

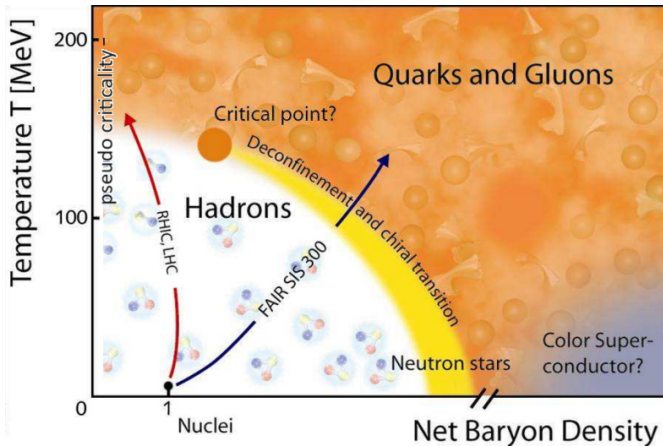
Thermodynamics of QCD with quarks and multi-quark clusters

Oleksii Ivanytskyi

D. Blaschke, M. Cierniak, OI, G. Röpke, EPJ A 2024 + more

EMMI Workshop - Aspects of Criticality
Wroclaw, 4 July 2024

From pseudo criticality to critical point

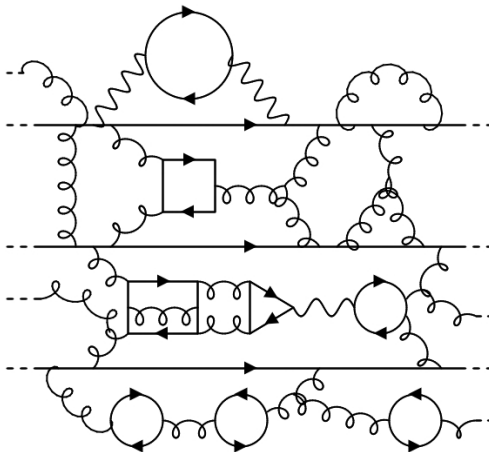


- **Introduction**
- **Density functional approach to quark matter**
- **Multiquark clusters**
- **QCD thermodynamics**

- **Introduction**
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- QCD thermodynamics

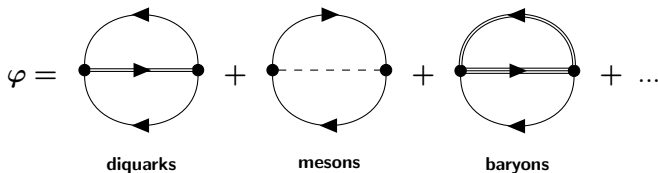
$$\mathcal{L}_{\text{QCD}} = \bar{q} \left(i \hat{D} - \hat{m} \right) q - \frac{1}{4} G^2$$

QCD: ..., sophisticated nonperturbative dynamics



This is just a three quark bound state, e.g. nucleon

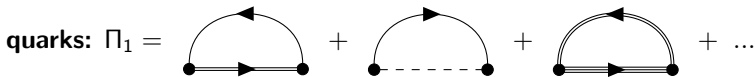
Cluster decomposition in two-loop approximation



G. Baym, L.P. Kadanoff, Phys. Rev. 124, 287 (1961); G. Baym, Phys. Rev. 127, 1391 (1962)

Two-loop self-energies & Dyson-Schwinger propagators

$$\Pi_n = \frac{\delta \varphi}{\delta S_n}, \quad (S_n)^{-1} = (S_n^{\text{free}})^{-1} - \Pi_n, \quad n = 1, 2, 3, \dots$$



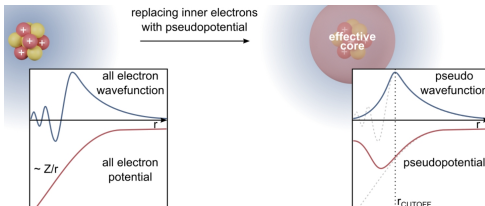
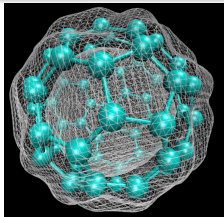
Dyson-Schwinger problem requires solving all S_n simultaneously

Mean-field approximation for quark propagators

The Dyson-Schwinger problem reduces to a subsequent solving S_n using $S_{m < n}$

- Introduction
- **Density functional approach to quark matter**
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Context: density functional theory



(Dirac)Brueckner-Hartree-Fock T-, G-matrix based theories



Density functional theory

- Many body problems
- Quantum chemistry
- Skyrme-type models for nuclear physics
- String Flip model for quark matter
- ...

