'Elia et al., PoS(LATTICE2022)184



Results at $\mu = 0$, $B > 0$:
Dashed line – no dressing, $T_c(B) > T_c(0)$
Solid line with symbols – dressing, $T_c(B) < T_c(0)$



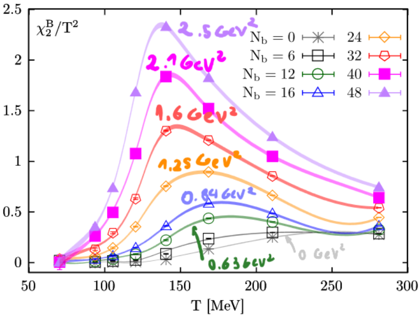
MS et al, work in progress

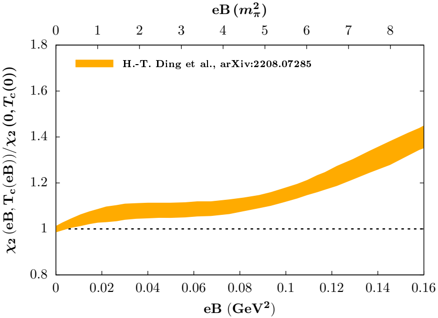


MS et al, work in progress

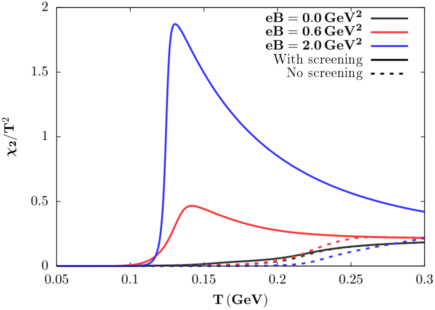
Baryon number fluctuations

$$\chi_n = \frac{\partial^n P}{\partial \mu_B^n}$$

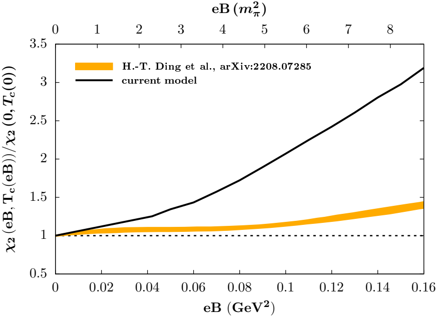


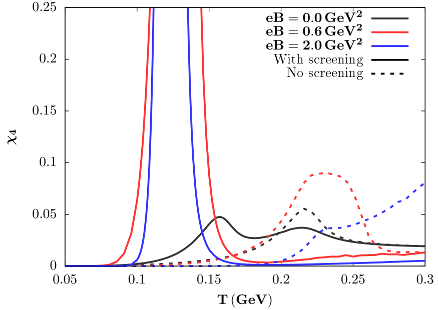


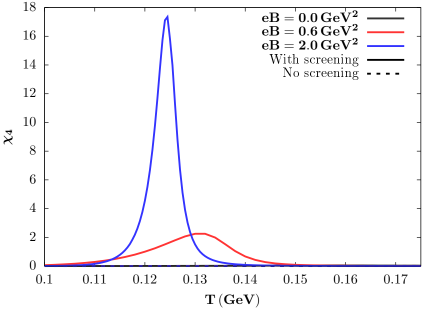
Left: H. T. Ding et al. Eur. Phys. J. A **57**, no.6, 202 (2021)
Right: H. T. Ding et al. Acta Phys. Polon. Supp. **16**, no.1, 1-A134 (2023)



MS et al, work in progress







MS et al, work in progress

Conclusions and outlook

- Effect of the screening of 4-quark interaction
  - $B = 0$: $T_C^{no\ screening} \approx 230\,\mathrm{MeV} \to T_C^{screening} \approx 160\,\mathrm{MeV}$
  - $\mu_B > 0$: CP located at lower μ_B than in PNJL models
  - $B \neq 0$: IMC due to screening, correct trend for χ_B^2

  No need for artificial rescaling of the parameters or fitting the coupling
- Future prospects
  - Effect too strong $\to$ additional contributions to gap eqautions?
  - Going beyond contact interaction, momentum dependence