Why? False quark dominance in hybrid quark-hadron EoS

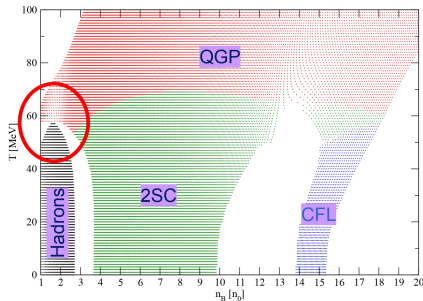
- Hadronic EoS consistent with astro (DDf4) + NJL model



False quark onset already @ $T \simeq 60$ MeV

- Hadron decays are energetically favorable

$$\begin{aligned} M_q &\simeq 330 \text{ MeV} \\ M_\omega &= 783 \text{ MeV} \\ M_\rho &= 775 \text{ MeV} \end{aligned} \Rightarrow \begin{aligned} M_{\text{meson}} &> 2M_q \\ \text{quarks are too light} \\ &\text{to be confined} \end{aligned}$$



Effective quark “confinement” is needed

Confining density functional @ $N_f = 2$

$$\mathcal{L} = \bar{q}(i\not{D} - m)q - \mathcal{U}, \quad \mathcal{U} = D_0 \left[(1 + \alpha) \langle \bar{q}q \rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5 q)^2 \right]^\kappa$$

O.Ivanytskyi, D. Blaschke, PRD 105, 114042 (2022)

D_0 - coupling, controls interaction strength

α - dimensionless constant, controls vacuum quark mass

$\langle \bar{q}q \rangle_0$ - χ -condensate in vacuum (introduced for the sake of convenience)

• Comparison to the NJL model

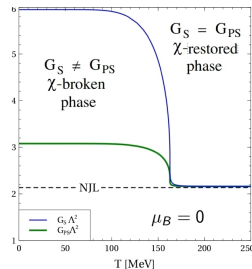
- $\kappa = 1$: NJL model

- $\kappa = 1/3$: String Flip model

$$\text{mean-field self-energy } \Sigma = \frac{\partial \mathcal{U}}{\partial \langle q^+ q \rangle} \propto \overbrace{\langle q^+ q \rangle^{-1/3}}^{\text{separation}}$$

C.J.Horowitz, E.J. Moniz, J. W. Negele, PRD 31, 1689 (1985)

G. Röpke, D. Blaschke, H. Schultz, PRD 34, 3499 (1986)

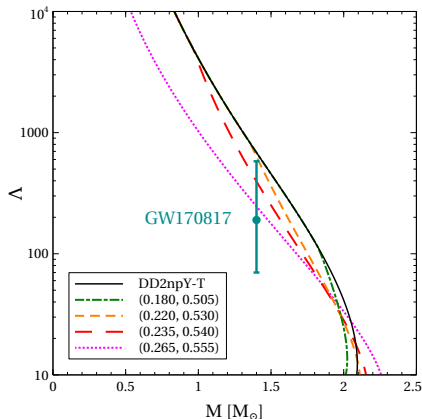
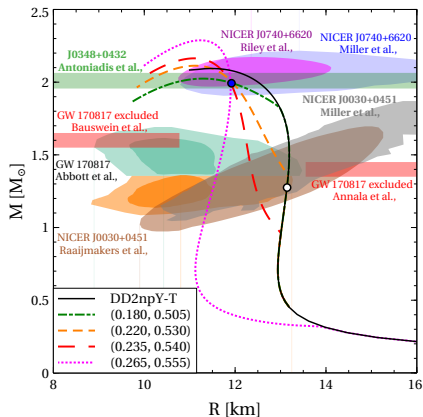