Appendix

Final set of gap equations:

$$M = m_0 + C_F \tilde{V}_0(M,\ell) \left[I_{vac} - \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(N_{th}(E,\ell,\bar{\ell},\mu) + \bar{N}_{th}(E,\ell,\bar{\ell},\mu) \right) \right]$$

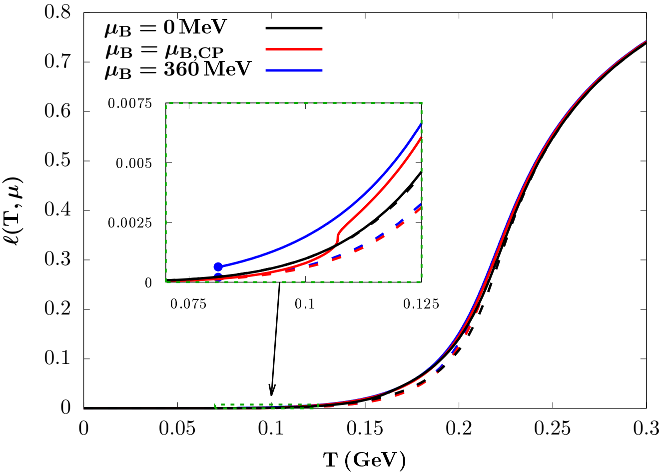
$$\tilde{V}_0(M,\ell,\bar{\ell}) = \frac{1}{V_0^{-1} + m_{el}^2(T,M,\ell,\bar{\ell})}$$

$$\frac{\partial}{\partial \ell} (\mathcal{U}_G + \mathcal{U}_Q) = 0$$

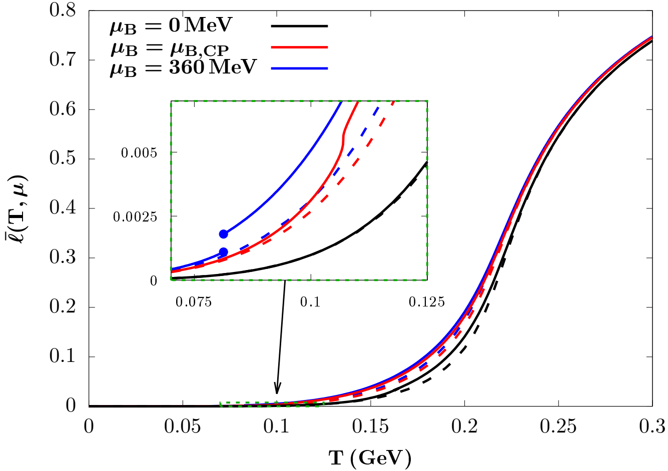
$$\frac{\partial}{\partial \bar{\ell}} (\mathcal{U}_G + \mathcal{U}_Q) = 0$$

Regularization

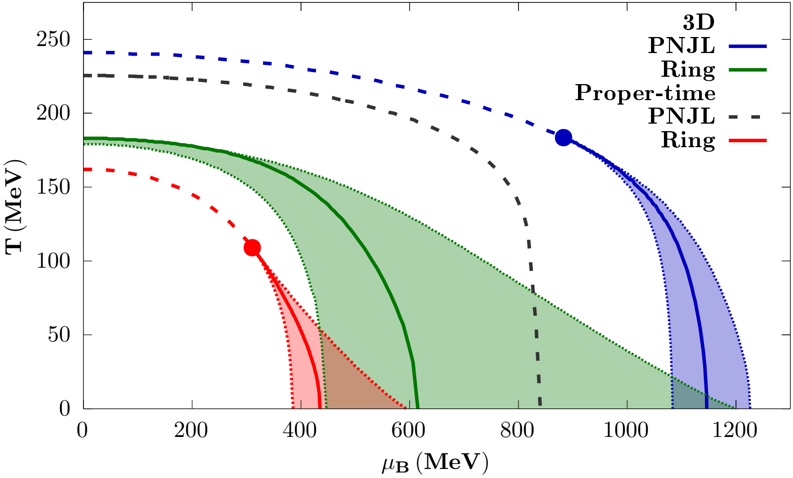
$$I_{vac} = \int \frac{d^3q}{(2\pi^3)} \frac{M}{2E} \to \int_{1/\Lambda^2}^{\infty} \frac{ds}{16\pi^2} \frac{1}{s^2} e^{-M^2 s}$$



$\mu_B = 0$ MeV
$\mu_B = \mu_{B,CP}$
$\mu_B = 360$ MeV
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.0075
0.005
0.0025
0
0.075
0.1
0.125
$\ell(T, \mu)$
0
0.05
0.1
0.15
0.2
0.25
0.3
T (GeV)



$\mu_B = 0$ MeV
$\mu_B = \mu_{B,CP}$
$\mu_B = 360$ MeV
$\bar{\ell}(T,\mu)$
T (GeV)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.15
0.25
0.3
0.005
0.0025
0.075
0.1
0.125

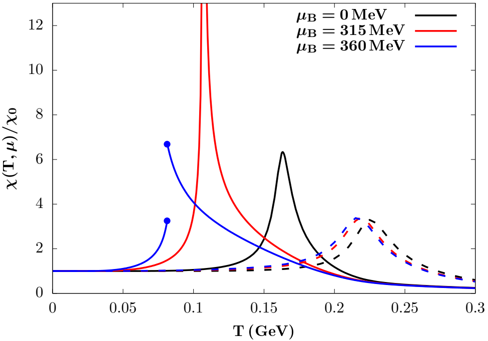


3D cutoff:

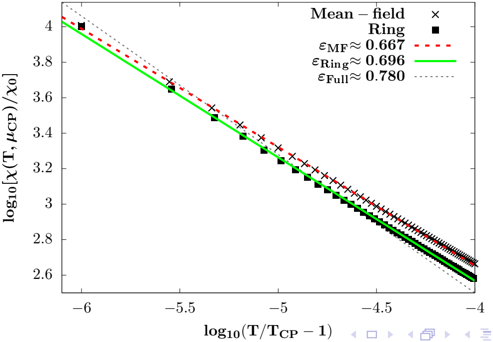
Screening: 1st order

PNJL: crossover → CEP → 1st order phase transition

Contact interaction → Regularization scheme dependence (both models!)



$\mu_B = 0$ MeV
$\mu_B = 315$ MeV
$\mu_B = 360$ MeV
$\chi(T,\mu)/\chi_0$
T (GeV)
0
0.05
0.1
0.15
0.2
0.25
0.3
2
4
6
8
10
12

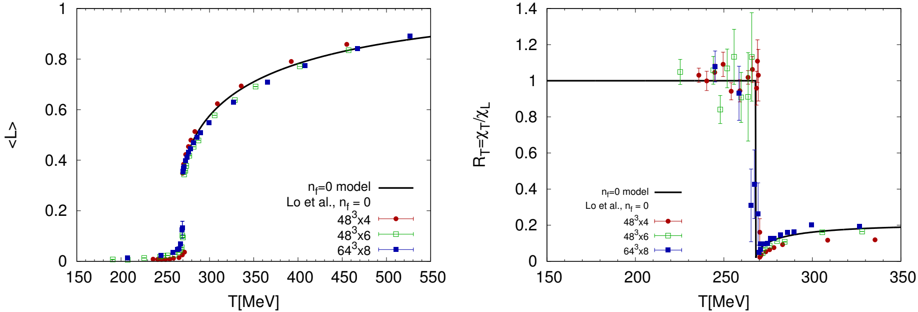


Mean − field
Ring
$\varepsilon_{MF} \approx 0.667$
$\varepsilon_{Ring} \approx 0.696$
$\varepsilon_{Full} \approx 0.780$
$\log_{10}[\chi(T,\mu_{CP})/\chi_0]$
$\log_{10}(T/T_{CP} - 1)$
−6
−5.5
−5
−4.5
−4
2.6
2.8
3
3.2
3.4
3.6
3.8
4

Pure gauge part → Polyakov loop potential[1]

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\ln M_H(\ell,\bar{\ell}) + \frac{1}{2}c(T)(\ell^3+\bar{\ell}^3) + d(T)(\ell\bar{\ell})^2$$

- Polyakov loop & fluctuations determined from LQCD



Quark-gluon interaction

$$\mathcal{U}_Q = -2T\int \frac{d^3q}{(2\pi)^3} 2\ln\left(1+3\ell e^{-\beta E}+3\ell e^{-2\beta E}+e^{-3\beta E}\right)$$

[1] *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88**, *074502 (2013)*

Electric mass

$$m_{el}^2 = -\frac{1}{2}N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0) = \frac{1}{2}N_f \times \int \frac{d^3q}{(2\pi)^3} 4\beta N_{th}(1 - N_{th})$$