• Dimensionality

$$\begin{aligned} [\mathcal{U}] &= \text{energy}^4 \\ [\bar{q}q] &= \text{energy}^3 \end{aligned} \quad \Rightarrow \quad [D_0]_{\kappa=1/3} = \text{energy}^2 = \left[\begin{array}{c} \text{string} \\ \text{tension} \end{array} \right]$$

$$\text{self energy} = [\text{string tension}] \times \text{separation} \Leftrightarrow \text{"confinement"}$$

Modeling neutron stars with quark cores @ $N_f = 2$



O.Ivanytskyi, D. Blaschke, PRD 105, 114042 (2022)

**Agreement with the observational constraints
on mass-radius relation and tidal deformability of neutron stars**

Confining density functional with Polyakov loop @ $N_f = 3$

$$\mathcal{L} = \bar{q}(i\not{D} + g\not{A} - \hat{m})q - \mathcal{U}_\chi - \mathcal{U}_\Phi, \quad \hat{m} = \text{diag}(m_u, m_d, m_s)$$

A_μ - homogeneous static gluon field in the Polyakov gauge

• Density functional

$$\mathcal{U}_\chi = D_0 \left[(1 + \alpha) \langle \hat{\mathcal{O}} \rangle_0 - \hat{\mathcal{O}} \right]^{1/3}, \quad \hat{\mathcal{O}} = \frac{1}{2} \sum_{a=0,8} [(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2]$$

D. Blaschke, O. Ivanytskyi, M. Shahrhaf, 2202.05061 [nucl-th]

• Polyakov loop potential

$$\Phi = \frac{1}{N_c} \text{Tr}_c \exp(i\beta A_0), \quad M_H = 1 - 6\bar{\Phi}\Phi + 4(\Phi^3 + \bar{\Phi}^3) - 3(\bar{\Phi}\Phi)^2$$

$$\frac{\mathcal{U}_\Phi}{T^4} = -\frac{1}{2}a\bar{\Phi}\Phi + b \log M_H + \frac{1}{2}c(\Phi^3 + \bar{\Phi}^3) + d(\bar{\Phi}\Phi)^2$$

T -dependence of a, b, c, d is fitted to the pure SU(3) gauge lattice data

P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88, 074502 (2013)

Expansion around mean-field solution @ $N_f = 3$

$$\mathcal{U}_\chi = \underbrace{\mathcal{U}_\chi^{MF}}_{0^{\text{th}} \text{ order}} + \underbrace{\bar{q} \hat{\Sigma} q - \langle \bar{q} \hat{\Sigma} q \rangle}_{1^{\text{st}} \text{ order}} - \underbrace{\sum_{f, f'} (\bar{f} f - \langle \bar{f} f \rangle) G_S^{ff'} (\bar{f}' f' - \langle \bar{f}' f' \rangle) - G_{PS} \sum_f (\bar{f} i \gamma_5 f)^2}_{2^{\text{nd}} \text{ order}} + \dots$$

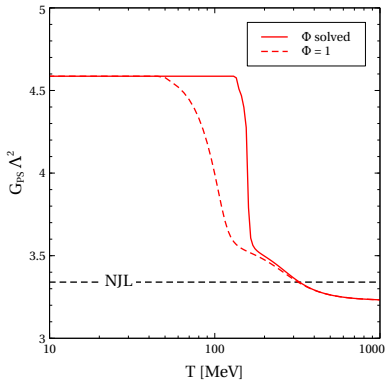
- Mean-field scalar self-energy

$$\hat{\Sigma} = \text{diag}(\Sigma_u, \Sigma_d, \Sigma_s), \quad \Sigma_f = \frac{\partial \mathcal{U}_\chi^{MF}}{\partial \langle \bar{f} f \rangle}$$

- **Effective medium dependent couplings**

$$G_S^{ff'} = -\frac{1}{2} \frac{\partial^2 \mathcal{U}_\chi^{MF}}{\partial \langle \bar{f} f \rangle \partial \langle \bar{f}' f \rangle}$$

$$G_{PS} = -\frac{1}{2} \frac{\partial^2 \mathcal{U}_\chi^{MF}}{\partial \langle \bar{f} i \gamma_5 f \rangle^2}$$

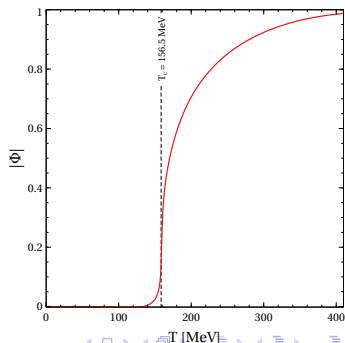
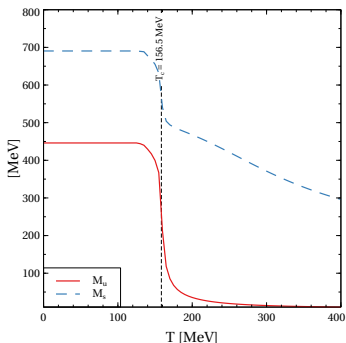


RDF setup

- Fitting vacuum phenomenology

$$\left\{ \begin{array}{l} M_\pi = 140 \text{ MeV} \\ F_\pi = 93 \text{ MeV} \\ M_K = 494 \text{ MeV} \\ F_K = 112 \text{ MeV} \\ T_c = 156.5 \text{ MeV} \end{array} \right. \Rightarrow \begin{array}{l} m_u = m_d = 4.4 \text{ MeV} \\ m_s = 134.8 \text{ MeV} \\ \Lambda = 636.1 \text{ MeV} \\ \sqrt{D_0} = 729.6 \text{ MeV} \\ \alpha = 1.44 \end{array}$$

- Effective masses and Polyakov loop

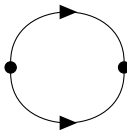


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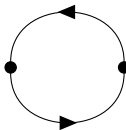
Generalized Beth-Uhlenbeck approach

- Large size clusters as correlations of the smaller size ones $\Rightarrow$ propagators

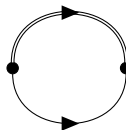
polarization
loops :



diquark



meson



baryon

- Phase shift of multi-quark clusters

$$S_n = |S_n| e^{i\delta_n} \Rightarrow \delta_n = \Im \ln S_n$$

- Generalized Beth-Uhlenbeck formula

$$\Omega_n = \frac{d_n}{\kappa_n} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} (tr_D)^{2-\kappa_n} \ln(\beta^{\kappa_n} S_n^{-1}) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}$$

$\kappa_n = 1$ - fermions, $\kappa_n = 2$ - bosons

G. Röpke, N.U. Bastian, D. Blaschke, T. Klähn, S. Typel, H. Wolter, NPA 897 (2013)

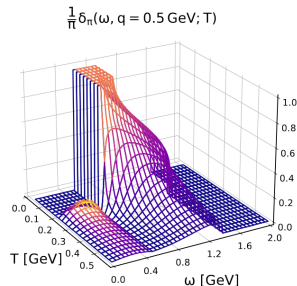
Phase shifts of multi-quark states

• Microscopic calculations for pions

K. Maslov, D. Blaschke, PRD 107, 094010 (2023)

- ① discontinuous jump at the on-shell energy below the dissociation temperature
- ② continuous growth at small energies above the dissociation temperature
- ③ continuous fall above the decay threshold
- ④ vanishing at high energies (Levonson's theorem)

M. Wellner, Am. J. Phys. 32, 787–789 (1964)

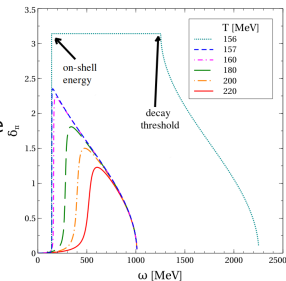


• Parametric model of $\delta = \delta(T, \omega)$

D. Blaschke, M. Cierniak, O. Ivanytskyi, G. Röpke, arxiv:2308.07950 [nucl-th]

- ① parametric expression reproduces all the properties of the microscopic calculations
- ② T-dependence of the hadron masses & widths agree with the microscopic calculations
- ③ requires hadron decay threshold given by quark masses $M_{u,d,s}$