External magnetic field $\to$ Landau quantization

$$2\int \frac{d^3p}{(2\pi)^3} \to \frac{|qB|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

$$E_k{}^2 = m^2 + p_z^2 + 2k|q_f B|\,,$$

Electric mass (per flavor)

$$m_{el}^2 = \frac{1}{2}\frac{|q_f B|}{2\pi}\sum_{k=0}^{\infty}(2 - \delta_{k,0}) \int \frac{dq_z}{2\pi} 4\beta N_{th}(E_k)(1 - N_{th}(E_k))$$

$$\approx \frac{1}{2}\frac{|q_f B|}{4\pi} \int \frac{dq_z}{2\pi} \frac{4\beta e^{\beta\sqrt{(q_z)^2+m^2}}}{(e^{\beta\sqrt{(q_z)^2+m^2}} + 1)^2}\,, \qquad |q_f B| \gg T^2$$

Coupling to the Polyakov loop → Statistical confinement

- Pure gluon system → Deconfinement order parameter
- Effective models → Accounts for non-preturbative gluon dynamics

$$N_{th}(E,\mu) \to N_{th}(E,\ell,\bar{\ell},\mu) = \frac{\ell e^{-\beta(E-\mu)} + 2\bar{\ell}e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}{1 + 3\ell e^{-\beta(E-\mu)} + 3\bar{\ell}e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}}$$

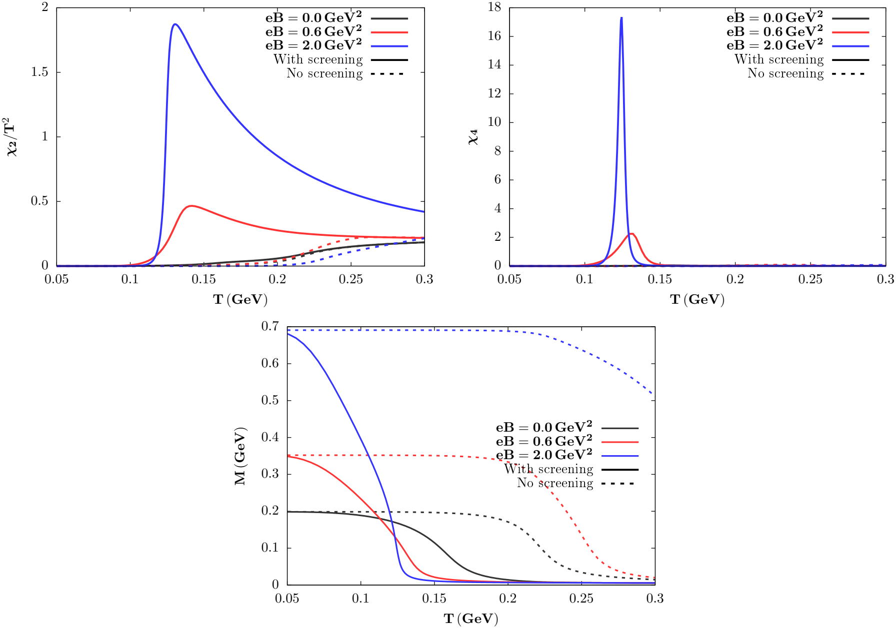
$$= \begin{cases} \dfrac{1}{1+e^{3\beta(E-\mu)}}, & \ell = \bar{\ell} = 0, \quad \text{baryon-like} \\ \dfrac{1}{1+e^{\beta(E-\mu)}}, & \ell = \bar{\ell} = 1, \quad \text{quark-like} \end{cases}$$

Two additional gap equations

$$\frac{\partial}{\partial \ell}(\mathcal{U}_G + \mathcal{U}_Q) = 0 \qquad \frac{\partial}{\partial \bar{\ell}}(\mathcal{U}_G + \mathcal{U}_Q) = 0$$

- $\mathcal{U}_G$ – pure gauge potential[6]
- $\mathcal{U}_Q$ – quark-gluon interaction

[6] P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D **88**, 074502 (2013)


eB = 0.0 GeV²
eB = 0.6 GeV²
eB = 2.0 GeV²
With screening
No screening
χ_2/T^2
T (GeV)
χ_4
T (GeV)
M (GeV)
T (GeV)