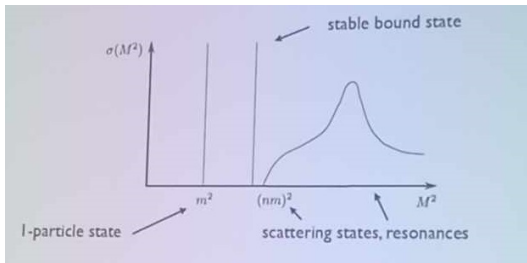
$$M_h^{\text{Th}} = N_h^u M_u + N_h^d M_d + N_h^s M_s$$



Beth-Uhlenbeck & spectral function

$$\Omega_n = \frac{d_n}{\kappa_n} \int \frac{d\mathbf{k}}{(2\pi)^3} \int d\omega (tr_D)^{2-\kappa_n} \ln(\beta^{\kappa_n} S_n^{-1}) \underbrace{\frac{\sin^2 \delta_n}{\pi} \frac{\partial \delta_n}{\partial \omega}}_{\text{spectral function}}$$

- step-up of phase shift $\Rightarrow$ bound state δ
- continuum of phase shift $\Rightarrow$ resonance tail

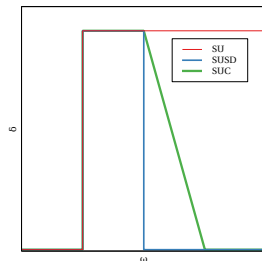


taken from Owe Philipsen's presentation @ Aspects of Criticality 2024

Beth-Uhlenbeck vs Hadron resonance gas

• Step-up (SU)

- ① is generated by the pole of $S_{n>1}$
- ② corresponds to a bound multi-quark state
- ③ is present only below the dissociation temperature
- ④ generates a HRG-like term in Ω

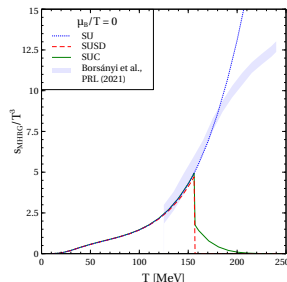


• Step-down (SUSD)

- ① rough account of the decay threshold
- ② partially/totally compensates HRG-like term in Ω

• Continuum (SUC)

- ① corresponds to a scattering multi-quark state
- ② partially compensates HRG-like term in Ω



Mass-spectrum

$$M_{n>1} = M_{n>1}^{\text{vacuum}} + A(T - T_c)\theta(T - T_c)$$

$$\Gamma_{n>1} = B\theta(T - T_c)$$

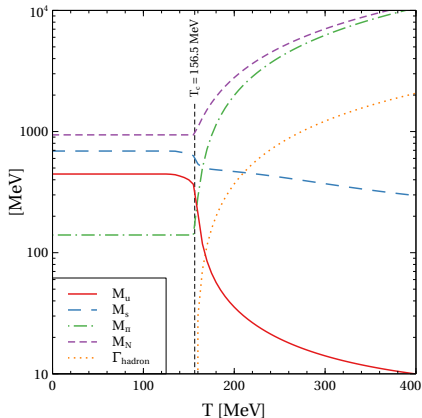
$T_c = 156.6 \text{ MeV}$, A, B - fitted to IQCD

- **Low T (χ -broken matter)**

- ① heavy quarks
- ② stable multiquark clusters
- ③ constant mass of multiquark states
- ④ zero width of multiquark states

- **High T (χ -symmetric matter)**

- ① light quarks
- ② unstable multiquark clusters
- ③ growing mass of multiquark states
- ④ growing width of multiquark states



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Thermodynamic potential

$$\Omega = \Omega_{\text{quarks}} + \mathcal{U}_\chi - \langle \bar{q} \hat{\Sigma} q \rangle + \mathcal{U}_\phi + \underbrace{\Omega_{\text{hadrons}} + \Omega_{\text{colored clusters}}}_{\text{multi-quark clusters}}$$

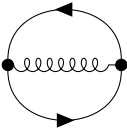
• Quarks

- Non-perturbative states at low momenta $k < \Lambda$

$$\Omega_{\text{quarks}}^{k < \Lambda} = -\frac{1}{\beta V} \text{Tr} \ln(\beta S_{\text{quarks}}^{-1})$$

S_{quarks} - quark propagator @ mean-field

- Perturbative states at high momenta $k > \Lambda$

$$\Omega_{\text{quarks}}^{k > \Lambda} = \frac{1}{2\beta V} \text{ (diagram) }$$


J.I. Kapusta, Finite Temperature Field Theory, Cambridge (1989)

- **Hadrons** - 62 meson, 60+60 baryon-antibaryon states with $M < 2.6$ GeV
- **Colored multi-quark states** - diquarks, tetraquarks, pentaquarks coupled to Φ

$$M^{\text{vacuum}} = \sum_f M_f^{\text{vacuum}} N_f - B \left(\sum_f N_f - 1 \right)$$

Entropy density

$$s = -\frac{\partial \Omega}{\partial T}$$

- **Low T**

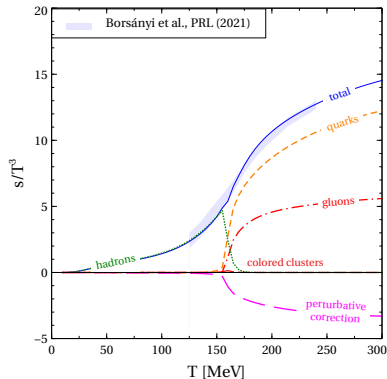
- ① hadron dominance

- **High T**

- ① quark-gluon dominance
 - ② negative perturbative contribution

- **Colored multi-quark states**
($\mu_B = 0$ only)

- ① suppressed by the Polyakov loop at high T
 - ② suppressed by high mass at high T



Chiral condensate

$$\langle \bar{f} f \rangle = -\frac{\partial \Omega}{\partial m_f} = \underbrace{2N_c \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{M_f}{E_f} (f_f^+ + f_f^- - 1)}_{\text{quarks}} + \underbrace{\sum_{n>1} \frac{d_n \sigma_n^f}{m_f} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} \frac{M_n}{\omega} (f_n^+ + f_n^-) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}}_{\text{multiquark clusters}}$$

• σ -factor

- ① σ_π^f, σ_K^f – defined from the GMOR relations
- ② other multiquark clusters

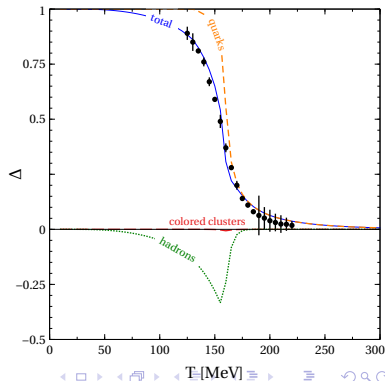
$$\sigma_n^f = m_f \frac{\partial M_n}{\partial m_f} = m_f N_n^f$$

J. Jankowski, D. Blaschke, M. Spalinski, PRD 87, 10 (2013)

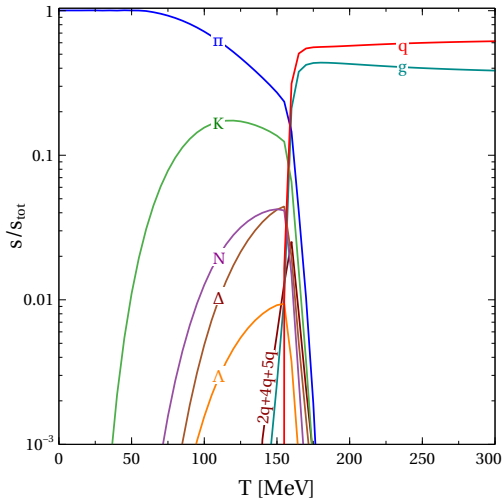
• Scaled chiral condensate

$$\Delta = \frac{m_s \langle \bar{l} l \rangle - m_l \langle \bar{s} s \rangle}{m_s \langle \bar{l} l \rangle_0 - m_l \langle \bar{s} s \rangle_0}$$

Almost constant quark term below T_c
Hadrons are necessary to reproduce
the IQCD data



Composition



Sharp switching between partonic & hadronic degrees of freedom

Freeze-out of hadrons

1. Sharp switching between hadrons and partons
2. Absence of colored mediators of interaction below T_c



Narrow range of the freeze-out temperatures for all hadrons?

Conclusions

- A unified EoS of strongly interacting matter based on a cluster decomposition approach
- Agreement with the lattice QCD data on entropy density and chiral condensate
(see EPJ A 2024 for baryon density and stay tuned for more)
- Sudden switching between partonic and hadronic degrees of freedom

Expansion around mean-field solution @ $N_f = 2$

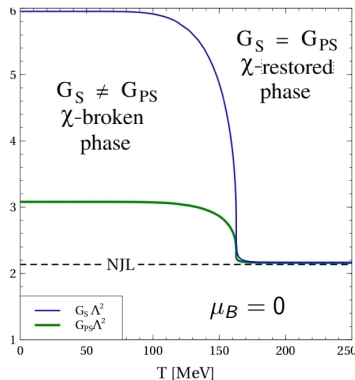
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{\text{th}} \text{ order}} + \underbrace{(\bar{q}q - \langle \bar{q}q \rangle) \Sigma_{MF}}_{1^{\text{st}} \text{ order}} - \underbrace{G_S (\bar{q}q - \langle \bar{q}q \rangle)^2 - G_{PS} (\bar{q}i\vec{\tau}\gamma_5 q)^2}_{2^{\text{nd}} \text{ order}} + \dots$$

- Mean-field scalar self-energy

$$\Sigma_S = \frac{\partial \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle}$$

- Effective medium dependent couplings

$$G_S = -\frac{1}{2} \frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle^2}, \quad G_{PS} = -\frac{1}{6} \frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}i\vec{\tau}\gamma_5 q \rangle^2}$$



Expansion around mean-field solution @ $N_f = 2$

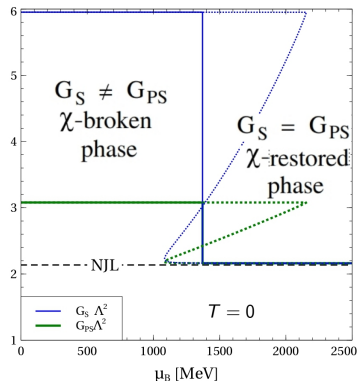
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{\text{th}} \text{ order}} + \underbrace{(\bar{q}q - \langle \bar{q}q \rangle) \Sigma_{MF}}_{1^{\text{st}} \text{ order}} - \underbrace{G_S (\bar{q}q - \langle \bar{q}q \rangle)^2 - G_{PS} (\bar{q}i\vec{\tau}\gamma_5 q)^2}_{2^{\text{nd}} \text{ order}} + \dots$$

- Mean-field scalar self-energy

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- Effective medium dependent couplings

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Comparison to NJL model @ $N_f = 2$

$$\mathcal{L} = \bar{q}(i\not{\partial} - \underbrace{(m + \Sigma_S)}_{\text{effective mass } m^*})q + G_S(\bar{q}q)^2 + G_{PS}(\bar{q}i\vec{\tau}\gamma_5 q)^2 + \dots + \mathcal{L}_V + \mathcal{L}_D$$

Similarities:

- current-current interaction
- (pseudo)scalar, vector, diquark, ... channels

Differences:

- high m^* at low T , $\mu \Rightarrow$ “confinement”

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 \Rightarrow m^* = m - \frac{2G_0}{3\alpha^{2/3}\langle \bar{q}q \rangle_0^{1/3}}$$

$\Downarrow$

$$m^* \rightarrow \infty \text{ at } \alpha \rightarrow 0$$

- medium dependent couplings:

$$\text{low } T, \mu, \Rightarrow G_S \neq G_{PS} \Rightarrow \chi\text{-broken}$$

$$\text{high } T, \mu, \Rightarrow G_S = G_{PS} \Rightarrow \chi\text{-symmetric}$$

$T = 0$

